Assessment of the human comfort of pedestrian footbridges utilising probabilistic response spectra

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Abstract. Nowadays, the design of pedestrian footbridges is associated to light weight structures with low natural frequencies and low structural damping rates. These facts have generated slender footbridges, sensitive to human dynamic excitations, and consequently changed the serviceability limit states associated to the design. Thus, the current design codes and technical guides recommend the use of deterministic models to assess the dynamic structural behaviour of footbridges. On the other hand, the pedestrian walking is related to a stochastic phenomenon and the dynamic force generated at each step depends of the weight, the step frequency and the step length of each pedestrian. This way, this investigation aims to develop a probabilistic approach to assess the steel and steel-concrete composite footbridges dynamic behaviour, based on the use of design response spectra. To do this, the developed analysis methodology has considered the stochastic nature of the pedestrian's walking, in order to evaluating the structural response having in mind excessive vibrations that may cause human discomfort. Based on the use of probabilistic methods, it becomes possible to determine the probability of the footbridge's peak acceleration values exceeding or not the human comfort criteria. The results obtained in this research work reveal that the peak acceleration values calculated through the deterministic methods can be overestimated in project situations.

Keywords: pedestrian footbridges, dynamic structural analysis, human comfort assessment.

1 Introduction

The pedestrian footbridges are more and more becoming the modern landmarks of urban areas. This way, the designers seem to continuously move the safety border, aiming to increase slenderness and lightness of the pedestrian footbridges. However, more and more footbridges are carried out as light weight structures with low frequencies and low damping [1-5]. These facts have generated very slender footbridges, sensitive to dynamic excitations, and consequently changed the serviceability limit states associated to the design.

The current design codes and project guides recommend the use of deterministic models to assess the dynamic behaviour of footbridges [2-3]. On the other hand, the effect of the uncertainties in mass, stiffness and damping of the investigated structure are relevant and lead to uncertainties on the values of the footbridges natural frequencies. Nevertheless, the human walking is a stochastic phenomenon and the dynamic force generated at each step depends of the weight, the step frequency and the step length of each pedestrian. This way, opposite to the deterministic approach, this work aims to present a probabilistic model to assess the dynamic behaviour of footbridges, when subjected to pedestrian walking, considering the stochastic nature of the human walking [1].

The probabilistic dynamic analysis methodology considered the uncertainty related to the pedestrian weight [W (N)], pedestrian step frequency [f_p (Hz)], and pedestrian step length [l_p (m)]. The proposed probabilistic model takes into consideration both inter-subject [Inter] and intra-subject [Intra] step variability [1]. These uncertainties in pedestrian weight, step frequency, step length, inter-subject and intra-subject step variability were modeled as random variables and included in the Monte Carlo simulations. The proposed probabilistic methodology is used to assess the dynamic response of steel and steel-concrete composite footbridges located at Rio de Janeiro/RJ, Brazil [1]. The results are presented and discussed based on design response spectra that allow the assessment of the vibration serviceability of footbridges. These results have demonstrated that the peak accelerations calculated using the deterministic methods [2-3] may be overestimated.

2 Maximum accelerations calculated by analytic methods

The human walking generates periodic loadings on the footbridge slabs and the two feet move alternately from one position to another and do not simultaneously leave the surface. The dynamic part of the load-time history of a single pedestrian can be modeled based on the Fourier series expansion that represents a combination of sinusoidal forces, whose frequencies are multiples of the step frequency, as presented in eq. (1).

$$F(t) = P\left[1 + \sum \alpha_i \cos(2\pi i\, f_p\, t + \phi_i\right] \tag{1}$$

Where:

$F(t)$: dynamic loading function (N);
P : pedestrian weight (N);
α_i : dynamic coefficient of the harmonic force;
i : harmonic multiple (1, 2, 3, etc.);
f_p : pedestrian step frequency (Hz);
t : time (s);
ϕ_i : phase angle to the harmonic i (rad).

