Geometrically Nonlinear 3D Topological Optimization: An Efficient MATLAB Code for the SESO Method

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Abstract. The study of Topological Optimization (TO) in three-dimensional structures with geometrically nonlinear formulation is scarce. This work aims to apply TO in elasticity problems extended to consider geometric nonlinearity, using the total Lagrangian formulation. To achieve this goal, we developed a numerical model in MATLAB, employing the finite element method with hexahedral elements. The TO method Smooth Evolutionary Structural Optimization (SESO) is used in conjunction with the Method of Moving Asymptotes to accelerate the optimization procedure, especially in the calculation of sensitivity factors. SESO is based on a bidirectional heuristic, systematically removing and adding elements with lower compliance compared to the maximum compliance of the structure. The results show that Smooth Evolutionary Structural Optimization Geometrically Non-Linear (SESO-GNL) is robust and efficient in solving classic problems from the literature.

Keywords: Topological Optimization; SESO, Geometric Nonlinearity; Moving Asymptote Method

1 Introduction

Topological optimization (TO) is a fundamental approach for the efficient distribution of material within a design domain, subject to volume, compliance, and von Mises stress constraints. This paper presents an extension of the Smooth Evolutionary Structural Optimization (SESO) method for geometrically nonlinear TO and compares the results with nonlinear TO for 3D structures using the SIMP method. It is particularly crucial to recognize the importance of geometric nonlinearity in TO design since some kind of structures in engineering are more efficient under the assumption of large deformations.

In geometrically nonlinear TO, excessive distortion in low-density elements can lead to convergence issues in Newton-Raphson iterations, affecting the final optimized result, Kemmler et al. (2005). Various methods have been proposed to address this distortion, including convergence criterion relaxation, Buhl et al. (2000), removal and reintegration of low-density elements, Bruns and Tortorelli (2003), the extremely soft hyperelastic material addition method, Luo et al. (2015), element connectivity parameterization, Moon and Yoon (2013), element deformation scaling, van Dijk et al. (2014), polyconvex constitutive models, Lahuerta et al. (2013), interpolation schemes over elastic energy density, Wang et al. (2014), and super element condensation methods, Hou et al. (2020).

For two-dimensional structures, the main references are Chen et al. (2018), Zhu et al. (2021) and Han et al. (2021). The proposed method can generate slightly non-symmetrical optimized results. In this context, some preliminary notions about symmetry in TO were outlined by Rozvany (2010), who also illustrated that optimal topologies were non-symmetrical in some cases involving buckling effects. The results presented in this paper are slightly non-symmetrical, considering geometric asymmetry, and can be seen as a further development of the asymmetry discussed in Buhl et al. (2000) and Rozvany (2010).

The SIMP method is the most widely used in TO for geometrically nonlinear problems, even though BESO exhibits a faster convergence rate. For instance, Werner et al. (2023) apply the BESO method to geometrically nonlinear TO in three-dimensional structures, considering random interactions of large forces typical in silicon. Zhao et al. (2023) present complete MATLAB codes for geometrically nonlinear three-dimensional TO for educational purposes. Two sets of MATLAB codes can be downloaded from the appendices: a 230-line code using the SIMP method and a 280-line code using moving morphable bars (MMB).

This research is relevant for engineers seeking effective methods for TO under large deformation conditions, offering insights into the importance of geometric nonlinearity and presenting an extension of the SESO method for Geometrically Nonlinear Topological Optimization (GNLTO). The results show that the proposed approach can lead to efficient optimized topologies, even when symmetry is not strictly maintained.

The remainder of the paper is organized as follows: Section 2 presents the 3D Geometric Nonlinearity: Finite Element Analysis. Section 3 introduces the formulation of Topology Optimization: Geometric Nonlinear Analysis. Section 4 discusses a numerical example, and Section 5 provides the conclusions.

2 3D Geometric Nonlinearity: Finite Element Analysis

2.1 Displacement-Strain Conversion Base Matrix: Hexahedral Elements

Geometric nonlinearity analysis is used to optimize structures that undergo large deformations. Hexahedral elements are used to discretize the 3D structural domain. Figure 1a shows the deformed hexahedral elements that need to be transformed into undeformed hexahedral elements as shown in Figure 1b.

