Enhancing Beam Analysis: Integrating 3D Topology Optimization with 2D Structural Analysis via Mesh Pixelation

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Abstract. A topology optimization procedure is adopted to design statically determinate rectangular beams with different heights. The aim is to verify the efficiency level that topology optimization procedure has in removing material from the structural member still preserving the flexural capacity. The design domain was described using the Solid Isotropic Material with Penalisation (SIMP) method. A 3D parametric model of the simply supported beams was implemented in the Rhinoceros3D (Computer-Aided Design) software and Grasshopper (Algorithm-Aided Design) plugin. The optimization was performed using the Topos plugin and the output was examined based on computational time efficiency and physically plausible generated shapes. After Topology Optimization, the structural behaviour was investigated using the Kiwi3D plugin. The complex 3D shape was simplified and converted to a 2D model using a pixelation procedure. A usual design load was specified for the beams. In all the cases analysed, the width and span are fixed. The results of the optimization procedure for beams with different heights are compared based on economic-sustainability criteria (volume of material) and load-bearing capacity (maximum bearing moment).

Keywords: topology optimization, structural analysis, SIMP, innovative design, concrete beams

1 Introduction

The Solid Isotropic Material with Penalization (SIMP) topology optimization method is based on the density of the structural material. This technique assumes that the differential equations governing the problem can be discretised using the finite element method and assigning local densities (ρ) to represent the presence ($\rho = 1$) or absence ($\rho = 0$) of constitutive material. Density values between zero and one make no physical sense as they do not permit to define exactly where the boundary is. This condition is known as “grayscale” and can be mitigated during the optimization process by using density penalty functions with cubic exponents or higher. Filters can often be used to improve the distinction of zones with and without material in a post-processing stage [1]. This enables material to be removed in region that are not under high stress levels, and material to be added in areas with a high demand for internal forces, leading to a less compliant design without losing its initial functionality despite the reduction of the volume and self-weight.

Notwithstanding the popularisation of the SIMP method, different techniques have emerged to perform topology optimization. One example of these techniques is the homogenisation method, in which microstructural heterogeneities are assessed in a local average sense and then expanded to the macro-scale using local expansion of macro-fields. In the case of composite materials, intermediate modulus of elasticity should be specified for expansion purposes to avoid large oscillations during the analysis (Wang [2]). Whether with SIMP, homogenisation or other associated techniques, Topology optimization has gained increased acceptance as part of the first stages in a conceptual design. Besides that, it can be employed simultaneously with structural modelling techniques or even size and shape optimization procedures, to investigate different systems. In the literature, many publications are dedicated to this research field [3-7].

There are some practical difficulties in the manufacturing procedure of optimized structures due to the usual complex and non-conventional final shapes. According to Walliston [5], some composite materials such as reinforced concrete elements suffer some drawbacks, such as the allocation of reinforcement bars in zones of discontinuous material layout. Thus, there is a demand not only for new analysis and optimization procedures, but also for the manufacturing of the outcoming elements. In the present study, we present a numerical procedure for predicting the structural behaviour of simply supported concrete beams before and after they have been topologically optimised.

This procedure is based on varying the height of the models, keeping the spans and thicknesses fixed, and then evaluating the results considering not only the change in flexibility but also the complexity of the final shapes. The topology optimization is performed considering 3D beams. A 2D model is created after the optimization to simplify the complex geometry. The procedure of converting the 3D mesh into a 2D surface is based on the mesh projection and pixelation. A structural analysis is performed considering the 2D model and using the plugin Kiwi3D that is based on Isogeometric Analysis (IGA). Further details of the proposed procedure will be provided in the following sections.

2 Methodology

The structural analysis of the parametrically optimized simply supported concrete beam was divided into 4 main stages.

