Active Suspension using a PID Controller with an Inerter Device

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Abstract.

This article is dedicated to a thorough investigation into the behavior of an active suspension that incorporates the PID controller in a quarter-car model, adding an inerter component to the system. Through a comprehensive analysis of a unit step track profile, the active suspension demonstrates its ability to precisely control wheel movements, with the primary aim of enhancing passenger comfort and vehicle drivability. By using the PID controller in conjunction with track sensors, the system is able to continuously assess road conditions, identifying the most effective movements to optimize the driving experience. The PID controller, which incorporates proportional, integral, and derivative components, plays a crucial role in minimizing error, integrating corrections, and adjusting response time as needed. The inerter, an innovative two-terminal mechanical device, is introduced to provide inertia without adding mass to the system, leveraging the difference in acceleration between the terminals. To validate and compare the performance of the active suspension with and without the inerter, detailed simulations are conducted using MATLAB/Simulink. The inclusion of the inerter in the active suspension system, along with the PID controller, aims to enhance overall system effectiveness, even though it entails a slight increase in manufacturing cost. However, the substantial benefits in terms of occupant comfort and vehicle safety fully justify this additional cost. This approach represents a significant advancement in the field of automotive suspension systems, offering an ideal balance between performance, cost, and user satisfaction.

Keywords: PID Controller, Active suspension, Vehicle suspension, Inerter, Vehicle dynamics

1 Introduction

Over the years, it is possible to observe an increase in research and technologies in the field of vehicle suspension [1]. These, in turn, focus primarily on enhancing comfort, performance, and ensuring a smooth and safe ride. In this regard, reducing the vertical acceleration that reaches passengers is a fundamental function of a vehicle's suspension system, thereby ensuring greater comfort during driving. Unlike a passive suspension, which generally consists only of springs and dampers [2], active suspension contains a force actuator that operates as a closed-loop control system. This actuator is a mechanical component integrated into the system and is activated by the controller. The controller uses information from sensors to calculate the amount of energy to be added or dissipated in the system, that is, it calculates the intensity of the force to be applied by the actuator, based on the road profile data provided by the sensors [3]. One advantage of active suspension over passive suspension is that the latter does not respond well to irregular terrain [4]. That is, the performance of passive suspension is influenced by the track profile to which it is subjected [5].

Currently, one of the most widely used controllers in industrial control systems is the PID controller. This PID controller (proportional-integral-derivative) consists of three parameters: proportional gain, integral gain, and derivative gain. Its performance is directly linked to the relative proportion between these three parameters [6]. The inerter, on the other hand, is a mechanical device that produces a force proportional to the relative acceleration between its two terminals, with the proportionality constant called inertance [7], measured in kilograms [kg]. This device, incorporated into the active suspension system, allows for a significant increase in the inertia of a dynamic system without the need to add physical mass. According to Matamoros-Sanchez et al. and He, H. et al. [8, 9], the inclusion of the inerter in an active suspension provided a great improvement in comfort due to reduced vibration, in addition to reducing the maximum force from the actuator. Furthermore, as pointed out by He et al. [10], the use of the inerter can reduce the active force by more than 48 percent.

This study aims to demonstrate that the use of the inerter, combined with active suspension controlled by a

PID, provides a smoother and more stable ride, resulting in a higher level of comfort for passengers. To validate that this combination can significantly improve the suspension's response to different road conditions, reduce vibrations and vertical acceleration transmitted to passengers, and increase the suspension system's efficiency without the need to add physical mass, computational simulations were carried out using MATLAB/Simulink.

This paper advances the state of the art by presenting a novel integration of the inerter device into active suspension systems, controlled by a PID algorithm, to achieve significant improvements in ride comfort and system efficiency. The scientific contribution lies in demonstrating, through computational simulations in MATLAB/Simulink, that the proposed system reduces vertical accelerations and vibrations without the need for additional physical mass, offering a more efficient solution compared to traditional active suspensions. This work contributes on this research area by validating the combined use of an inerter and active control under varying road conditions in the perspective of vehicle vertical dynamics. Future studies should address this combination in more comprehensive models that also encompass lateral and longitudinal dynamics.