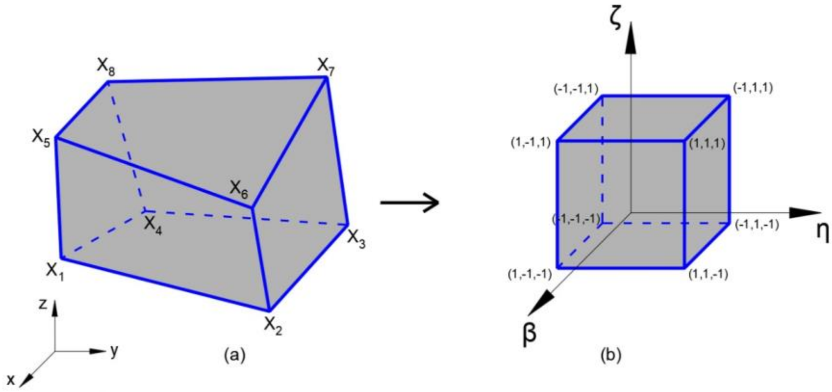


Figure 1. Hexahedral Element - (a) Deformed and (b) Undeformed

For this purpose, the Lagrangian coordinate functions are used to construct the structural deformation, and the interpolations for displacements (u) and coordinates are expressed by the isoparametric shape function N_i, given by:

$$u = \sum_{i=0}^{8} N_i u_i \tag{1}$$

$$x = \sum_{i=0}^{8} N_i x_i \tag{2}$$

where u_i is the displacement and e x_i are the nodal coordinates. The strain gradient is defined as $F = 1 + \nabla_0 u$, and the Lagrangian strain E is expressed as:

$$E = \frac{1}{2}(F^T F - 1) \tag{3}$$

Thus, considering the second-order tensors E_{kl} and S_{ij} , which represent the Lagrangian deformation E and he Piola–Kirchhof stress tensor *S*, respectively, and the fourth-order tensor D_{ijkl}, which represents the elastic constitutive matrix *D*:

$$S_{ij} = D_{ijkl} E_{kl} \tag{4}$$

where *D* is the constitutive tensor. The Young's modulus *E* is a function in relation to the density of the element ρ_e expressed by:

$$D = \left[\rho_{min} + \rho_e^p (1 - \rho_{min})\right] D_0 \tag{5}$$

where ρ_{min} is normally set to 10^{-4} to avoid numerical singularity; $p = 3$ is the penalty factor for the SIMP model. E_0 is the Young's modulus for a given isotropic material. In the SESO method, the design variable is the element. Therefore, eq. (5) was replaced by:

$$E = [\gamma + x_e(1 - \gamma)] * E_0 \tag{6}$$

where γ is given by:

$$\gamma = \frac{\sum_{i=1}^{n}\left(\frac{1}{1+e^{-\beta * x_i}}\right)}{n} \tag{7}$$

where γ performs the same function as ρ_{min} in SIMP . In this article is used $\beta = 2$.

2.2 Residual of Equilibrium Equation

When considering geometrically nonlinear finite element analysis in topology optimization, the equilibrium state of the structure is given by:

$$\iiint_{\Omega_0} \bar{u}^T f^b d\Omega + \iiint_{\Gamma_s} \bar{u}^T t d\Gamma = \iiint_{\Omega_0} S : \bar{E} d\Omega \tag{8}$$

where $\bar{u}$ is the virtual displacement, f^b and t are respectively the body and surface forces and $\bar{E}$ is the form the variational form of the strain. Thus, by mathematically manipulating Eq. (8), the equilibrium for the linear system can be expressed as:

$$R(u) = f_{int} - f_{ext} = 0 \tag{9}$$

where $R(u)$ is defined as the residual force vector, f_{int} is the internal force vector and f_{ext} represents the external force vector

3 Topology Optimization: Geometric Nonlinear Analysis

3.1 Formulation

The mathematical formulation for GNLTO with the objective of minimizing compliance subject to volume restrictions via SESO is given by:

$$Min\ C = (f_{ext})^T u$$
$$s.t. \begin{cases} R = f_{int} - f_{ext} = 0 \\ V(\boldsymbol{x}) = \sum_{i=1}^{n} x_i V_i - V^* \leq 0 \\ x_i = 1\ ou\ x_i = 1e - 9 \end{cases} \tag{10}$$

where C is the compliance (objective function), V_i is the volume of the element, V^* represents the volume fraction. $\boldsymbol{x}$ are the design variables, $x_i = 1$ e $x_i = 10^{-9}$ are, respectively, the values that represent the solid and empty material for the design variables, n is the number of elements, $\boldsymbol{u}$ displacement vector.