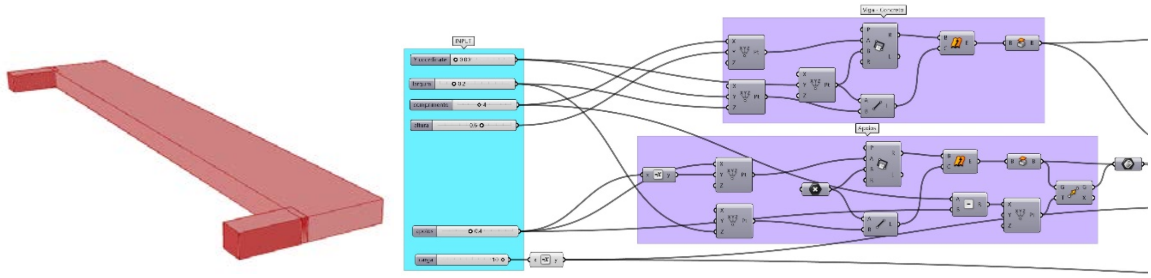
- Construction of the parametric 3D structural element taking advantage of the Rhinoceros3D (Computer-Aided Design) software and Grasshopper (Algorithm-Aided Design) plugin. The advantages of the software include a user-friendly interface, algorithmic design, compatibility with other softwares, script integration (e.g. Python) and extensive toolset, plugins and add-ons.
- 3D topology optimization using the Topos plugin. The main advantages is the integration with Grasshopper, customizable parameters, visualization tools and iterative design process.
- Conversion of the 3D optimized model into a 2D surface. In this case, the 3D mesh is projected in the 2D plane, and the mesh is pixelated using squares to cover all the surface of the element. The pixels are merged to create the surface. This step is introduced since the Topos plugin works only with 3D elements.
- Structural analysis performed in the Kiwi3D plugin. It enables the integration of Isogeometric Analysis (IGA) into the CAD environment in one workflow. The plugin uses NURBS (Non-Uniform Rational B-Splines) for representing geometry, providing high precision and flexibility in modelling complex shapes and surfaces.

2.1 3D parametric model

The parametric 3D beam model, including its supports (boundary conditions) and loading, was developed using Rhinoceros 3D (CAD) with the aid of the Grasshopper (AAD) visual programming plugin. Parametric modelling allows instantaneous changes of the geometry, giving flexibility and allowing an iterative design process. Table 1 shows the input parameters for the beams. Figure 1a presents an example of the 3D digital beam. The geometric parameterization of the beam in Grasshopper plugin is sown in Figure 1b.

Table 1. Set of input parameters for the 3D beams

Parameter	Dimensions
Square columns sides (supports)	0.15 m
Beam width	0.20 m
Span	5.0 m
Height of the columns	independent
Height of the beam	variable



a. b.

Figure 1. 3D Modelling of the Beam: a. Solic geometry in Rhino 3D, b. Grasshopper algorithm.

All the geometric parameters were fixed, except for the height of the beam section, which varied accordingly to allow the simulation of different scenarios. A total of 7 models were analysed, considering height increments of 10 cm.

2.2 Topology Optimization

The geometric model was coupled with the Topos plugin, which is based on the SIMP methodology. The input variables for the plugin are separated into specific parameters that vary according to the expected design results. The input parameters are shown in Table 2. Figure 2 presents the model generated by Topos with the discretization of the mesh.

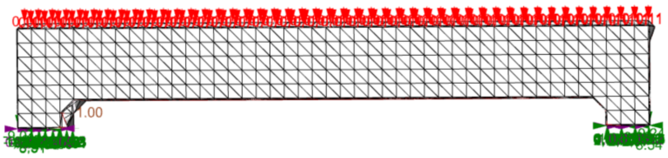


Figure 2. 3D Model generated in the plugin tOps including the discretization of the mesh.

The mesh represents the Boundary Domain, from which the material will be removed according to the given Density Value. For this study, a value of 0.60 was adopted. The closer the value is to 1, the greater is the amount of concentrated material per mesh cell and lesser material are removed. The mechanical properties (Young modulus and Poisson ratio) are also provided.

For the Boundary Conditions, the geometry of the supports is specified. The loading is also defined. In this case, for all models, a uniformly distributed load was applied at the top surface of the beam. The load intensity was defined considering usual design standards for residential buildings and includes the overload, self-weight and live load.

The Optimus Parameters are responsible for controlling the optimization by means of a series of predefined control values. The number of iterations was chosen experimentally based on direct observation of the performance between consecutive iterations aiming to find an average value that was adopted for all models. The value of the final volume of the structural element was fixed for the analysis. The Penalty variable is responsible for controlling penalties in intermediate densities. In this case, the maximum value was adopted to guarantee the lowest presence of intermediate densities in the beam and a provide a more defined geometry. The variable Change controls how much the material density can change between consecutive iterations. The standard value provided by the plugin was adopted, which makes the process more gradual and with lower risks of instabilities. The Sensitivity Filter Radius controls the densities close to each other and is responsible to avoid abrupt variations nearby. Thus, the larger it's the radius of action, the more smoothing is the iterative process. For the analyses, the intermediate value was adopted based on experimental criteria.

The Optimus component depicted in Figure 3 is responsible for gathering all the input data and create a single model. After that the ISO mesh is generated. Once the optimization has been carried out, the plugin provides the performance characteristics of the iterative process and the final optimized geometry in the mesh configuration. Figure 4 shows the optimized beam, highlighting in white (mesh) the final configuration and in red the initial geometry.