2 Mathematical Formulation

In this study, a simplified quarter-car model is investigated. Figure 1 illustrates this model, which is frequently used in the literature due to its simplicity and effectiveness in describing the vertical dynamics of the vehicle [11]. Additionally, the optimized inerter configuration suggested by Shen [12], known as the ISD (Inerter-Spring-Damper) system, is considered. This configuration, in turn, is derived from the structure of a Tuned Inerter Damper, proposed by Lazar et al. [13], originally designed to reduce vibrations caused by seismic excitation in building foundations.

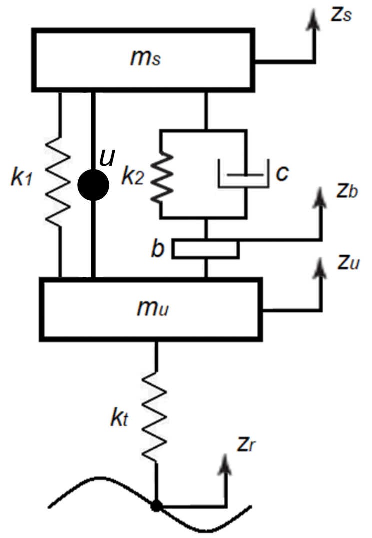

Figure 1. Quarter-Car Model

Initially, the time-domain equations of motion for a quarter-car model with passive suspension and inerter [14] are presented, where:

$$\begin{cases} m_s\ddot{Z}_s + k_1(Z_s - Z_u) + F = 0 \\ m_u\ddot{Z}_u + K_t(Z_u - Z_r) - k_1(Z_s - Z_u) - F = 0 \\ F = b(\ddot{Z}_b - \ddot{Z}_u) = k_2(Z_s - Z_b) + c(\dot{Z}_s - \dot{Z}_b) \end{cases} \tag{1}$$

- m_s : sprung mass;
- m_u : unsprung mass;
- F : resistance force time variation produced by the inerter;
- b : inerter's inertia;
- c : viscous damping coefficient of the damper;
- K_t : equivalent stiffness of the tire;
- $k1$: stiffness of the main suspension spring;

- $k2$: stiffness of the auxiliary spring;
- Zs : displacement time variation of the sprung mass;
- Zb : displacement time variation of the inerter;
- Zu : displacement time variation of the unsprung mass;
- Zr : displacement relative to the road profile time variation.

In the case of an active suspension, its behavior can be simulated computationally using the block diagram presented in Figure 2, where the transfer function G(s) in Eq. 4 is obtained from the equations of motion of the active system (Eq. 2), which include the addition of a control force u acting between the sprung and unsprung masses as can be verified on Figure 1. These equations are given by:

$$\begin{cases} m_s\ddot{Z}_s + k_1(Z_s - Z_u) + F + u = 0 \\ m_u\ddot{Z}_u + K_t(Z_u - Z_r) - k_1(Z_s - Z_u) - F - u = 0 \\ F = b(\ddot{Z}_b - \ddot{Z}_u) = k_2(Z_s - Z_b) + c(\dot{Z}_s - \dot{Z}_b) \end{cases} \tag{2}$$

where u is the control force.

The control force applied by the actuator provided by the PID control algorithm is given by:

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)\,d\tau + T_d\frac{de(t)}{dt}\right) \tag{3}$$

where $e(t)$ is the difference between the system output and the reference input. $\frac{1}{T_i}$ is the integral gain, and T_d is the derivative gain.

The open-loop transfer function relating the output relative to the displacement of the sprung mass to the input road profile is given by:

$$G(S) = \frac{A}{B + C + D + E + F} \tag{4}$$

where:

$$A = (S^3bc + b(k1 + k2)S^2 + Sck1 + k1k2)kt$$
$$B = bc(M_s + M_u)S^5$$
$$C = ((M_s + M_u)k1 + (M_s + M_u)k2 + ktM_s)bS^4$$
$$D = c(bkt + (M_s + M_u)k1 + ktM_s)S^3$$
$$E = (kt(k1 + k2)b + k2((M_s + M_u)k1 + ktM_s))S^2$$
$$F = S(ck1kt) + k1k2kt$$

