3D TOPOLOGICAL OPTIMIZATION FOR MULTIPLE LOAD CASES

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Abstract. Topological optimization seeks to determine the optimal distribution of material within a design domain (2D/3D) to optimize structural performance following imposed design constraints. This material distribution can vary continuously, resulting in complex shapes that are lighter than the original structure and take better advantage of the strengths of the materials. The incorporation of multiple loads in topological optimization increases the complexity of the problem, as it requires simultaneous considerations of different types of loads and their interactions with the structure in problems that have different degrees of freedom. This can include analyzing load combinations due to self-weight, other load combinations, or even dynamic and thermal loads. This work developed and implemented a structural 3D topological optimization model that takes into account these multiple loads in MATLAB language. Practical examples of optimization cases considering multiple loads are presented, as well as comparisons with results already existing in the literature to demonstrate the accuracy of the proposed algorithm. The results demonstrate a different final topology when considering independent or concurrent loads.

Keywords: multiple load cases, topology optimization, three-dimensional finite elements, BESO.

1. Introduction

Topological optimization aims to identify the most efficient structural configurations within a defined domain by systematically adding or removing material. This ensures the structure can withstand predefined loads and specific boundary conditions, focusing on minimizing material usage while balancing weight and stiffness. Although topological optimization is relatively recent, numerous studies have addressed increasingly complex practical problems.

The Bi-directional Evolutionary Structural Optimization (BESO) algorithm, developed by Huang and Xie [1], evaluates material parts by adding and removing them from the domain to replace where they are most efficient for structure formation. Unlike other methods, BESO finds an optimal configuration without considering the initial setup and is independent of the finite element mesh. However, traditional BESO is typically limited to ideal applications, as real structures that are subjected to one type of load, which limits its effectiveness in real-world scenarios where multiple loads are present.

Without topological optimization, a project might have excess material in some regions or a lack thereof in others, leading to inefficient use of material strength. Increased component weight needs a more robust part, resulting in higher energy consumption due to high inertia and difficulty in movement. Topological optimization can enhance machine performance and produce lighter parts that better use material strength. Optimized structures can withstand similar or greater loads than non-optimized structures, possibly with fewer parts. However, optimized material distribution can result in complex geometries that make production complicated. Real-world applications require simultaneous analysis of various load types and combinations, including dynamic and thermal loads, introducing several degrees of freedom. Consequently, considering multiple loads in topological optimization becomes crucial for realistic projects.

This work aims to develop and implement a 3D structural topology optimization model accounting for multiple loads. Finite element analysis is implemented for three-dimensional elements with eight nodes using MATLAB (2014) and the BESO methodology. Practical optimization examples considering multiple loads are presented, with comparisons to existing literature results, as well as a final larger example, all to demonstrate the proposed algorithm's accuracy.

2. Literature Review

Topology optimization enables the design of structures that meet performance requirements by considering various loads, ensuring strength without wasting material or adding excess weight. Considering multiple loadings results in more realistic design solutions and provides a comprehensive view of structural behavior under various operational scenarios.

One of the first significant works addressing multiple loads in optimization was by Young et al. [2]. They implemented a BESO formulation with a limit stress criterion for 3D problems, considering multiple loads through the weighted average of Compliance for each combination. They checked stress constraints (von Mises) for each combination. However, the work lacked important details, such as the material stress limit used.

Bendsøe and Sigmund [3] presented the Solid Isotropic Material with Penalization (SIMP) method, which eliminates elements with low influence on stiffness by applying penalties to their densities and elastic moduli, making them virtually empty in an iterative process. The method uses Finite Element Analysis (FEA), creating a mesh that divides the domain into finite elements of 8 nodes. Extended to handle multiple loads, it minimizes the weighted average of elastic strain energy (Compliance) for each load. However, SIMP produced intermediate density areas, or "gray areas," and was prone to overestimate Compliance in these elements.

Zhou and Li [4] optimized 2D structures under multiple loads using an orthotropic fiber-reinforced composite model. Instead of averaging load compliance, they aligned fiber orientations with principal stress orientations and arranged fiber densities according to the strains along these orientations for the load combinations. The iterative process continued until no more shear stresses were present in the main elements, forming axially stressed bar structures. The optimization bound the elastic matrix to approximate optimal structures for each load case.

