Robust topology optimization of trusses with automatic grouping of cross-sections

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Abstract. The ground structure method is a topology optimization strategy that tends to generate complex structural solutions to structural optimization problems. One of the main causes of this complexity is the large number of different cross-sections present in the topologies optimized by this method, in which the solutions obtained often have dozens of bars with different cross-sections. This excessive diversity of cross-sections makes the solutions obtained impractical for the manufacture and assembly of real structures, which require standardized cross-sections for economic and constructive viability. Therefore, in order to improve the workflow during the design stage, we propose the development of a strategy that allows the number of cross-sectional areas available to be restricted during the optimization process. The proposed method acts independently of the number of members present in the initial structure and ensures that the optimized solution has a predefined number of cross-sections with equal areas. To do this, an initial optimization is carried out to obtain an ordered vector of optimized design variables with a higher tolerance. From this ordering, the groupings of members that will be associated with the same design variable are defined, which generates a new optimization problem with a reduced number of design variables. The material of the structure is again redistributed among the members and the new optimization problem is solved. Structures were optimized with a robust compliance minimization formulation that introduces uncertainty in the loading directions. This formulation naturally increases the complexity of the final structure compared to a nominal formulation. As a result, simpler topologies are obtained that benefit the construction process by reducing the number of elements with different cross-sections in the structure. Numerical examples are presented to demonstrate the efficiency of the strategy developed.

Keywords: Topology optimization, Grouping of cross-sections, Robust optimization

1 Introduction

Topology optimization is a technique used to determine the optimal distribution of material within a domain to enhance specific structural properties, such as stiffness or natural frequency [1]. The origins of this methodology trace back to Michell's pioneering work in 1904 [2], which introduced an analytical method for minimizing the volume of structures under given loads. Over the years, this technique has significantly evolved, extending its applications to fields such as material development [3] and concrete structures [4].

Despite these advancements, current topology optimization techniques often yield complex and constructively challenging solutions [5–7], hindering their practical application. Typically, solutions optimized through the ground structure method include numerous bars with varying cross-sectional dimensions, posing significant construction challenges.

To address this issue and enhance the constructability of ground structure optimization solutions, this work proposes a methodology to simplify topologies by grouping bars with identical cross-sectional areas. To validate the proposed approach, a robust topology optimization strategy is employed to generate solutions, showcasing the effectiveness and reliability of the developed method.

2 Worst case optimization

Robust optimization is a field that addresses uncertainties within the optimization process [8]. In structural engineering, these uncertainties can pertain to various parameters, such as the direction and magnitude of forces [9] or material properties [10]. There are several methods to achieve robust solutions to optimization problems, including deterministic and stochastic methods [11–13].

This work focuses on uncertainties associated with nodal loads within the structure. Specifically, given a node's position and the maximum loads that can act in each direction, the structure is optimized to minimize the compliance of the worst possible combination of these loads. The approach utilized involves minimizing the maximum eigenvalue of a specific objective function, described in details by [11]. This problem is known to be non-differentiable. To address this, a smoothing methodology for the objective function proposed by [9] is employed to make it differentiable. Therefore, the regularized robust optimization problem for a structure with a single loaded node in two-dimensional space is defined by 1.

$$\begin{aligned}
\min_{\boldsymbol{x}} \quad & C(\boldsymbol{x}) = \frac{t_{xx}(\boldsymbol{x}) + t_{yy}(\boldsymbol{x})}{2} - \sqrt{\left(\frac{t_{xx}(\boldsymbol{x}) - t_{yy}(\boldsymbol{x})}{2}\right)^2 + t_{xy}(\boldsymbol{x})^2 + \mu^2} \\
\text{s.t.} \quad & g(\boldsymbol{x}) = \boldsymbol{L}^\top \boldsymbol{x} - V_0 \leq 0 \\
& x_{\min} \leq x_i \leq x_{\max} \quad i = 1, ..., n \\
\text{with} \quad & \boldsymbol{K}(\boldsymbol{x})\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{F}
\end{aligned} \tag{1}$$

