Multi-material topology optimization of 2D structures using the SESO and SIMP method with reliability

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Abstract. Topology optimization is essential for the efficient distribution of structural form within the desired design domain, ensuring that structural elements are positioned to resist the requested forces and the specific boundary conditions for which they were designed. Therefore, the main objective of the article is the multi-material topological optimization of a variety of two-dimensional structural problems, based on literature benchmarks, such as Michell structures and the MBB beam. These problems are approached considering a multi-material perspective, to distribute different materials throughout the structure, thus seeking their optimal solution. The methodology was generated based on the use of the finite element method for structure analysis and computing techniques were implemented for multi-material topology optimization (MMTO), Smoothing-ESO (SESO), and Solid Isotropic Material with Penalization (SIMP) via MATLAB. Furthermore, a reliability analysis is incorporated to deal with uncertainties, using Reliability-Based Topology Optimization (RBTO) with the First-Order Reliability Method (FORM), dealing with the random variables involved, such as geometry, modulus of elasticity, volume fraction, compliance, and loading, such as normal and lognormal probability distributions. The results obtained show a satisfactory convergence between the two topological optimization methods studied, thus highlighting the potential for applying these techniques to various structures as an effective tool in the search for economic efficiency in structural design.

Keywords: Multi-material topology optimization, SESO and SIMP, Reliability

1 Introduction

The development of a structural project in the civil, mechanical, and aeronautical areas requires the consideration of several essential factors. It is essential to carefully approach the boundary conditions and loads acting on the structure to develop effective structural solutions. A crucial aspect of the project, prepared by the engineer, is the configuration of the structural topology.

Given the above, Topological Optimization (TO) has been gaining ground when addressing the efficient distribution of materials within the project domain. In most cases, it aims to reduce structural weight, simultaneously minimizing its compliance.

Simultaneously, with the advancement of technology in numerical simulations using the finite element method, TO techniques have been widely studied and methodologies developed, highlighting the methods ESO (Evolutionary Structural Optimization) proposed by Xie and Steven [1], Solid Isotropic Material with Penalization (SIMP) proposed by Bendsøe and Kikuchi [2] and SESO (Smoothing ESO) proposed by Simonetti *et al.* [3].

In addition TO with a single material, an even more promising strategy is the integration of multiple materials to create the desired structure. This approach offers the advantage of increasing stiffness in areas requiring high-performance materials while reducing costs by using lower-performance materials in regions with lower loads. Adopting this technique makes it feasible to achieve an efficient and economical composition of structural stiffness and stability. Despite the complexity involved in manufacturing these components, current 3D printing technology stands out as a practical solution. As demonstrated by Gaynor *et al.* [4], 3D printing allows the precise and efficient production of structures with multiple materials, overcoming traditional manufacturing challenges.

Recently, the study of topological optimization with multi-materials (MMTO) using the SIMP methodology has gained prominence in several studies, as evidenced by Wan *et al.* [5], Li *et al.*[6], Ozcakar *et al.* [7] and Zuo and Saitou [8]. However, to date, the SESO methodology for multi-material topology optimization has not yet been explored in any study, especially when integrated with a structural reliability analysis. This article seeks to fill this gap by investigating the most appropriate two-dimensional structural topology for the numerical examples presented.

Furthermore, it proposes a comparison between the high-performance tools SIMP and SESO, providing an innovative approach, simultaneously analyzing both methods coupled with structural reliability analysis, to guarantee economic efficiency and structural safety.

2 Topology Optimization with Multi-material

2.1 SIMP method

The SIMP method was created in 1988 by Bendsøe and Kikuchi and improved by Rozvany and Zhou [9]. This method aims to distribute the material within a design domain efficiently, guaranteeing the boundary conditions, restrictions, and objective functions imposed on the various engineering problems. To achieve this objective, the method uses a penalty criterion that guides the removal of material in unnecessary regions, thus optimizing the use of available material. The flowchart shown in Fig. 1 represents the implementation process of the SIMP method.