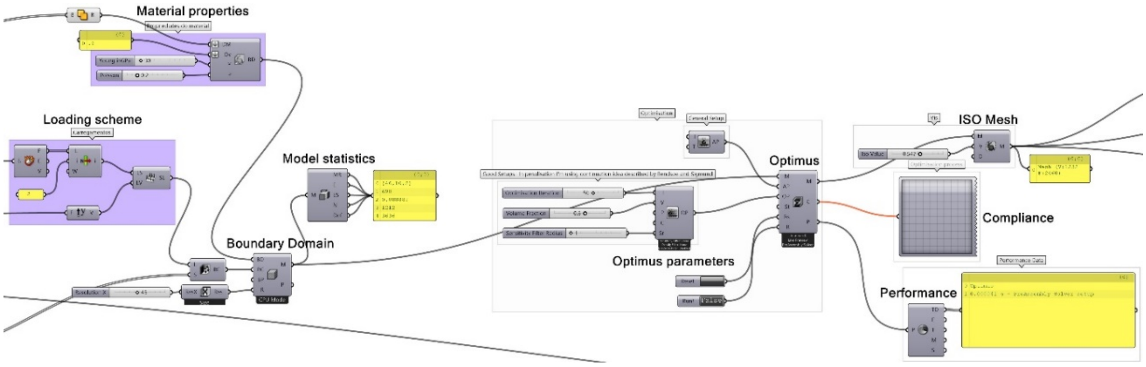


Figure 3. General view of the tOps plugin.

Table 2. Input parameters defined in the plugin Topos.

Parameter	Component	Value
Density Value	Boundary Domain	0.6
Young Modulus	Boundary Domain	30 GPa
Poisson Ratio	Boundary Domain	0.20
Load	Boundary Conditions	30 kN/m²
Iteration	Optimus Parameters	15
Volume Fraction	Optimus Parameters	65%
Penalty	Optimus Parameters	2
Change	Optimus Parameters	0.005
Sensitivity Filter Radius	Optimus Parameters	1,5

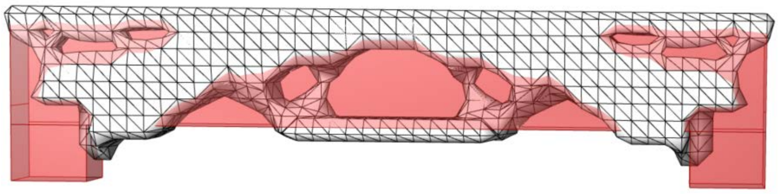


Figure 4. Mesh representation of the topology optimized beam.

2.3 3D mesh transformation to 2D surface

The shape of the 3D beam was reparameterized to obtain a 2D shell-like element. The purpose is to obtain a simplified geometry, avoiding irregularities and voids in the beam's width and, thus, creating a more suitable shape for practical applications. Internal cavities are disregarded to create a constant width and allow the fabrication of the element using formworks or 3D printing. The strategy consisted in the 2D projection of the 3D mesh. After that, all the points on the projected mesh are identified and new points are generated to improve the accuracy of the procedure. Then, the projected surface is pixelated by the creation of rectangles centered on the predefined points. Lastly, the rectangles are joined to create a uniform surface. The representation of the procedure is depicted in Figure 5 and the algorithm is shown on Figure 6.

The dimensions of the rectangles indicate the precision of the final geometry to be generated. The smaller the rectangles are, the more precise the final geometry. The rectangles should be capable of intersecting each other to allow the solid union and formation of a uniform 2D surface. The number of points also directly impacts the smoothing of the final 2D surface. The greater the number of points, the greater the accuracy of the methodology.

a. b.

Figure 5. Optimized beam projected in-plane, a. points generation, b. pixelation (generation of rectangles).