where

$$t_{xx}(\boldsymbol{x}) = \boldsymbol{u}_x(\boldsymbol{x})^\top \boldsymbol{F}_x \tag{2}$$

$$t_{yy}(\boldsymbol{x}) = \boldsymbol{u}_y(\boldsymbol{x})^\top \boldsymbol{F}_y \tag{3}$$

$$t_{xy}(\boldsymbol{x}) = \boldsymbol{u}_x(\boldsymbol{x})^\top \boldsymbol{F}_y \tag{4}$$

$$\mu = \beta \left(\frac{t_{xx}^0 + t_{yy}^0}{2}\right) \tag{5}$$

In the above equations, $\boldsymbol{K}$ is the structure's stiffness matrix; $\boldsymbol{F}$ is the nodal forces vector; $\boldsymbol{u}(\boldsymbol{x})$ represents the nodal displacements; $\boldsymbol{L}$ is the vector of element lengths; $C(\boldsymbol{x})$ is the objective function; $\boldsymbol{u}_x(\boldsymbol{x})$ and $\boldsymbol{u}_y(\boldsymbol{x})$ are the nodal displacement components in x and y directions, respectively; $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are the load components in x and y directions, respectively; μ is a smoothing parameter defined in terms of the initial t_{xx}^0 and t_{yy}^0 components and a dimensionless factor β.

3 Automatic grouping of cross-sections

Alcazar et al. [9] developed a methodology that enables the association of the areas of several elements with a single design variable. Consequently, elements linked to the same design variable will have identical area values at the end of the optimization. This approach also naturally reduces computational cost by decreasing the number of design variables.

The design variables of the original formulation, $\boldsymbol{x}$, are expressed as a function of a reduced vector of auxiliary variables, $\boldsymbol{y}$, using a transformation matrix $\boldsymbol{D}$. Therefore,

$$\boldsymbol{x}(\boldsymbol{y}) = \boldsymbol{D}\boldsymbol{y} \tag{6}$$

The transformation matrix $\boldsymbol{D}$ is a binary matrix that maps each element of the vector $\boldsymbol{y}$ to a single element of the vector $\boldsymbol{x}$. This implies that the box constraints are equivalent for both variables.

To introduce this constraint into the robust optimization problem, Eq. 1 is modified as follows:

$$
\begin{aligned}
\min_{\boldsymbol{y}} \quad & C_{gr}\left(\boldsymbol{x}(\boldsymbol{y})\right) = \frac{t_{xx}\left(\boldsymbol{x}(\boldsymbol{y})\right) + t_{yy}\left(\boldsymbol{x}(\boldsymbol{y})\right)}{2} - \sqrt{\left(\frac{t_{xx}\left(\boldsymbol{x}(\boldsymbol{y})\right) - t_{yy}\left(\boldsymbol{x}(\boldsymbol{y})\right)}{2}\right)^2 + t_{xy}\left(\boldsymbol{x}(\boldsymbol{y})\right)^2 + \mu^2} \\
\text{s.t.} \quad & g\left(\boldsymbol{x}(\boldsymbol{y})\right) = \boldsymbol{L}^\top\left(\boldsymbol{x}(\boldsymbol{y})\right) - V_0 \leq 0 \\
& y_{\min} \leq y_i \leq y_{\max} \quad i = 1, ..., n \\
\text{with} \quad & \boldsymbol{K}\left(\boldsymbol{x}(\boldsymbol{y})\right)\boldsymbol{u}\left(\boldsymbol{x}(\boldsymbol{y})\right) = \boldsymbol{F}
\end{aligned}
\tag{7}
$$