In this research work the analytical solution proposed by Živanović [4] was considered for investigate the pedestrian footbridges peak acceleration, having in mind that for this purpose the first vibration mode of the structure can be represented by a half sine wave function sen ($\pi v_p t/L$). This way, the equation that governs the first vibration mode of the investigated footbridge is presented in eq. (2).

$$a(t) + 2\xi\omega\, v(t) + \omega^2 u(t) = \frac{1}{m}\alpha P\, sen(2\pi f_p t)\, sen(\frac{\pi v_p}{L} t) \tag{2}$$

Where:

$a(t)$: peak acceleration (m/s²);
ξ : damping coefficient;
m : footbridge modal mass (kg);
α : dynamic coefficient;
P : pedestrian weight (N);
f_p : pedestrian step frequency (Hz);
l_p : pedestrian step length (m);
v_p : pedestrian step velocity [$v_p = f_p\, l_p$] (m/s);
L : footbridge span (m).

3 Probabilistic analysis methodology: response spectra

In this research work, the proposed probabilistic dynamic analysis methodology assumes that the independent random variables associated to the pedestrian weight [W (N)], pedestrian step frequency [f_p (Hz)], and pedestrian step length [l_p (m)] follow a normal distribution [1]. The developed analysis methodology was computationally implemented and the generated design response spectra allow structural engineers the vibration serviceability assessment of pedestrian footbridges.

Hausdorff et al. [5] have demonstrated that the step frequency is not constant during walking. Each pedestrian walks freely with a preferential step frequency. However, at each step, the frequency varies because the pedestrian cannot repeat the same motion with the same frequency. This step-by-step variation is called the intra-subject step variability, whereas the step frequency variation between different people represents the inter-subject step variability.

In this study, the proposed probabilistic model takes into consideration both inter-subject [Inter] and intra-subject [Intra] step variability [1]. Table 1 shows the mean values (μ) and respective standard deviations (σ) of the random variables adopted in the Monte Carlo simulations [2,4,6,7] to assess the dynamic structural behaviour of simply supported footbridges.

Table 1. Statistical parameters used to simulate the pedestrian walking

Variable	Units	Mean *(μ)*	Standard Deviation (*σ*)	References
Weight	N	727	145	Ingólfsson [6]
f_p	Hz	2,00	0,2	Bachmann [2]
l_p	m	0,71	0,071	Zivanovic [4]
Intra	Hz	-	3,0% de μ	Brownjohn [7]

Thus, for each pedestrian, a preferential step frequency was randomly generated. At each step period, the step frequency varied by 3% around the preferential pedestrian's step frequency to account the intra-subject step variability [7]. The dynamic load factors were calculated using the mathematical formulations proposed by Rainer et al. [8] and Young apud Hauksson [9].

In sequence, Fig. 1 presents the developed analysis methodology in details. It must be emphasized that there are significant differences between the analysis methodologies I [4] and II [1]. In fact, analysing the Fig. 1 it is possible to conclude that when the methodology I (M-I) is utilised in this investigation, the effect of the intravariability is considered only to the calculation of the dynamic coefficients (α_i) [Step 3: see eq. (1)], and used to determine the peak accelerations [Step 4: see Fig. 1]. On the other hand, when the analysis methodology II (M-II) is investigated, the intravariability is considered on the dynamic coefficient (α_i) calculations and on the sinusoidal harmonic function sen ($2\pi f_p$) [Step 3 and Step 4: see eq. (1)]. After that the peak acceleration percentile 95% (a95%) is determined, based on Step 5, as illustrated in Fig. 1.

4 Investigated pedestrian footbridges

In this research work two pedestrian footbridges located at the Rio de Janeiro city (Rio de Janeiro/RJ, Brazil) were investigated, aiming to compare the results calculated based on the use of the proposed analysis methodologies I and II (see Fig. 1), and the results determined based on experimental tests developed on the analysed pedestrian footbridges [10,11].