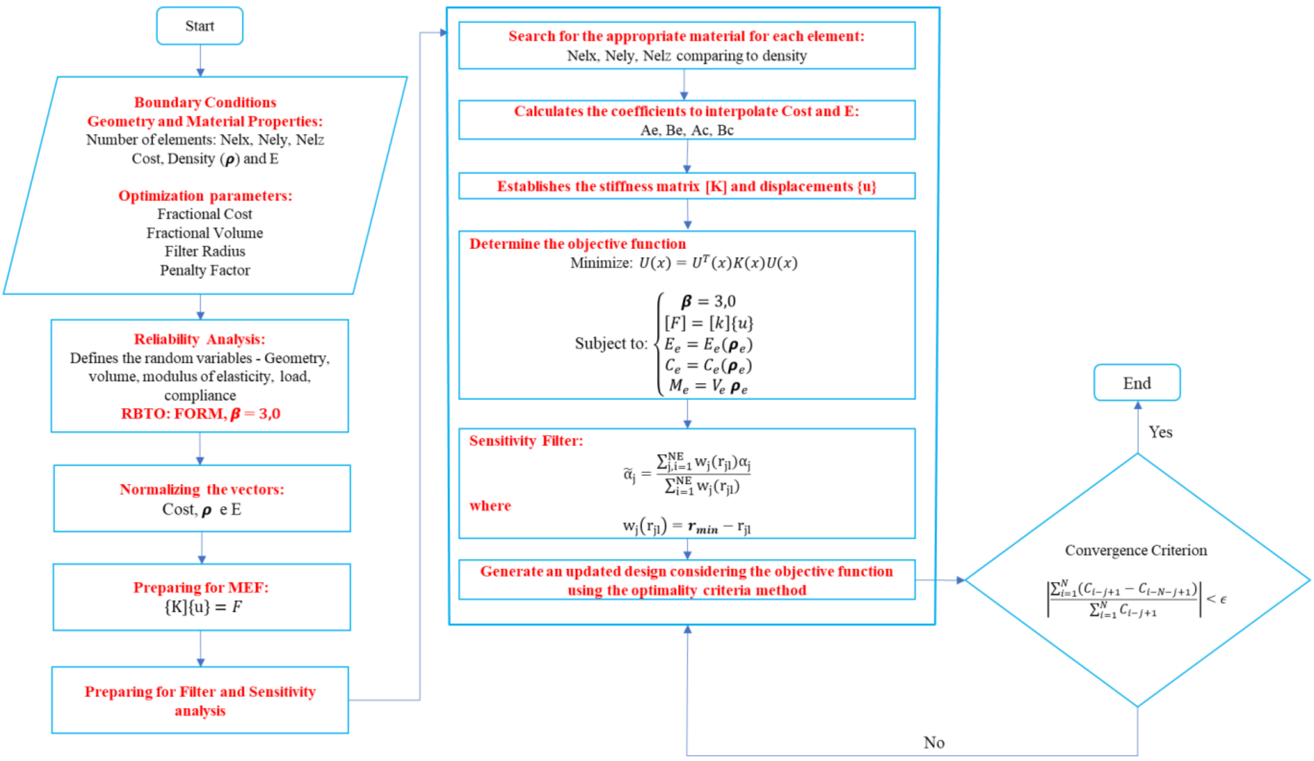


Figure 1 - Flowchart of the optimization process using the SIMP method

Due to the simplicity of implementation and a range of practical approaches, SIMP has gained popularity. Subsequently, Zuo and Saitou [8] proposed an improvement in the SIMP method, introducing a new interpolation to solve multi-material problems. The problem with two and three phases is demonstrated by Eq. (1) and Eq. (2), respectively:

$$E(\rho) = \rho^P E_1 + (1 - \rho^P) E_2 \tag{1}$$

$$E(\rho_1, \rho_2) = \rho_1{}^P (\rho_2{}^P E_1 + (1 - \rho_2{}^P) E_2) \tag{2}$$

where: $\boldsymbol{\rho}$ is the artificial density [0,1], P is the penalty coefficient, E_1 and E_2 are the elastic moduli of the two phases, when $E_2 = 0$ is taken as the empty phase.

2.2 SESO method

ESO Smoothing (SESO) was proposed by Simonetti *et al.* [3] as an alternative for smoothing ESO, assigning each element a stiffness value instead of removing elements based on the equation $C_e < RR * C_e^{max}$ where RR is the rejection ratio, C_e e C_e^{max} are the element compliance and maximum compliance, respectively. SESO takes into account that some elements will be removed from the structure and others will be returned to the structure at each iteration, based on the weighting factor given by the ratio between the C_e / C_e^{max} within the domain Γ.

The flowchart shown in Fig. 2 represents the process of implementing the SESO method considering multi-material problems.