The sensitivities of the objective function and the volume constraint are,

$$
\frac{\partial \boldsymbol{g}\left(\boldsymbol{x}(\boldsymbol{y})\right)}{\partial \boldsymbol{y}} = \boldsymbol{D}^\top \frac{\partial \boldsymbol{C}\left(\boldsymbol{x}(\boldsymbol{y})\right)}{\boldsymbol{y}} \tag{8}
$$

$$
\frac{\partial \boldsymbol{g}\left(\boldsymbol{x}(\boldsymbol{y})\right)}{\partial \boldsymbol{y}} = \boldsymbol{D}^\top \frac{\partial \boldsymbol{g}\left(\boldsymbol{x}(\boldsymbol{y})\right)}{\boldsymbol{y}} \tag{9}
$$

Based on the matrix $\boldsymbol{D}$, it is possible to determine which bars will be grouped into a single design variable during the optimization (Eq. 10 and Fig. 1) [9]. To automatically generate $\boldsymbol{D}$, the process begins with the conventional optimization of problem 1. From the obtained result, all elements e_i with area $x_i \leq a_{tol}$ are removed. If the removal of thin elements causes the structure to lose equilibrium [14], the value of a_{tol} should be reduced. The remaining elements from this first filtering move to a second stage, which starts with sorting these elements in ascending order of area. Then, based on the defined number of element groups, N_g, the interval $[a_{\min}, a_{\max}]$ is divided into equal N_g intervals. Each element is allocated to the interval that contains its respective area. In this way, each interval contains associated elements, thus forming the groups corresponding to the new design variables. That is, all elements within the same interval will have the same area throughout the optimization process. With the groups formed, the matrix $\boldsymbol{D}$ can be assembled according to the scheme illustrated in Fig. 1.

$$
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} \tag{10}
$$

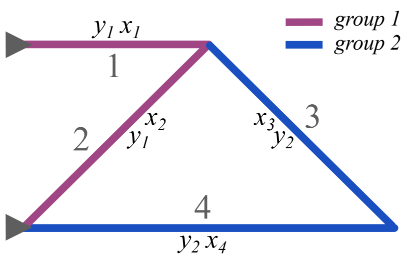

Figure 1. Example structure with four elements and two groups. Elements 1 and 2 are associated with group 1 and elements 3 and 4 are associated with group 2.

4 Numerical results

Two numerical examples were made using the methodology described in 3. In both cases, a ground structure composed of equidistant nodes in x and y directions with full connectivity for the Tower domain (Fig. 2 a)), meaning all nodes are interconnected by truss elements, and 5 level of connectivity for the Square domain (Fig. 2 b)). The OC (Optimality Criteria) [1] method was used as the optimizer. The following parameters values were adopted for both examples: modulus of elasticity $E = 1$, forces $F_x = 1$ and $F_y = 1$, material volume $V_0 = 1$, tolerance for OC convergence $\epsilon = 10^{-8}$, and a move parameter $move = 10^4 A_0$, where A_0 is given by Eq. 11, and a regularization coefficient $\beta = 0.1$. The solution was implemented using Julia 1.10.4 for the solver and Python 3.12.4 for post-processing.