The first analysed structural model (FB-I: m = 10,674.24 kg; f_{01} = 3.83 Hz; ξ = 1,5% [10]) is located at the Osvaldo Aranha Street, route of great importance and very heavy traffic of vehicles along the day, next to the new Maracanã Stadium, in the city of Rio de Janeiro/RJ, Brazil [10]. The footbridge is made of a steel concrete composite structural system, and presents two spans with length equal to 29.5m and 24m, respectively, and a total length of 53.5m [10], as presented in Fig. 2a.

The second analysed footbridge (FB-II: m = 34,490 kg; f_{01} = 1.85 Hz; ξ = 0.23% [11]) is associated to a simply-supported inverted-queen-post-truss steel footbridge spanning 68.6 m, with all beams and columns made of steel [11]. The deck floor is made of precast concrete slabs 7 cm thick, and the two parallel trusses are braced at their top and bottom. The cross-sections of the truss members, the reinforcement beams and the primary members of the bracing systems are made of steel with double-U profiles welded at theirs flanges to form a box section [11], as illustrated in Fig. 2b.

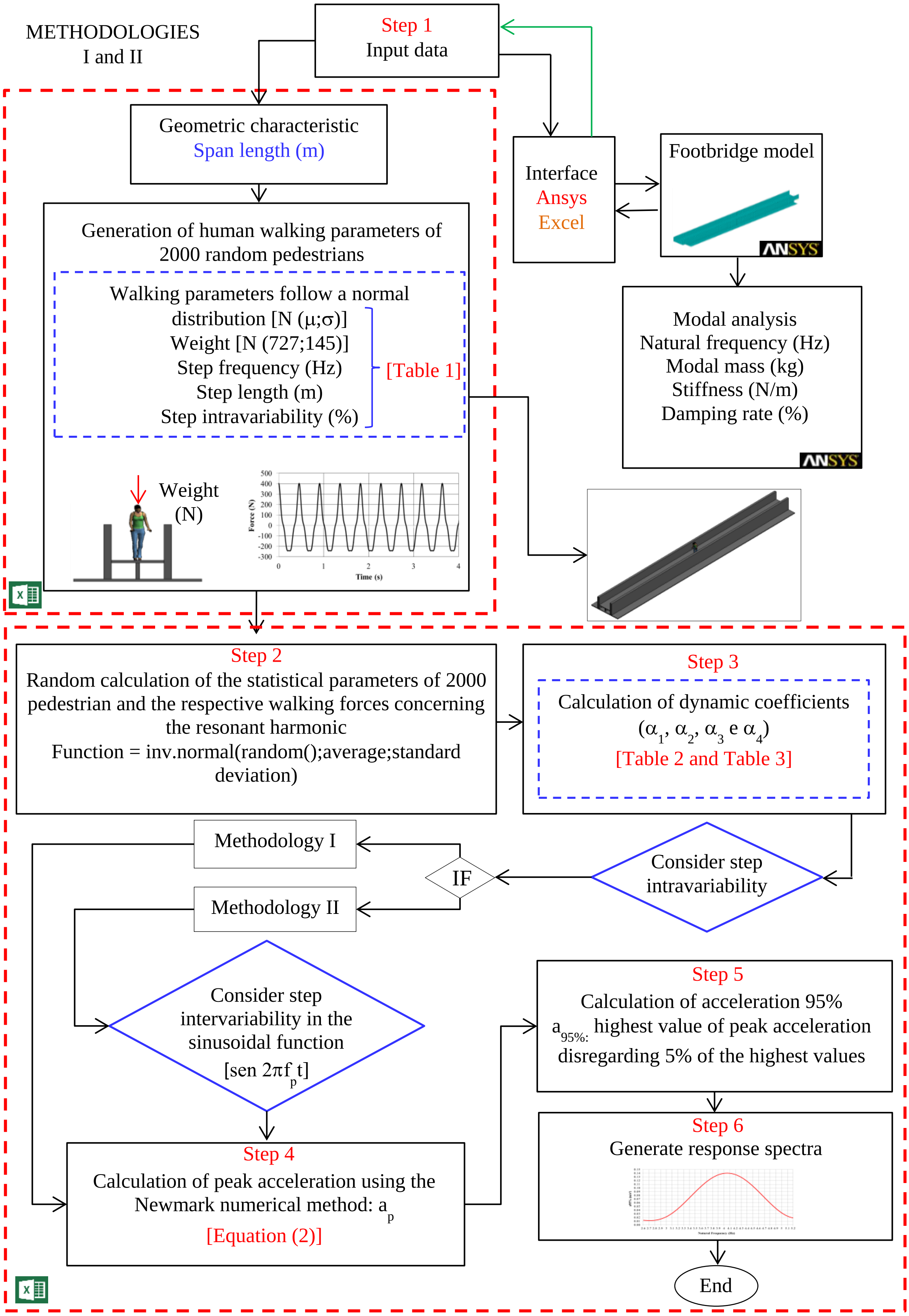


Figure 1. Proposed analysis methodology to generate the project response spectra

a) Steel-concrete composite footbridge: FB-I b) Steel footbridge: FB-II

Figure 2. Investigated pedestrian footbridges (FB-I: L = 53.5 m; FB-II: L = 68.6)

5 Probabilistic design response spectra