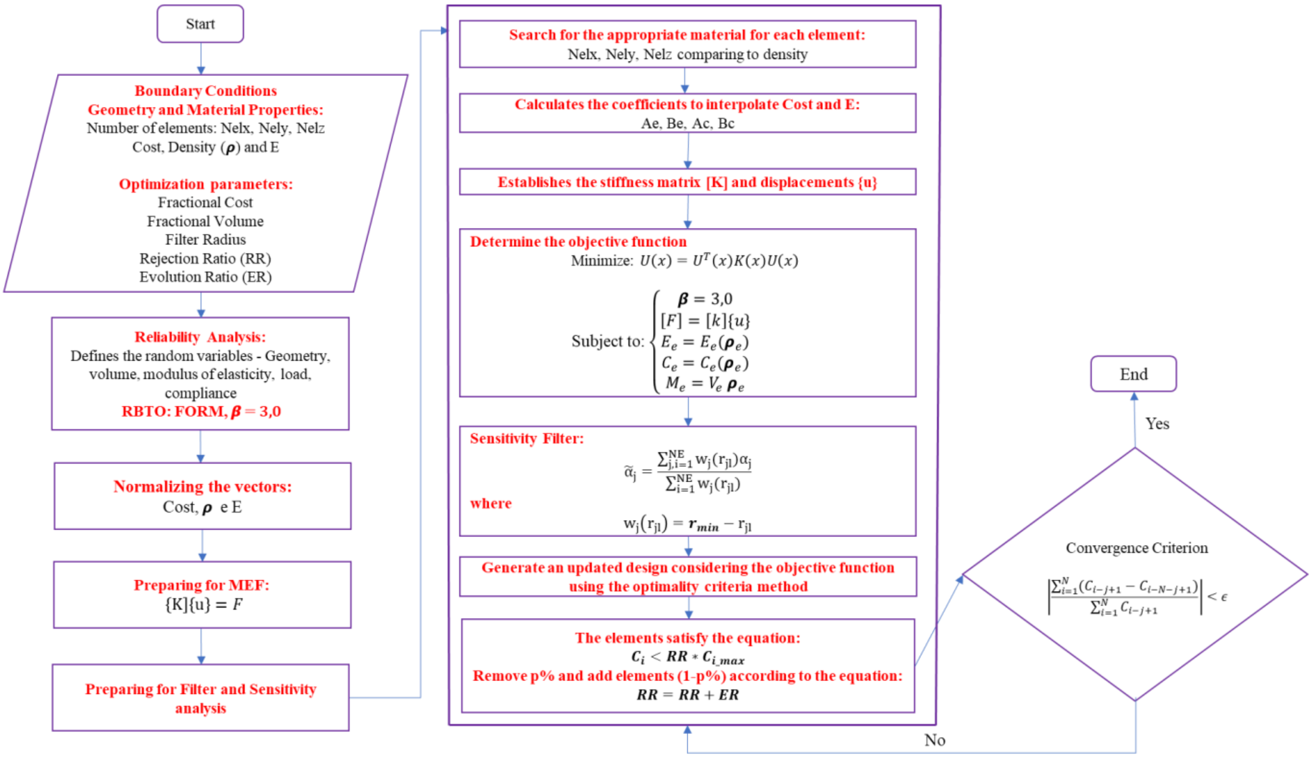


Figure 2 - Flowchart of the optimization process using the SESO method

3 Reliability Analysis: First-Order Reliability Method (FORM)

Structural reliability analysis is based on probabilistic modeling of uncertainties inherent to processes associated with resistance and stresses. Thus, aiming to maximize safety levels in the optimization approach, and based on several works in the literature (Silva *et. al.* [10]; Simonetti *et. al.* [11], the RBTO (Reliability-Based Topology Optimization) was approached using the FORM method (First Order Relayability Method) to deal with the random variables involved. Thus, the FORM method covers the Taylor series expansion, approaching the failure function iteratively until the result converges to the design point, given this minimum distance from the origin to the failure function, known as the reliability index ($\boldsymbol{\beta}$), (Haldar and Mahadevan [12]). Therefore, the random variables were used: N_{elx} (length), N_{ely} (width), massfrac (fractional mass), E (modulus of elasticity), compliance and strength, all considering a probabilistic distribution of the normal type, adopting a reliability index

$\boldsymbol{\beta}$=3.0. The equation used to find the distance from the origin to the failure function $d_{\boldsymbol{\beta}}$, i.e., $\boldsymbol{\beta}$, is given below according to Kharmanda and Olhoff [13], given by:

$$d_\beta = \sqrt{\sum_{i=1}^{n}\left(\frac{y_i - x_i}{\sigma_i}\right)^2}\ ,\ i = 1, \dots, n \tag{3}$$

where x_i is the mean of the random variables, y_i is the random variable and σ_i is the standard deviation equal to 1.0.