3.2 Sensitivity Number: Considering Geometric Nonlinearity

Assuming that the design variables do not influence the external load, the sensitivity of the objective function in relation to the design variables is given by:

$$\frac{\partial C}{\partial x_e} = (f_{ext})^T \frac{\partial u}{\partial x_e} \tag{11}$$

To determine the sensitivity $\frac{\partial u}{\partial x_e}$ the adjoint method is used. In simple terms, this method works by calculating an adjoint variable, which is then used to find the gradient of the objective function with respect to the design variables. The main advantage of this method is computing the gradient using the adjoint variable is often much more computationally efficient than directly calculating the derivatives.

By introducing a vector of Lagrange multipliers λ and assuming that equilibrium has been reached, the term $\lambda R = 0$ can be added to the objective function $C = (f_{ext})^T u$ without altering it. Thus, the modified sensitivity is given by:

$$\frac{\partial C}{\partial x_e} = (f_{ext})^T \frac{\partial u}{\partial x_e} + \lambda \left(\frac{\partial R}{\partial u_e} \frac{\partial u}{\partial x_e} + \frac{\partial R}{\partial x_e} \right) \tag{12}$$

Mathematically manipulating eq. (12) and using linearization in eq. (8) according to the Newton-Raphson (NR) method and neglecting the term $\frac{\partial u}{\partial x_e}$ we arrive at:

$$(f_{ext})^T = -\lambda K_T \tag{13}$$

where $K_T = \frac{\partial R}{\partial u}$ e K_T is the global stiffness matrix.

4 Examples

In engineering, most structures that need to be optimized for stiffness do not undergo large displacements. This is because structures are generally designed to withstand loads within specific limits, and large displacements can indicate non-stability or unsafety issues. However, in cases where large displacements are expected, such as in structures subjected to plastic deformations or in impact analyses, it is important to consider these effects in structural optimization. In such cases, nonlinear analysis methods may be necessary to obtain accurate results. Here, we use a material with a Young's modulus E=3 GPa and a Poisson's ratio assumed to be ν=0.4. The following structural engineering examples focus on topology optimization based on minimizing compliance. The geometry and boundary conditions for numerical applications are represented in each case. Additionally, all numerical examples were processed on a Core i7-2370 notebook, 8th generation, with a 2.8 GHz CPU and 20.0 GB RAM..

4.1 Example 1 – Double-clamped beam

The design domain and boundary conditions of the double-clamped beam are illustrated in Fig. 2. This example is used to validate the MATLAB SESO-GNL code for topology optimization of three-dimensional structures. The domain was discretized into a mesh with dimensions $L = 60,\ H = 20\ and\ W = 2$, totaling 2,400 hexahedral finite elements, according to Liu et al. (2014). The volume fractions for comparison are $V = 0.25\ and\ V = 0.365$, and external force of magnitude $F = 1e6N$ was applied to the double-clamped beam. The results of both linear and geometrically nonlinear topology optimization using the SESO method with the Method of Moving Asymptotes (MMA) optimizer of Svanberg (1987) are shown in Fig. 3a and 3b, and Fig. 3c and 3d, respectively and compared with the SIMP proposed by Zhao et al. (2023).

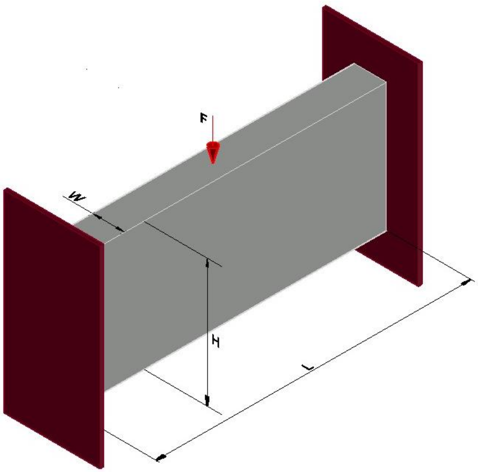
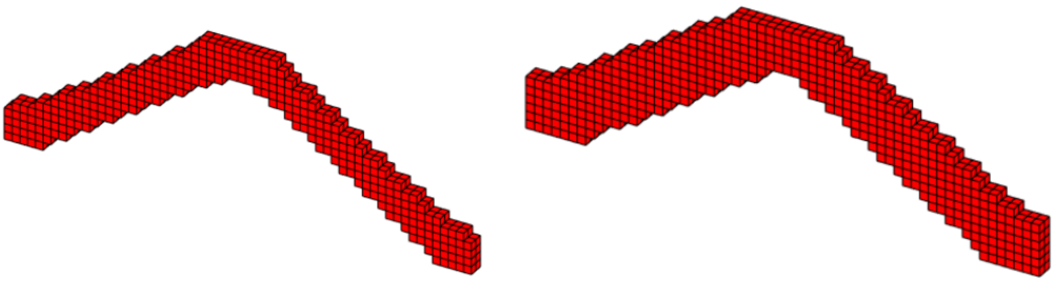
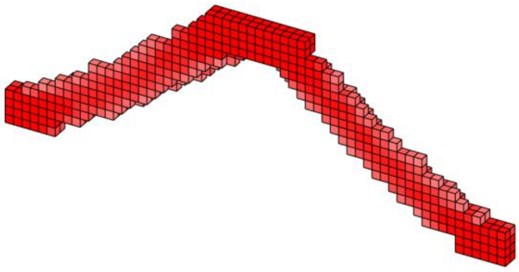