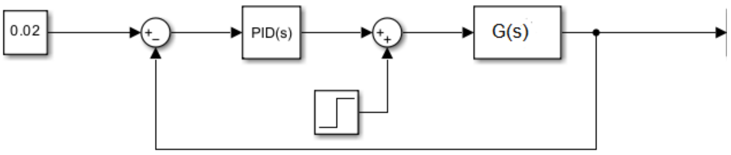

Figure 2. Block Diagram

The simulations and analyses presented in this paper were performed using MATLAB/Simulink student suite, a widely used computational platform for modeling, simulating, and analyzing dynamic systems.

3 Results

The quarter-car model with active suspension and coupled inerter properties are listed on Table 1 [12] [15]. Table 2 presents the gains of the PID controller obtained through a tuning algorithm embedded in Matlab/Simulink.

Table 1. Parameters Used

Properties	**Symbol**	**Value**	**Unit**
Sprung Mass	m_s	320	[kg]
Unsprung Mass	m_u	45	[kg]
Spring Stiffness	k_s	22000	[N/m]
Tire Stiffness	k_u	190000	[N/m]
Inerter Stiffness	k_2	10000	[N/m]
Damping Coefficient	c	500	[Ns/m]
Amplitude	Amp	0.02	[m]

Source: Shen, 2016 [12]; Melo and Avila, 2018 [15]

Table 2. PID Controller Parameters

Parameter	Designation	Value
Proportional Gain	K_p	6
Integral Gain	K_i	0.18
Derivative Gain	K_d	0.02

Figure 3 shows the time evolution of the displacement of the passive suspension with and without an inerter when subjected to a step road profile. Specifically, the presence of the inerter in the suspension system contributes to a reduction in response peaks. Although the overshoot is slightly higher in the system with the inerter, this additional component helps minimize steady-state errors, meaning that the system can return to its equilibrium state more effectively after the initial disturbance.

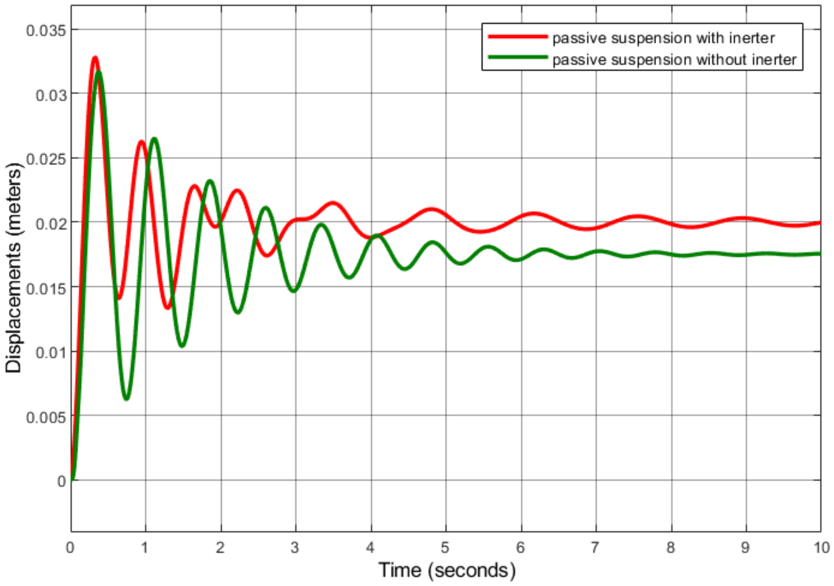

Figure 3. Displacement time history of the sprung mass considering passive suspension with and without inerter.

In Fig. 4, the displacement time evolution of the sprung mass is compared for two active suspension systems: the system with an inerter, represented by the blue line, and the system without an inerter, represented by the orange line. In the active system without an inerter, a significant overshoot and oscillatory behavior are observed before the system stabilizes. On the other hand, in the active system with an inerter, there is a significant reduction in overshoot and nearly instantaneous stabilization. This performance highlights the effectiveness of the inerter in improving the dynamic response of the suspension system, as the inclusion of the inerter not only reduces response peaks but also accelerates system stabilization, providing a smoother and more controlled transition after the application of external forces. In the active suspension system using a PID controller and an inerter, the overshoot is much smaller and the stabilization occurs almost immediately.