4 Numerical Examples

The numerical examples covered in this work include two benchmarks from the literature, the Michell-type structure and the MBB beam, in which the design constraints have cost and modulus of elasticity, with the objective function being the minimization of compliance, given by Eq. (4):

$$\begin{cases} Minimize\ c = u^T K u = \sum_{e=1}^{N} E_e u_e^T k_0 u_e \\ \qquad E_e = E_e\ (\boldsymbol{\rho_e}) \\ \qquad C_e = C_e\ (\boldsymbol{\rho_e}) \\ \qquad Ku = P \\ subject\ to: \quad M = \sum_{e=1}^{N} V_e \rho_e \\ \qquad C = \sum_{e=1}^{N} V_e \rho_e C_e \\ \qquad M \le \varepsilon M\ M_0 \\ \qquad C \le \varepsilon c\ C_0 \end{cases} \tag{4}$$

where: E_e and C_e are the elasticity modulus and cost of the nth element; ρ_e is the normalized density of the nth element; c is compliance; K, u and P are the global stiffness matrix, displacement vector and force vector, respectively; V_e is the volume (in 2D) of the nth element; M and C are the mass and cost of the current design domain, respectively; M_0 and C_0 are the mass and cost of the fully populated design domain with $\rho_e = 1$, respectively; εM and εc are the prescribed mass and cost fraction, respectively.

The materials used in both examples were data provided by the author Zuo and Saitou [8], for validation purposes, given in Table 1.

Table 1 - Material data in the optimization of the Michell structure (Zuo and Saitou [8])

Name	Density	E	Cost (P)	Color
Empty	0	0	0	White
A	0,4	0,2	0,5	Blue
B	0,7	0,6	0,8	Red
C	1,0	1,0	1,0	Black

4.1 Michell-type structure

The design domain and boundary conditions are shown in Fig. 3. The structure domain was discretized with a 100x50 mesh, totaling 5000 quadrilateral finite elements. This structure was subjected to a concentrated load with F = 1 N applied at three different points, located at positions (25.0), (50.0) and (75.0). The Michell-type structure was topologically optimized using several methods and the results were compared with those obtained by Zuo and Saitou [8], as illustrated in Fig. 4. The history of the optimization procedure of the Michell-type structure was performed with a fixed number of 123 iterations, similar to the study by Zuo and Saitou [8]. The SIMP method and the SESO method were used, both with RBTO, as shown in Fig. 5a and 5b, respectively.

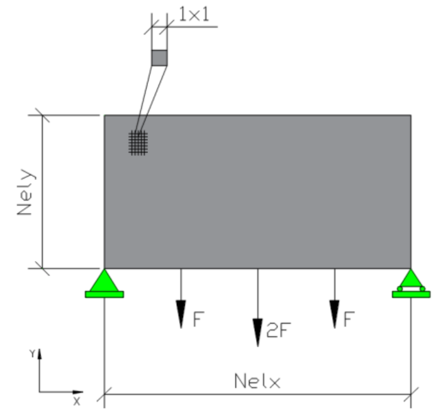
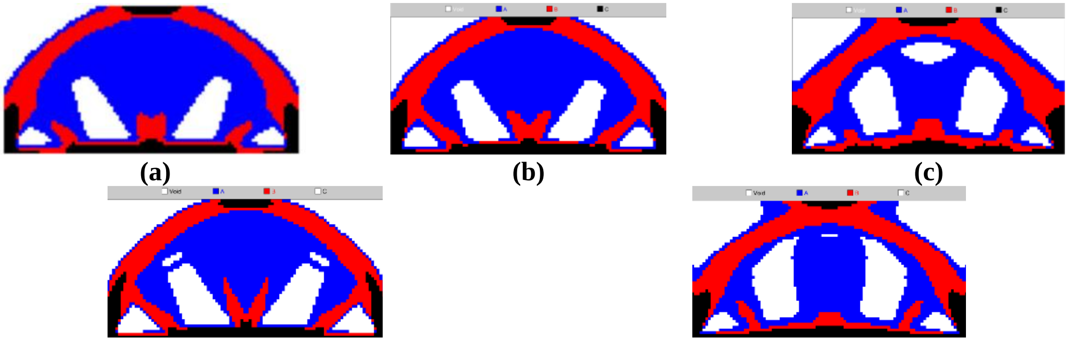