Figure 2. Design domain and boundary conditions of slender beam.

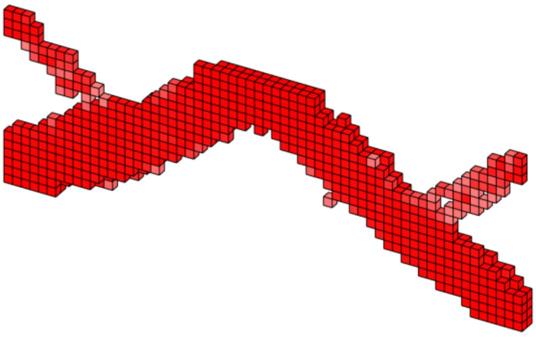
In Fig. 3e and 3f the optimal topologies using the SIMP method are shown. This example demonstrates the effectiveness of the SESO-GNL method for nonlinear geometric analysis. It is worth mentioning that the parameters used in MMA for the three analyzes are: asyinit = 0.5, asyincr = 1.2 and asydecr = 0.7.



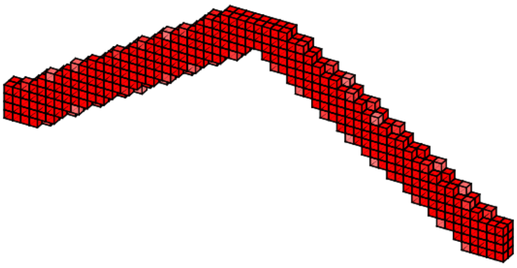
(a) $V = 0.250$ (b) $V = 0.365$



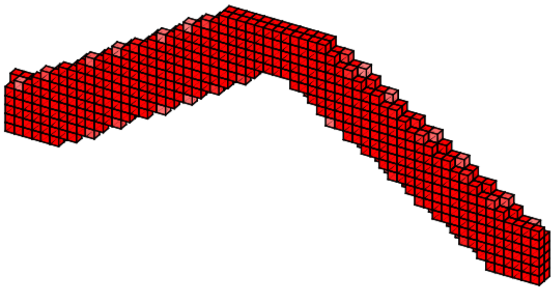
(c) $V = 0.250$



(d) $V = 0.365$



(e) $V = 0.250$



(f) $V = 0.365$

Figure 3. Optimal topologies: (a and b) linear SESO, (c and d) SESO-GNL and (e and f) SIMP-GNL

5 Conclusions

This paper proposes the extension of the SESO method for topology optimization procedures subjected to large geometrically nonlinear deformations, using structural stiffness as the criterion. The SESO-GNL code implemented was proposed in MATLAB and the procedure includes a filtering technique applied to achieve mesh independence and stable algorithm convergence, proving to be efficient and effective for geometrically nonlinear analysis. Additionally, a sigmoid function was used to determine the optimization parameter aimed at preventing singularity in the stiffness matrix. The geometrically nonlinear finite element analysis is derived for 3D structures, and the model sensitivities are also derived using the adjoint method. It is concluded from the analyzed example, the double clamped beam, that there is symmetry in linear FEA, while the results using geometrically nonlinear analysis are slightly asymmetric for the 3D structure. Future work involving more complex loads, such as uncertain or multiple loads, should be analyzed.

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Authorship statement.

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