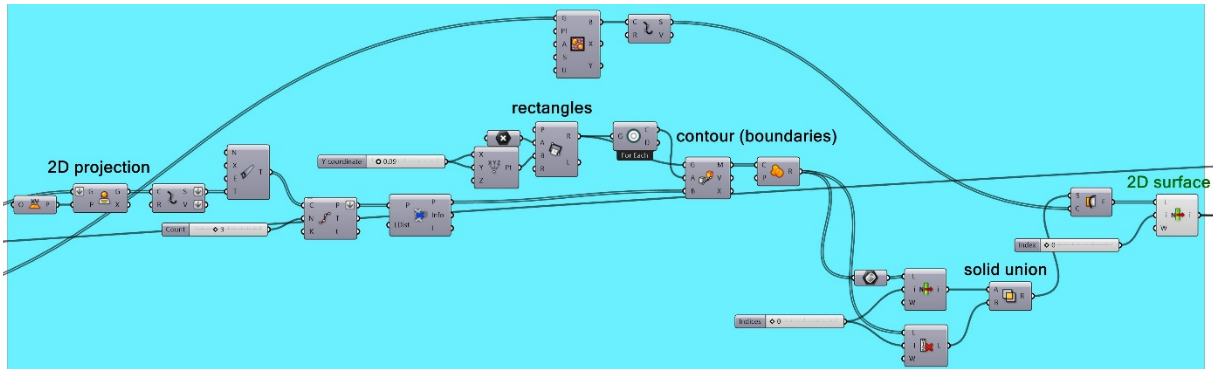


Figure 6. Algorithm for converting 3D mesh into a 2D surface.

2.4 Structural Analysis

For the structural analysis, the plugin Kiwi3D was adopted. The plugin is based on isogeometric finite element (FE) analysis, which allows complex geometries to be processed. The degrees of freedom do not lie on the surface of the element but on the control points of the geometry. The special characteristic of the plugin lies in the usage of Non-Uniform Rational B-Splines (NURBS) as basis functions for the finite elements. NURBS are used for the geometry description in the software Rhino/Grasshopper. The plugin applied FEM-kernel is Carat++, a FE program for structural simulation, structural shape optimization and form finding.

The shell element generated in Grasshopper after the topology optimization was entered in Kiwi3D. A point was established on the centre of the bottom side of the shell to measure the vertical displacements of the beams. The material properties are the same as the initially one provided to the Topos plugin. The supports were remodelled at the beam ends to accommodate the new optimized 2D geometry. Nonlinear displacement analysis can then be performed, obtaining positioning at the lower centre point of the beam.

3 Results and discussion

The beams were divided into 7 models with the same properties, except for the cross-sectional height. The models assumed, as a simplification, a homogeneous distribution of the material (isotropic behavior). The midspan deflection was measured and the results were compared considering the structural analysis before and after the topology optimization. The results are shown in table 3.

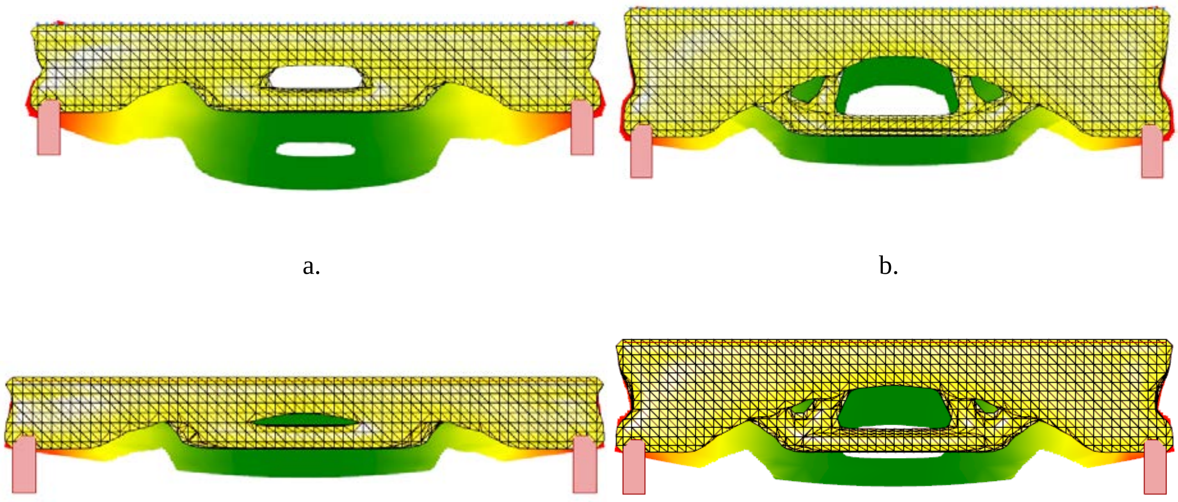
Table 3. Comparison of the deflections of the beams.