Initially, it is worth highlighting that the results obtained based on the proposed probabilistic analysis methodology was calculated from 2,000 analyses associated with pedestrian crossings. Therefore, the probabilistic response spectra were constructed based on an extensive parametric study, based on the calculation of maximum acceleration values (peak accelerations), corresponding to each pedestrian crossing, varying the fundamental frequency in the vertical transverse direction in the range of 0.5 Hz to 10 Hz of the reference footbridges, FBI and FB-II (see Figs. 2a and 2b) and the structural damping (ξ) of 0.5% to 2.0%.

The peak acceleration values were determined using analysis methodologies I and II (M-I [4] and M-II [1]), see Fig. 1. The calculation of the dynamic coefficients (α_i), see eq. (1), was considered based on the use of the mathematical formulations proposed by Rainer et al. [8] and Young apud Hauksson [9], presented in Tab. 3 and Tab. 4. The 95% percentile values ($a_{95\%}$) were determined from the calculated probabilistic peak accelerations. The 95% percentile corresponds to the value with a 95% probability of occurrence, that is, by calculating the maximum structure dynamic response 100 times, it is expected that in 95 cases the peak acceleration values will be equal to or lower than the representative value of the 95% percentile ($a_{95\%}$).

Table 2. Dynamic coefficients proposed by Rainer et al. [8]: footbridge vertical direction

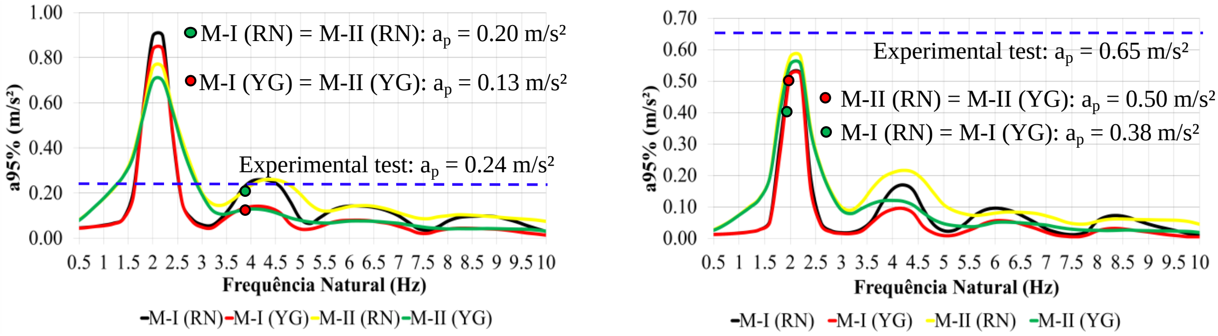
Dynamic coefficients (α_i)
$\alpha_1 = -0.22169f_p^3 + 1.11946\ f_p^2 - 1.44748\ f_p + 0.5967$
$\alpha_2 = -0.012037(2\ f_p)^3 + 0.1494(2\ f_p)^2 - 0.53146(2\ f_p) + 0.6285$
$\alpha_3 = 0.00009068(3f_p)^5 - 0.0021066(3fp)^4 + 0.018364(3f_p)^3 - 0.077278(3f_p)^2 + 0.17593(3f_p) - 0.1477$
$\alpha_4 = 0.00051715(4\ f_p)^4 - 0.014388(4\ f_p)^3 + 0.14562(4\ f_p)^2 - 0.6018469$