$$A_0 = \frac{\sum_{i=1}^{n} L_i}{V_0} \tag{11}$$

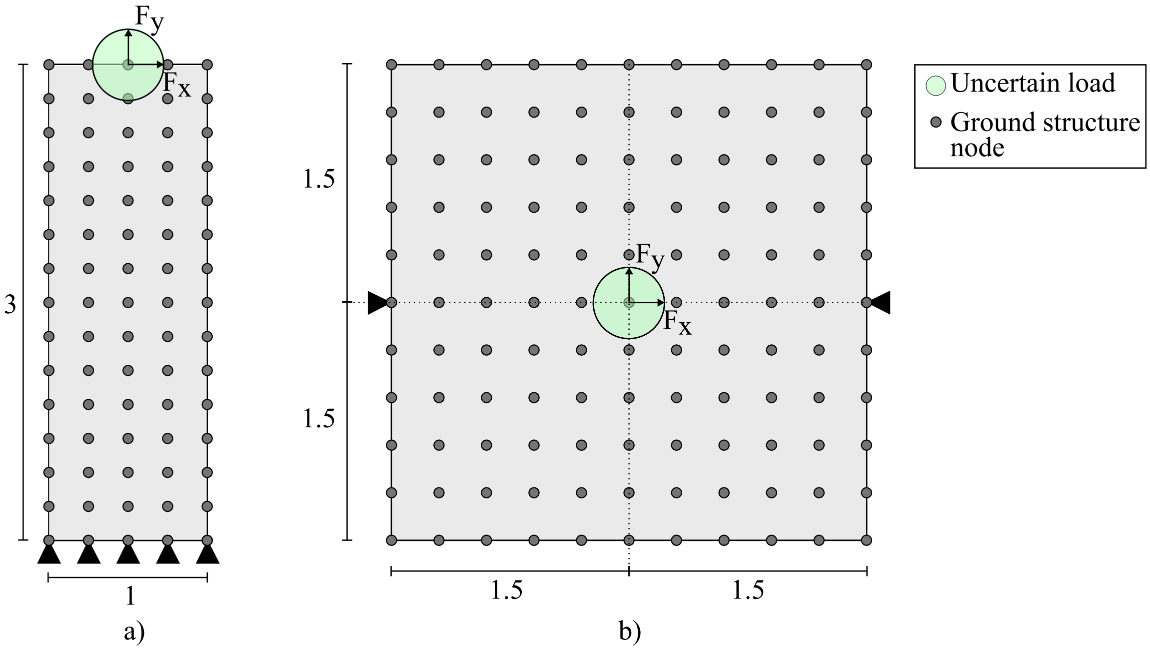

Figure 2. Domains of the numerical examples. a) tower domain and b) square domain.

4.1 Tower domain

In this example, a rectangular domain with a base of 1 and a height of 3 is presented in Fig. 2 a). The ground structure for this case was generated by positioning 5 nodes along the base and 15 nodes along the height of the domain, totaling 1718 elements.

Fig. 3 shows the results obtained by optimizing this domain. Note that when there are no groups defined (Figs. 3 a) and 4 a)), the optimizer has more freedom to generate bars with varied areas, resulting in a large number of distinct cross-sections, as shown in Fig. 4 a). In Figs. 3 b) and 4 b), a considerable simplification of the solution is observed due to the reduction in the number of different cross-sections, although the topology remains the same. In Fig. 4 b), the steps indicating the 5 sections of elements corresponding to the optimization problem with grouped bars are clearly visible. Despite the elements not having optimal areas compared to the reference without groupings, it is noticeable that the areas of each group remained close to the original values. It is important to note that with the limitation imposed for the grouped elements to be optimized together, the compliance tends to increase as the number of groups is reduced (Tab. 1).