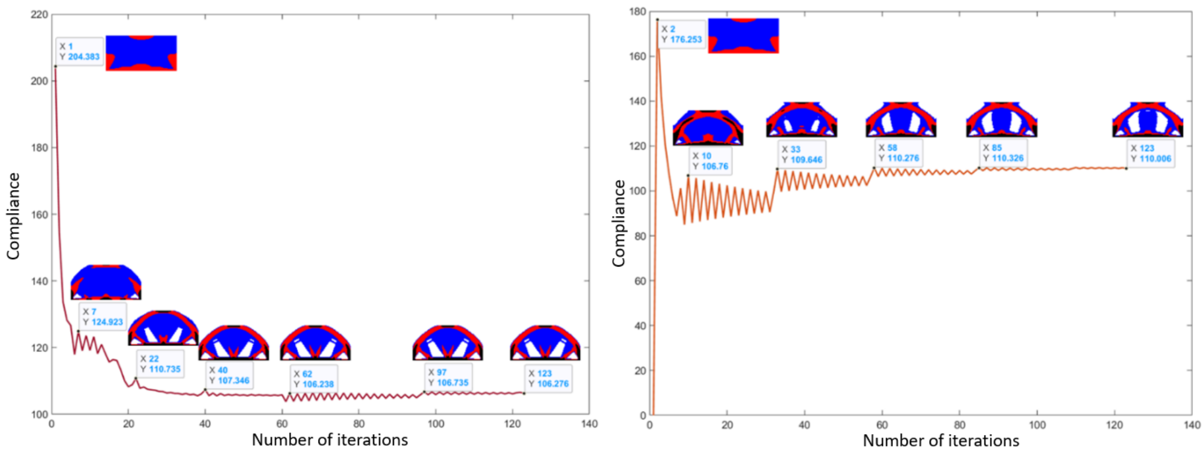
Figure 3 - Design of the Michell Structure with boundary conditions - 2D



(a) (b) (c)

(d) (e)

Figure 4 - Michell-type structure: (a) Zuo and Saitou [8]; (b) SIMP; (c) SESO; (d) SIMP with RBTO; (e) SESO with RBTO.



(a) (b)

Figure 5 – Compliance by iteration number for Michell-type structure:
(a) SIMP with RBTO; (b) SESO with RBTO.

4.2 MBB Beam Structure

The design domain and boundary conditions are shown in Fig. 6. The structure domain was discretized with a 60x20 mesh, totaling 1200 quadrilateral finite elements. This structure was subjected to a concentrated load of 1 N, located at the midpoint of the lower side. The optimal configurations of the MBB beam are illustrated in Fig. 7. The history of these configurations and the evolution of compliance during the optimization process are shown in Fig. 8. The SIMP method and the SESO method were used, both with RBTO, as shown in Fig. 8a and 8b, respectively.