Lu and Tong [5] tackled the multiple loads problem by averaging the Compliance of each load, weighted by predefined values, using SIMP for topology optimization. Their work also optimized fiber orientations in composite materials and allowed for multiple materials using the MIST (Moving Iso-Surface Threshold) strategy.

Stragiotti et al. [6] aimed the minimization of the volume of 3D truss structures in aeronautical structures under multiple load cases, with constraints in maximum stress, and buckling load factors. Their optimization involved a two-step process: sequential linear programming followed by heuristic methods to refine the initial solution. Then, they applied IPOPT (Interior Point Optimizer) to a complete nonlinear optimization problem, ensuring all constraints remained valid. This work is closest to the 3D topology optimization of systems but leans more towards parametric shape optimization than pure topology optimization.

In any of the papers surveyed, the computational cost of the FE mesh is an issue, since even for regular structured meshes, simple structures demands intensive computation for assembling and solving large system of equations. In the present work, a modified incomplete Cholesky preconditioner with conjugate gradient method, present in Matlab is used to alleviate the computation burden. There are BESO versions in the literature that circumvents these issues by using unstructured FE meshes, not implemented here.

3. Theoretical Foundation

1.1. Topology Optimization

Topology optimization aims to define the ideal distribution of material, creating an optimal layout, within a design domain for a defined objective like stiffness and an amount of available material. Therefore, it optimizes structural performance following imposed design constraints, as stated by Jaouadi and Lahmer [7]. This material distribution can vary continuously, resulting in complex shapes, lighter than the original structure, and which take better advantage of the materials' strength, reducing the amount of deformation energy stored for this structure within the available material.

In this work, a 8-node isoparametric hexahedral three-dimensional FE with 2×2×2 Gaussian integration is used. Here, previously existing topological optimization formulations in the literature are not used due to the fact it is not possible to modify integration rules or use non structured meshes. This eight-node element offers a balance between accuracy and computational efficiency with flexibility for future improvements in the code.

1.2. BESO method considering multiple load cases

The Bi-directional Evolutionary Structural Optimization (BESO) method, presented by Huang and Xie [1], defines methods and material criteria for a structure be topologically changed, being the addition or removal of material happening simultaneously until the conditions for convergence are met. Multiple load cases are situations where structures suffer in reality. There are several possibilities for actions on the structure, such as loads acting simultaneously or independently, applied forces that does not act in a single place or direction, moving from one region to another, and even supposing that this force was punctual, there would be a chance of it occurring along different time instants, among other countless occurrences. Therefore, topological optimization seeks the best way that such a structure can be designed to support all multiple loading situations.

Huang and Xie [1] proposed that the objective function and sensitivity number equations should be minimally changed, as the collaboration of each element must be taken into account for all applied loads. In this way, a weight factor is proposed in each loading case, thus modifying them as shown in eq. (1) to (4). The idea is to use a weighted sum of compliances in each of the load cases, according to its relevance.

$$Minimize \quad f(x) = \sum_{k=1}^{M} w_k C_k \tag{1}$$

$$Subject\ to \quad V^* - \sum_{i=1}^{N_{elem}} V_i x_i = 0 \tag{2}$$

$$x_i = 1 \quad or \quad x_{min} \tag{3}$$

$$\alpha_i^e = \frac{1}{2} x_i^{p-1} \sum_{k=1}^{M} w_k (u_i^T K_i u_i)_k \ , \tag{4}$$

where M is the total number of load cases considered, w_k is the weight factor assigned to each load case and C_k and the Compliance average calculated for each load case. The value imposed for the weight factor ranges from zero to one and its sum must be equal to 1. It is observed that the higher w_k, the more relevant the load combination is concerning other loads, i.e., its choice depends on importance and prevalence (maybe in time) the load combination has to the structure to be optimized.

The discretization of continuous structures using lower-order bilinear or trilinear finite elements can result in discontinuous sensitivity numbers, causing issues such as the checkerboard pattern (Jog and Harber [8]). This pattern, characterized by interspersed filled and empty elements, hinders optimal structure design. Another issue is the mesh dependence, where different finite element mesh refinements create varying topologies. A finer mesh can produce many smaller members than intended in the final design. According to Bendsøe and Sigmund [3], ideally, mesh refinement should improve finite element modeling and boundary descriptions without adding unnecessary detail or distinct qualitative differences.