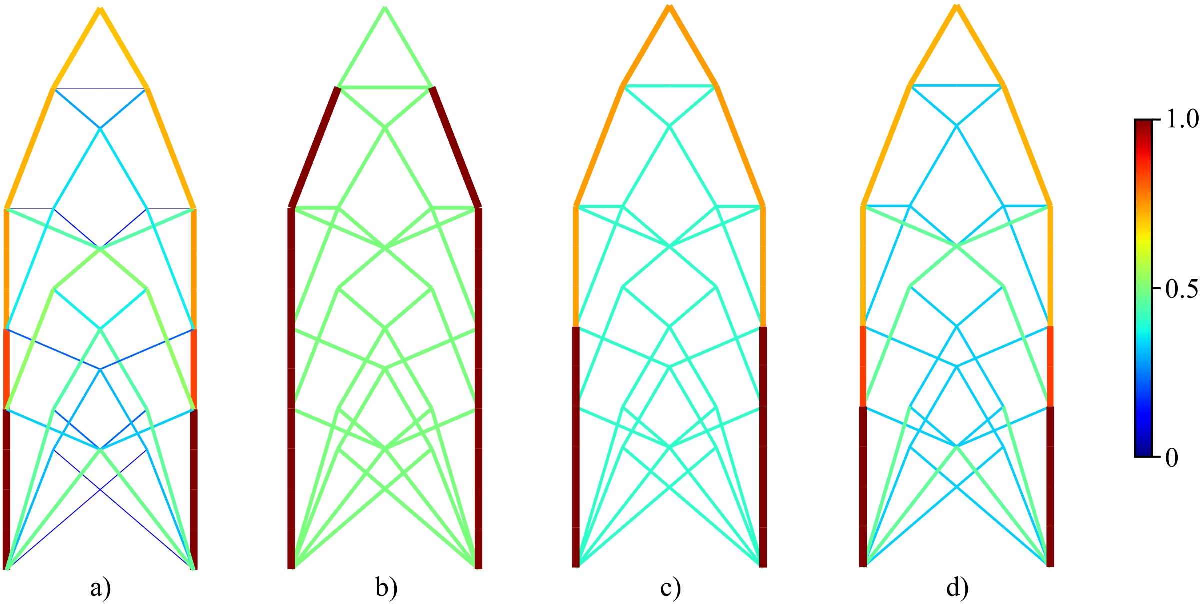

Figure 3. Topologies for the tower example obtained with varying numbers of groups: a) without grouping, b) with 2 groups, c) with 3 groups, and d) with 5 groups.

Table 1. Tower numerical results.

Case	Groups	Compliance	Iterations before grouping	Iterations after grouping
a	-	202.39	3419	-
b	2	248.19	3419	8
c	3	220.81	3419	5
d	5	213.42	3419	12

4.2 Square domain

This example, with a domain formed by a square with a side of 3 and an uncertain load applied at the centroid, has symmetry with respect to both Cartesian axes. This symmetry is sufficient for the objective function of this problem to be non-differentiable [9]. The nodes are equidistant in x and y and positioned on a grid with 15 positions in each direction, totaling 4184 elements in the initial ground structure.

Although this problem has a simple final solution, solving it is more complex due to its non-differentiable nature and the large number of elements present in the initial ground structure. The results presented in Fig. 5 show that the reduction of the structure's complexity through section grouping occurs effectively. As expected, similar to the behavior observed in Sec. 4.1, in this case there is also an increase in compliance as the number of groups decreases. It is important to emphasize that defining the number of groups limits the maximum number of distinct cross-sections that can appear in the solution. However, the final number of distinct cross-sections may be less than or equal to the defined value.

Furthermore, in both examples, the number of iterations required in the second stage of the process is very low, as it begins with an already optimized structure. Similar to the example in 4.1, the proposed strategy successfully simplifies the solution by reducing the number of distinct cross-sections.

Table 2. Square numerical results.

Case	Groups	Compliance	Iterations before grouping	Iterations after grouping
a	-	24.77	1142	-
b	2	24.97	1142	6
c	4	24.78	1142	5