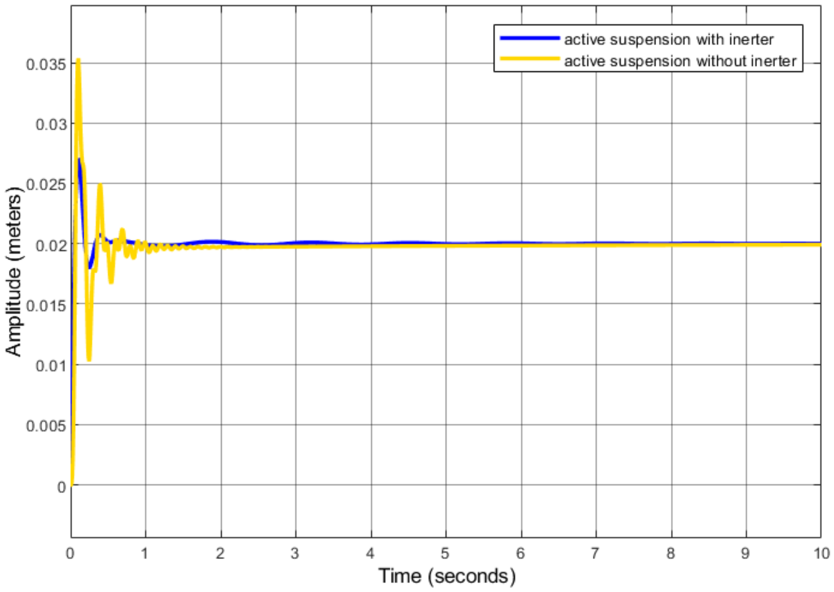

Figure 4. Displacement time history of the sprung mass considering active suspension with and without inerter.

Figure 5 presents the sprung mass displacement time history, comparing passive versus active suspension systems when coupled with an inerter device. The blue line represents the active system with an inerter, while the green line shows the passive system without an inerter. It can be observed that the incorporation of the actuator reduces the overshoot of the response and virtually eliminates oscillatory behavior, with the response quickly converging to the proposed reference value.

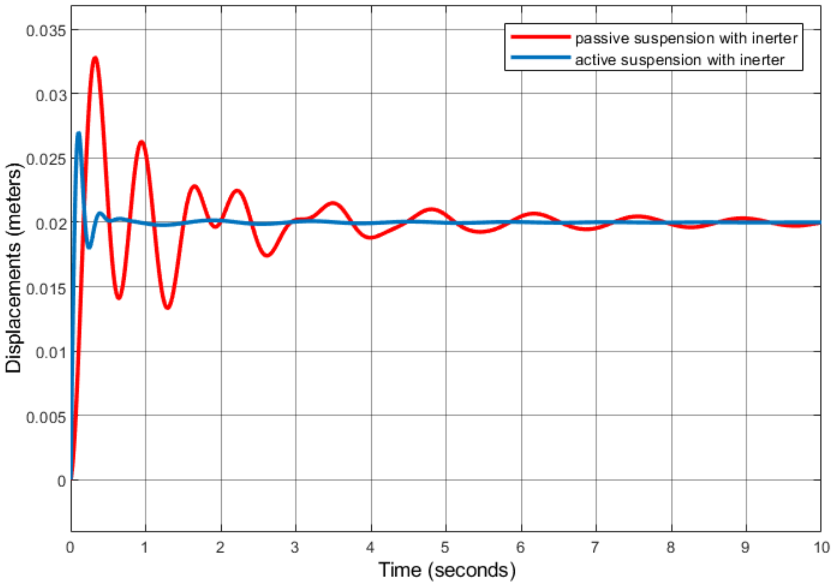

Figure 5. Displacement time history of the sprung mass considering active and passive suspension with an inerter attached.