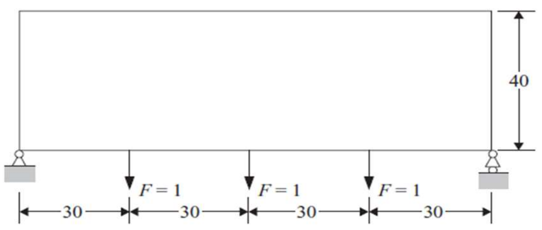
To address these issues, a filter is applied that considers the sensitivity of adjacent elements. First, the nodal sensitivity number is calculated as a weighted average of the elementary sensitivity numbers, with weights based on proximity to the node. This filter creates a minimum radius length scale within the mesh, identifying nodes that significantly influence element sensitivity. The filter then smooths the nodal sensitivity numbers into elementary sensitivity numbers across the domain. Elemental sensitivity is defined as a weighted average of nearby elements' sensitivities, with closer elements having greater influence, ensuring a more coherent and optimal structural design.

4. Numerical Examples

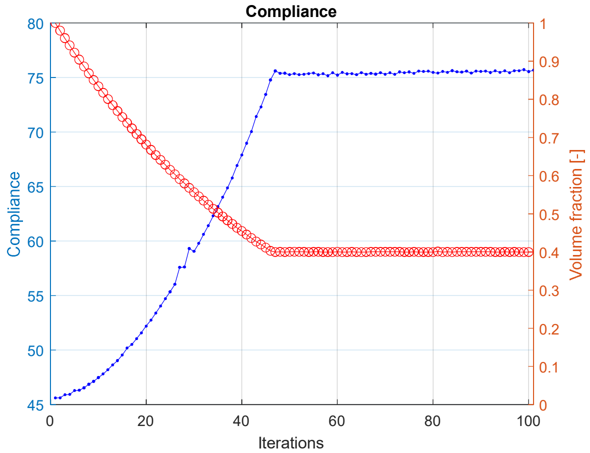
The problems presented here are all static, and use the BESO method. They are modelled in three dimensions, and optimize multiple loads. The first example was taken from Huang and Xie [1] and the second was inspired by the paper by Young et al [2]. Two cases will be considered in each problem to gauge the importance of considering or not multiple load cases have in the final topology, where in the first case (a) there will be the application of simultaneous loads and in the second case (b) there will be the application of independent loads.

1.3. Problem 1 – Beam simply supported with 3 load cases

Original Problem 1 is a simply-supported beam that supports three loads of 1 N located at the bottom. Figure 1(a) represents Problem 1. It has a rectangular domain, in millimeters, of 120×40 four-noded flat isoparametric elements, presented by Huang and Xie [1]. It was used the 3D eight-noded isoparametric hexahedral FE with the same discretization. The parameters used in this case are $E = 1$ Pa, v=0.3, $V_{frac} = 0.4$, $ER = 0.02$, $p = 3$, $r_{min} = 3$, $\rho_{min} = 0.001$ kg/m³, $h = 0.001$ m (element edge), and $w_k = 1/3$. The objective function is the Compliance, in [Nm].



(a)



(b)

Figure 1. (a) Domain, boundary conditions, and loadings of Problem 1. (b) Compliance values along iterations for case (a) in the present work (3D).

Figures 2(i) and 2(ii) show, respectively, the final topology obtained by Huang and Xie [1] (in black) and the final topology obtained here (in cyan). The convergence graph obtained in this work is presented in Figure 1(b) where the progress of the objective function is indicated with blue dots and the volumetric fraction is presented with red circles throughout the iterations. Figures 2(iii) and 2(iv) depict, respectively, the final topology by Huang and Xie [1] (in black) and the final topology achieved with the algorithm (in cyan).