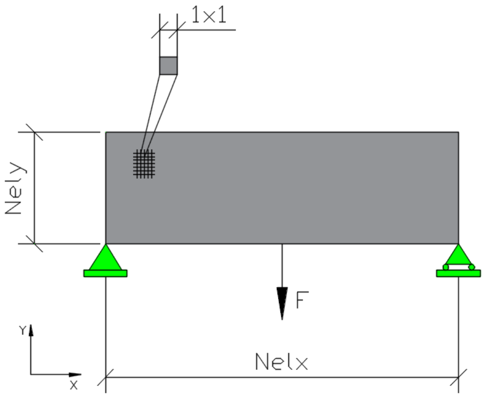
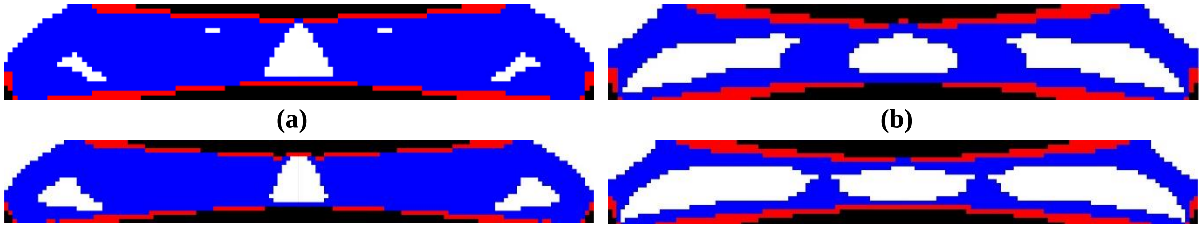
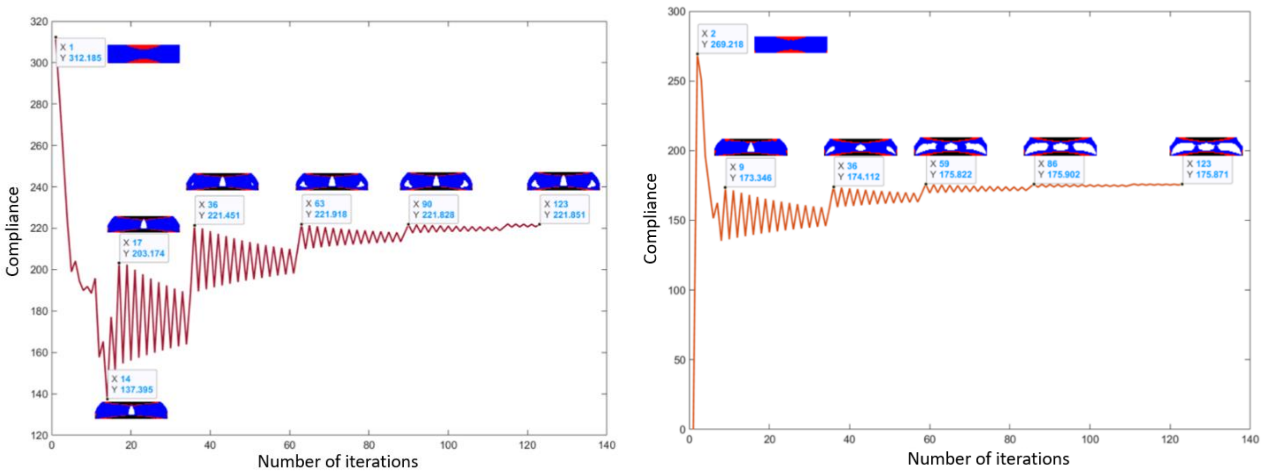


Figure 6 - Design of the MBB Beam Structure with boundary conditions - 2D



(a) **(b)** **(c)** **(d)**

Figure 7 - MBB Beam Structure, MMTO: (a) SIMP; (b) SESO; (c) SIMP with RBTO; (d) SESO with RBTO.



(a) (b)

Figure 8 – Compliance by iteration number for MBB beam structure: (a)SIMP with RBTO; (b) SESO with RBTO.

5 Conclusions

The topological optimization and reliability analysis, based on Michell-type structures and MBB beam, showed that:

i) Optimization of the Michell structure with material C (black) resulted in a compliance of 277 (Zuo and Saitou [8]). With three materials, compliance was reduced to 233.

ii) The SIMP and SESO methods presented results similar to those of Zuo and Saitou [8], with SESO using more high-performance materials (B and C) and showing more empty spaces.

iii) The integration of reliability analysis with MMTO reduced compliance in both methods, with SESO achieving better results than SIMP.

iv) In regions with high loads, MMTO prefers material C, which is denser and more resistant, despite the higher cost, while material B is used for transitions, and material A, which is more economical, in regions with light loads.

v) There are few studies on the integration of MMTO with reliability analysis. SIMP and SESO methods proved to be efficient, reducing costs and project compliance.

vi) The SIMP and SESO methodologies, when applied to MMTO, have shown promise in two-dimensional structures. A significant step forward for this research would be to expand these approaches to developing three-dimensional structures. This would not only allow for more realistic modeling of civil engineering applications, but would also enable the optimization of the use of various commercial materials, such as steel, concrete, and aluminum. This expansion could lead to more efficient and economically viable solutions in complex projects, such as large structures like bridges and buildings.

Acknowledgements. This study was financed in part by the Coordination for the Improvement of Higher Education Personnel - Brazil (CAPES) - Finance Code 001, CNPq (National Council of Scientific and Technological Development) under Grant Number 316771/2023-6, FAPEMIG under Grant number TEC-PPM-00692-18, Federal University of Ouro Preto (UFOP) and FUNDAÇÃO GORCEIX.

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