Model	Beam height	Deflection (before optimization)	Deflection (after optimization)
1	0.45 m	0.785 mm	0.439 mm
2	0.55 m	0.472 mm	0.294 mm
3	0.65 m	0.285 mm	0.176 mm
4	0.75 m	0.200 mm	0.134 mm
5	0.85 m	0.141 mm	0.106 mm
6	0.95 m	0.107 mm	0.086 mm
7	1.05 m	0.084 mm	0.059 mm

From Table 3, one can notice that as the height of the beam section increases, the displacement is smaller. Besides, the displacement of the optimized beam for the same model is lower when compared to the original model without optimization. This can be attributed to the change of the beam shape, resulting in a modification of the stress distribution and the structural behaviour of the beams. More analyses should be performed to corroborate the results, including a full non-linear FE analysis, considering the material properties and the failure criterion.

The change in the geometry of the beams after optimization can be seen in Figure 3. The 3D mesh of the beam is highlighted in yellow with a grid of black line divisions. The equivalent deformed 2D projection of the beam is represented in the background on an amplified scale, in green, yellow and red according to the respective deformation intensities.

One can observe from Figure 6 that the flow of stresses tends to change as the height of the cross-section of the beams increases, modifying the final geometry for the same density value. Beams with similar heights tend to behave very similarly, as occurred to models 2 and 3, and models 6 and 7.



a. b.

c. d.

Figure 6. Deformed shape of the optimized beams after the structural analysis: a. Model 3, b. Model 7, c. Model 2, d. Model 6.

Moreover, the number of branches tends to become more numerous as the height of the section increases, which is in line with the theory of previous study of Montaute [8]. The shape tends to approximate to a truss form, with well-defined compression and tension regions. A higher compression stress flow is developed near the supports and at the top of the beam. High tension stresses arise on the centre of the bottom side of the beam, where there is maximum bending moment. Therefore, these regions remained with a solid mesh after optimization in all simulated cases.

The results are compared in Figure 7. A pronounced and non-linear decay of the midspan deflection can be observed for beams with increasing heights. It is worth mentioning that the simplifications on the geometry of the beam led to discrepancies in the measurements of the midspan deflection obtained by the plugins Topos and Kiwi3D. For smaller beam heights, the displacements of the optimized and non-optimized section. diverge with greater amplitude. The percentage error of the 3D model optimized by the Topos plugin and the simplified 2D shell is between 15% and 20%. It was observed that the maximum deflection did not follow a linear relationship with the section height.

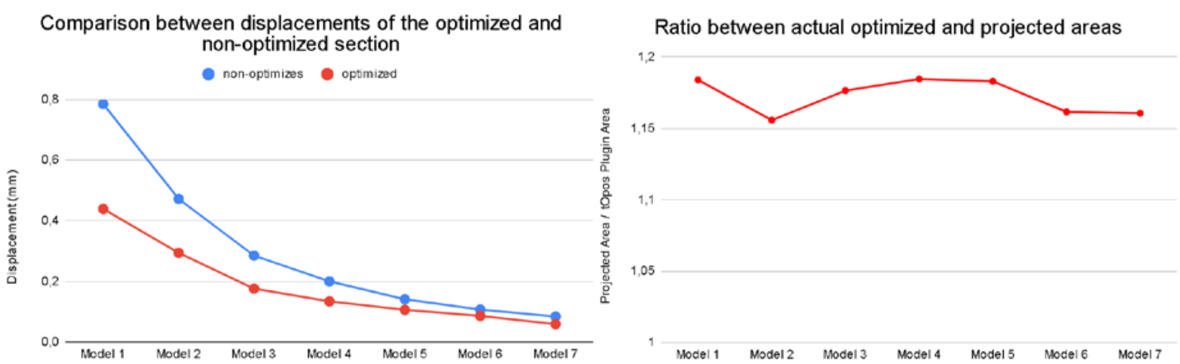


Figure 7. Comparison between geometries with and without optimization. Mechanical response (left) and material variation (right). (Geometric simplifications have been adopted.)

4 Conclusions

The investigation of the topology optimization of concrete beams is relevant as the current procedures considers a homogeneous and isotropic material, which is different from the non-linear behaviour found in real materials used in engineering. The results of the analyses using the plugin Topos demonstrated that the final geometry of the beams is close to the topology optimization theory of previous studies [8]. The geometric shapes were characterized by the formation of branches, with a clear division between compression and tension regions.

Before the structural analysis, some geometric simplifications were introduced in the optimized beam to simplify the non-conventional and complex shape. In this case, the 3D mesh was projected on the 2D plane and the mesh was pixelated using squares to cover all the surface of the element. The pixels were merged to create the surface NURBS for further analysis using the plugin Kiwi3D. As a result, a difference was observed in the measurements of the midspan deflections. The difference between the 3D optimized geometry and the 2D shell is more evident in the models with lower section heights, since they have smaller and closer void regions. A pronounced and non-linear decay of the midspan deflection was observed for beams with increasing heights.

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