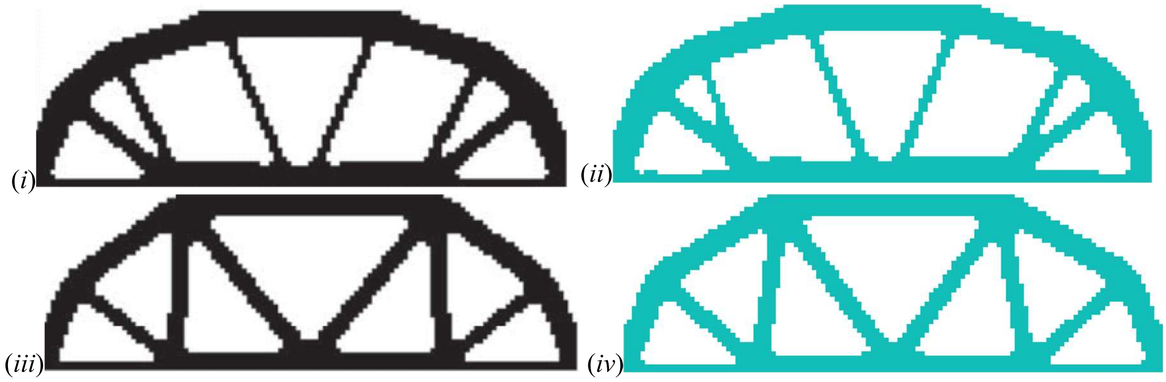


Figure 2. Final topologies in Problem 1, for case (a): (i) Haung and Xie [1]-2D and (ii) present work (3D), and for case (b): (iii) Haung and Xie [1]-2D, and (iv) present work (3D).

Figures 2(i) and 2(ii) are very similar to each other, except for the second void that appears on both the left and right of the topologies that is larger in Figure 2(i) compared to Figure 2(ii). Now in Case (b), a great similarity between Figures 2(iii) and 2(iv) is found, with no significant differences.

1.4. Problem 2 – Embedded-free beam with 2 load cases and tension constraint

Problem 2, sketched in Figure 3 is a fixed-free beam in which two loads of 15 kN are applied. This problem was inspired by an example from Young et al [2]. It does not exactly reproduce that example because of lack of information in the original paper. It has dimensions 160×40×100 mm. Eight-node isoparametric hexahedral 3D FE are used with a mesh size of 48×30×12 (17,280 elements). The material is steel (high carbon) and the yield stress limit is $\sigma_{lim} = 250$ MPa. The parameters used in the optimization are: $E = 2.1 \times 10^{11}$ Pa, *ν=0.3*, $ER = 0.005$, $p = 3$, $r_{min} = 3$, $\rho_{min} = 0.001$ kg/m³, $h = 0.001$ m(element edge size) and $w_k = 1/2$. The objective function remains the Compliance. The volumetric fraction is free here, being a by-product when the stress limit constraint is met for all elements. For stresses below design resistance, the algorithm removes material from the working volume following the Compliance sensitivity criterion. For stress values above design resistance, the algorithm starts to add material, following the same sensitivity criteria. Thus, iterations continue until the stress constraint is met, with the addition or subtraction of elements $|\sigma_{vM} - \sigma_{lim}| \leq tol$.

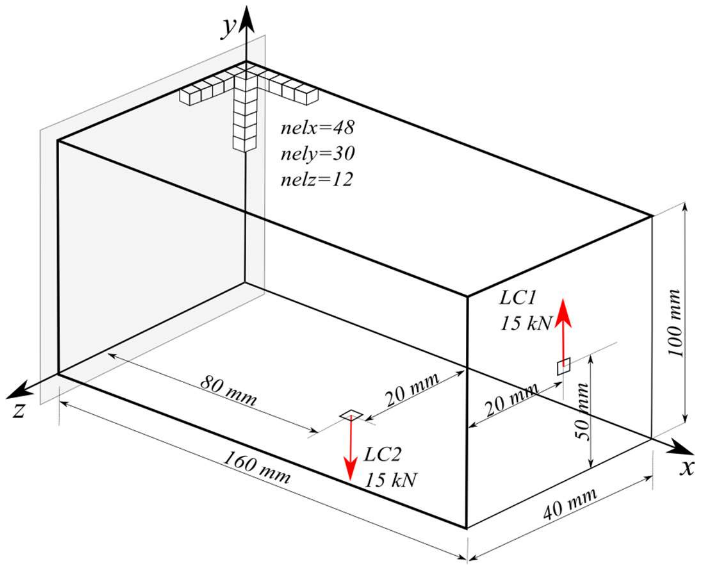


Figure 3. Domain, boundary conditions of Problem 3. (Adapted from Young et al, 1999).