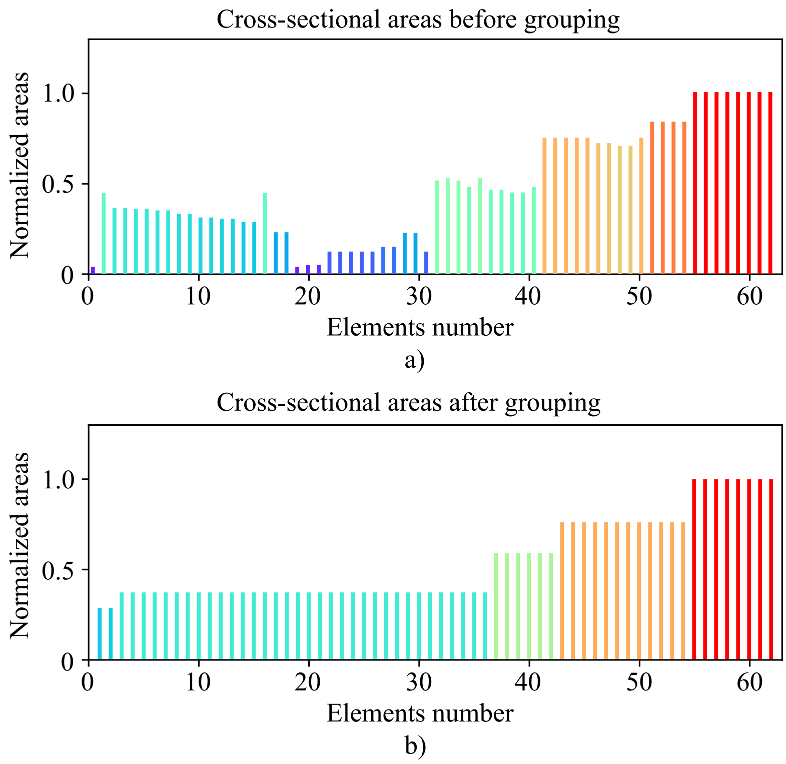

Figure 4. Distribution of cross-sectional areas of elements in the tower solution: a) without grouping (Fig. 3 a)), and b) with 5 groups (Fig. 3 b).

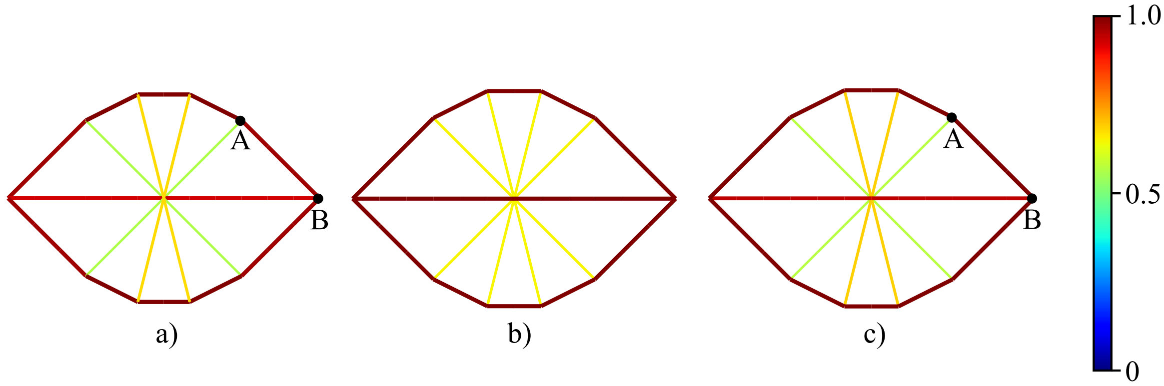

Figure 5. Topologies for the square example obtained with varying numbers of groups: a) without grouping, b) with 2 groups and c) with 4 groups. The cross-sectional areas of the segments $\overline{AB}$ (and their corresponding symmetrical segments) in a) and c) are different.

5 Conclusions

This study presented the development and application of an automatic grouping strategy for cross-sections, combined with a deterministic robust optimization formulation (worst load case). The proposed approach incorporates a regularized formulation for robust optimization, aiming to smooth the objective function and thus avoid points of non-differentiability in the original objective function. The results obtained from numerical examples demonstrate the effectiveness of the proposed methodology in reducing the complexity of optimized structures by reducing the number of different cross-sections. This feature is particularly relevant for approximating the numerical solution to a real-world constructive application, where the number of available cross-sections is naturally limited. The methodology developed in this work proved effective in grouping cross-sections of the same area, even within the context of a robust optimization formulation, which presents a considerably more complex optimization process. Future work may explore the extension of this methodology to dynamic structure optimization problems and structures with predefined cross-sectional dimensions.

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