Figure 6 presents the control force u(t) (Eq.3) considering the system with and without the inerter device. It can be observed that initially, the actuator applies a force of 0.36 N. It is noticeable that the system with an inerter, represented by the blue line, exhibits both a smaller force overshoot and faster stabilization compared to the system without an inerter. This observation is crucial as it highlights the effectiveness of the inerter in improving the control system's response, indicating greater efficiency in disturbance mitigation and maintaining system stability.

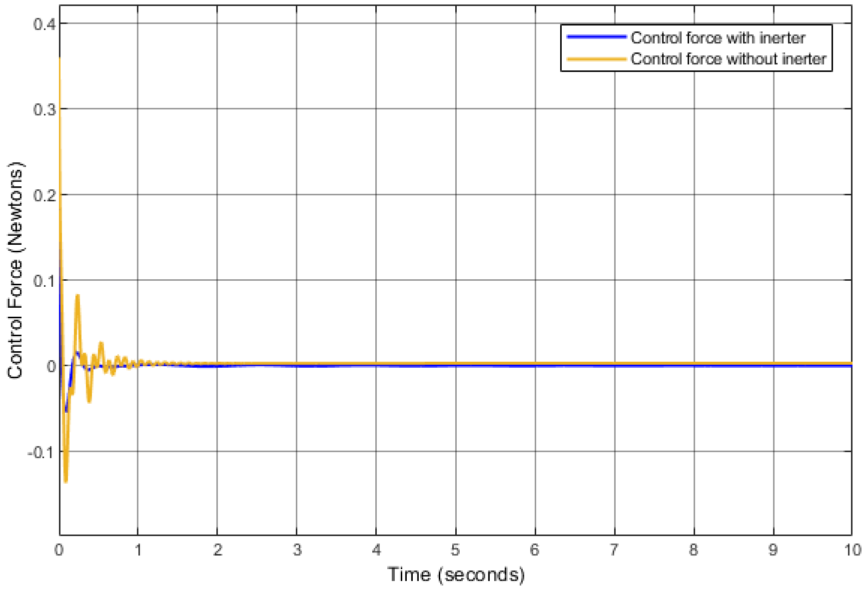

Figure 6. Time history control force with and without an inerter

4 Conclusion

In this study, an inerter was incorporated into a PID-controlled active suspension system, leading to significant improvements in suspension performance. The inerter proved to be a key component in optimizing dynamic responses, effectively reducing vibrations and enhancing passenger comfort. This advancement represents a promising contribution to the evolution of vehicle suspensions, offering a practical and effective solution to current challenges in the automotive industry.

It is important to note, however, that the model used in this study was limited to representing only vertical dynamics. For a more comprehensive validation of this approach, further studies that consider multiple degrees of freedom are required, allowing for a thorough analysis of lateral and longitudinal dynamics as well.

The results obtained underscore the effectiveness of combining a PID controller with an inerter in an active vehicle suspension system. Detailed simulations demonstrated that integrating the inerter significantly enhances suspension performance, particularly in terms of passenger comfort.

An analysis of different suspension configurations revealed that active suspension systems with an inerter exhibit optimized responses to terrain irregularities, characterized by shorter stabilization times and a considerable reduction in oscillation peaks. Compared to traditional passive suspensions, the PID-controlled active suspension with an inerter offers a smoother and more comfortable driving experience.

Furthermore, the inerter's ability to increase system inertia without adding physical mass stands out as a critical advantage. This feature allows for optimized suspension performance without compromising the vehicle's overall weight, striking an essential balance between energy efficiency and cost-effectiveness in modern vehicles. The inerter, therefore, emerges as a crucial component in the pursuit of more advanced and efficient suspension systems.

In conclusion, this study suggests that the use of an inerter in conjunction with active suspension offers a viable and effective solution to current automotive challenges. However, it is important to emphasize that the present model was limited to vertical dynamics. Future studies should focus on more complex models that incorporate multiple degrees of freedom, providing a more accurate and comprehensive analysis of vehicle behavior across a wider range of operating conditions.

Acknowledgements. The authors acknowledge the financial support from the University of Brasília (UnB) and the Federal District Research Support Foundation (FAPDF).

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