To verify the safety level of the applied loads, a steel resistance reduction factor ($\gamma_m = 1.10$) was used as well as a load safety factor ($\gamma_f = 1.5$). The characteristic load in fact is p_k= 10 kN and therefore the design load was defined as $p_d = p_k \, . \gamma_f = 15$ kN. For the yield stress, a characteristic or nominal resistance is assumed as $f_{yk} = 275$ MPa, therefore, the design resistance must be $f_{yd} = f_{yk} / \gamma_m$=250 MPa. Unlike Problem 1, here the stress constraint will be checked at all integration points of the FE mesh using the von Mises σ_{vM} stress criterion.

The graph in Figure 4(a) indicates the final topology. Figure 4(b) shows that the final volume fraction resulted in 0.075 after 745 iterations. The final Compliance obtained was 1.547 Nm.

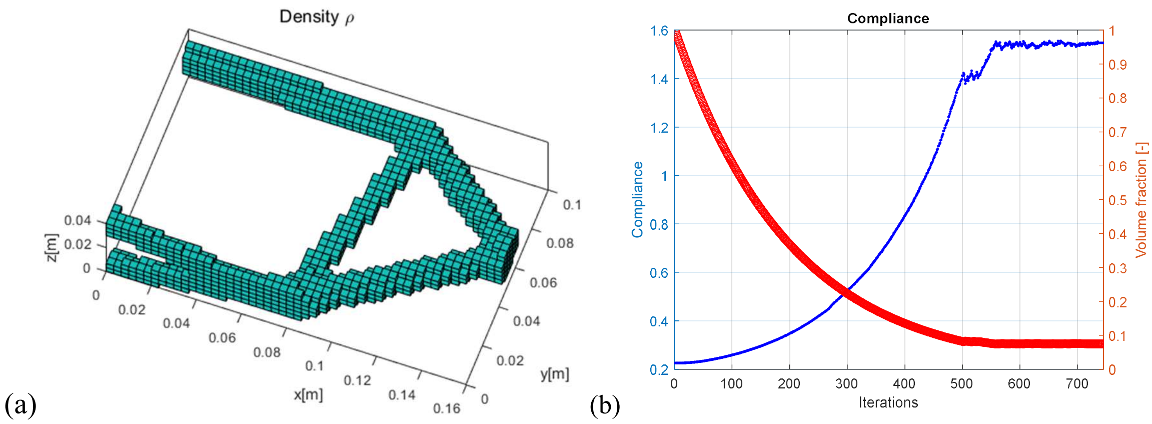


Figure 4. Graph of (a) final topology and (b) volumetric fraction and Compliance × Iteration achieved in Case (a) of Problem 2.

Figure 5(a) shows the final displacements and von Mises stresses. Figure 5(b) shows that the maximum von Mises stress (in all elements) did not exceed the established strength limit f_{yd}. Figure 6(a) indicates the final topology achieved. Figure 6(b) shows the resulted final volumetric fraction of 0.084 after 998 iterations. The respective final Compliance was 2,154 Nm.

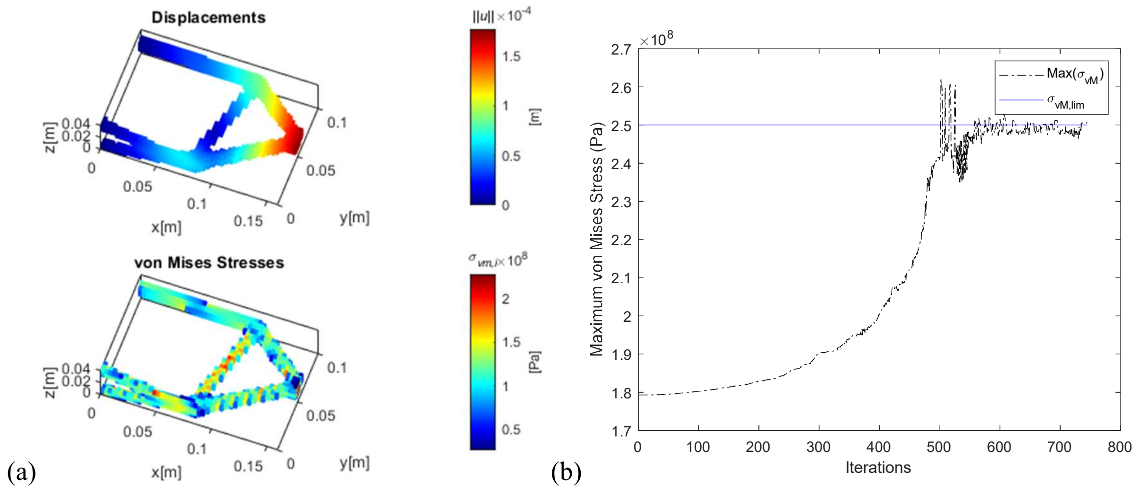


Figure 5. (a) von Mises displacements and stresses. (b) Limit stress and maximum von Mises stress in the structure in Case (a) of Problem 2.

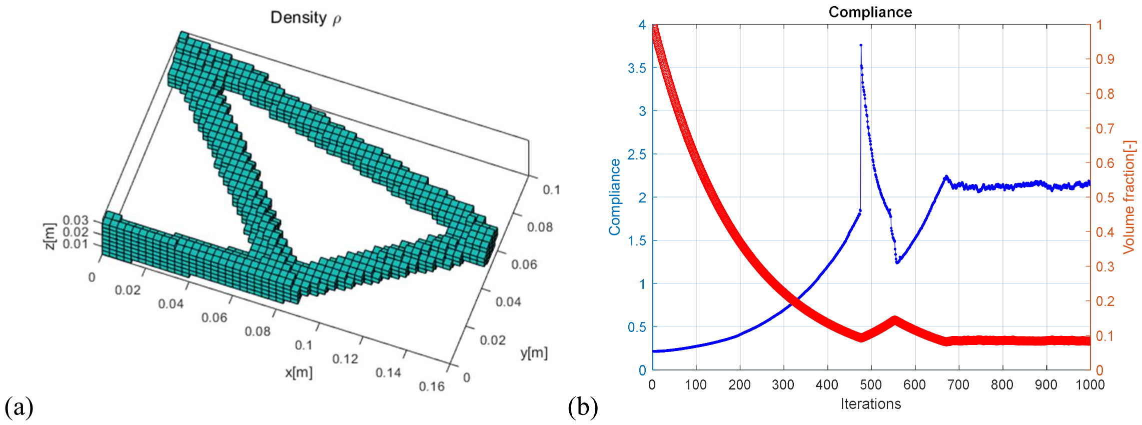


Figure 6. Graph of (a) final topology and (b) volumetric fraction and Compliance × Iteration achieved in Case (b) of Problem 2.

Figure 7(a) shows the final displacements and von Mises stresses. Figure 7(b) shows that the maximum von Mises stress in the final structure did not exceed the established limit, remaining within the specified tolerance. The peak of the stress in the graph that appears in Figure 7(b) near the 500th iteration refers to the loss of elements in the topology similar to the truss, which in the end, was reduced to 4 main parts (main bars) presented in the topology in Figure 7(a).

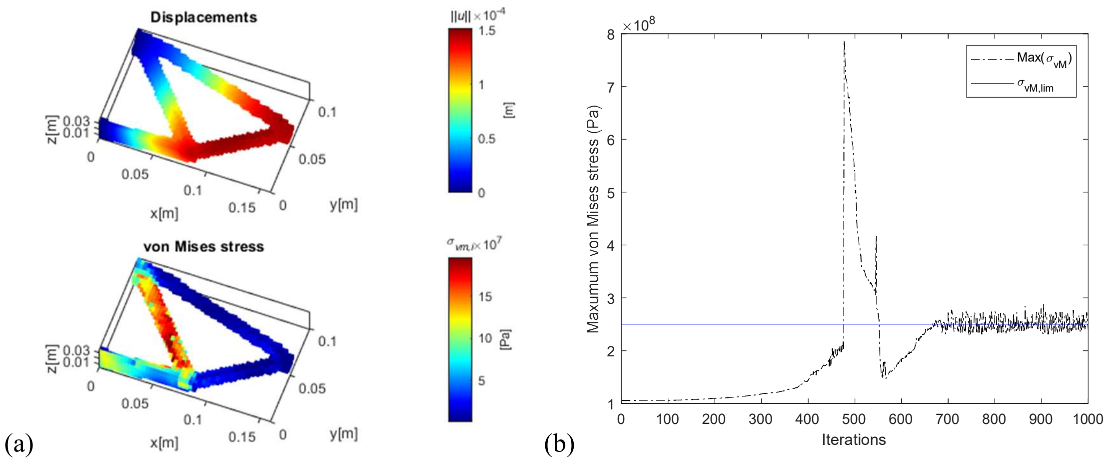


Figure 7. (a) von Mises displacements and stresses. (b) Limit stress and maximum von Mises stress in the structure in Case (b) of Problem 2.

5. Conclusions

The work aimed to implement a 3D topology optimization model in structures subject to multiple loads. The MATLAB software (2014) was used combined with the BESO methodology presented in this work, thus creating practical examples based on the literature (Huang and Xie [1] and Young et al [2]). The proposed algorithm proved to be efficient, obtaining results similar to those found in previous works.

The topologies resulting from Problem 1 were very close to those found in Huang and Xie [1] both in topology and in values of the objective function. It was perceived as a small change in voids of the lateral region, but nothing that would harm the comparison with the result obtained in this work.

Problem 2 focused on presenting a 3D with multiple loads example. Due to the lack of articles with problems in three dimensions, the work of Young et al [2] was used as a basis, which, despite being older, contained examples similar to the intended in this paper. Unfortunately, there were no sufficient information for comparisons with the methodology here presented, due to the lack of information about the materials used and their strength limit, these being the main factors for the non-comparison of results, unlike what was presented in Problem 1.

Notably, in all of the examples of the cases analyzed, (a) simultaneous loads and (b) independent loads, they allowed to demonstrate that the consideration or not of load combinations result in significant differences in the final topologies. This indicates that, in the presence of multiple loads, the inclusion of this approach is necessary to obtain a structure with the lowest Compliance and that still meets the stress limits for all multiple loads. Future works foresee the inclusion of new optimization techniques, other types of large systems of equations solutions to make the analysis faster, the inclusion of self-weight, and changing the objective function to minimize the maximum stress, instead of Compliance only.

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6. References

[1] X. Huang and Y. M. Xie, "Evolutionary topology optimization of continuum structures: Methods and Applications". Wiley, 223 p., 2010.

[2] V. Young, O. M. Querin, G. P. Steven and Y. M. Xie, "3D and multiple load case bi-directional evolutionary structural optimization (BESO)". Structural Optimization 18, p.183-192, 1999.

[3] M. P. Bendsøe and O. Sigmund, "Topology Optimization". Springer-Verlag, Berlin, Heidelberg GmbH, 2nd. Edition, 370p. 2002.

[4] K. Zhou and X. Li, "Topology optimization of structures under multiple load cases using a fiber-reinforced composite material model". Computer. Mech. V.38, p.163-170. doi: 10.1007/s00466-005-0735-9.

[5] Y. Lu and L. Tong, "Topology and orientation optimization of multi-material hinge-free composite compliant mechanisms under multiple design-dependent loadings". Structural and Multidisciplinary Optimization, 67:17, 2024. doi: 10.1007/s00158-023-03723-5.

[6] E. Stragiotti, F. X. Irisarri, C. Julien and J. Morlier, "Efficient 3D truss topology optimization for aeronautical structures". Structural and Multidisciplinary Optimization 67:42, 2024. doi: 10.1007/s00158-024-03739-5.

[7] Z. Jaouadi and T. Lahmer, "Topology optimization of structures subjected to multiple load cases by introducing the Epsilon constraint method". Internal Report. T. Lahmer, K. Gürlebeck Editors. Institut für Strukturmechanik, 2015.

[8] C. Jog, R. B. Harber and M. P. Bendsøe, "Variable-topology shape optimization with a constraint on the Perimeter". Proceedings of 20th ASME Design Automation Conf., September 11–14, 1994, Minneapolis, ASME publication DE-Vol. 69-2, pp.261–72. 1994.