Table 3. Dynamic coefficients proposed by Young apud Hauksson [9]: footbridge vertical direction

Dynamic coefficients (α_i)
$\alpha_1 = 0.37\ (f_p - 0.92)$
$\alpha_2 = 0.054 + 0.0044f_p$
$\alpha_3 = 0.026 + 0.0050f_p$
$\alpha_4 = 0.010 + 0.0051f_p$

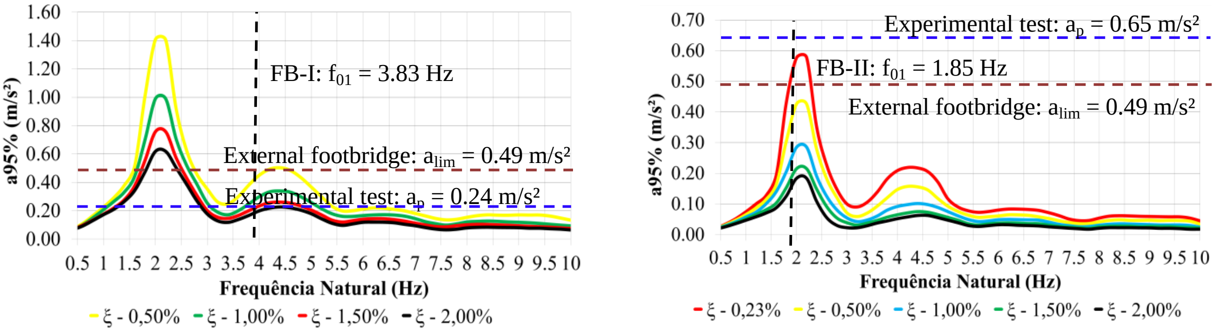
6 Analysis and discussion results

Having in mind the assessment of the 95% percentile values ($a_{95\%}$), determined based on the use of methodologies I and II (M-I [4] and M-II [1]) using the mathematical formulations proposed by Rainer et al. [8] and Young apud Hauksson [9], the experimental results are presented and delimited by dashed lines (see Figs. 3a and 3b). Following the investigation, the results comparison is presented considering the damping coefficient (ξ) variation from 0.5% to 2.0%, as well as the analysis of the human comfort criterion recommended by design standards for external footbridges ($a_{lim} = 0.49$ m/s² [3]) (see Figs. 4a and 4b).



(a) FB-I: L = 53.5 m; ξ = 1.50% (b) FB-II: L = 68.6 m; ξ = 0.23%

Figure 3. Response spectrum for peak accelerations



(a) L = 53.5 m; ξ = 0.5% to 2.0% (b) L = 68.6 m; ξ = 0.5% to 2.0%

Figure 4. Response spectrum for peak accelerations

Based on the results presented in Figs. 3a and 3b, it is possible to conclude that the 95% percentile values ($a_{95\%}$), determined using the proposed design response spectra (M-I [4] and M-II [1]) are lower when compared to the experimental tests peak accelerations.

It must be emphasized that when the FB-I (Harmonic resonant: 2nd; see Figure 2a) is considered in the investigation, the $a_{95\%}$ values determined based on the use of the design spectra were the same for the two analysis methodologies (M-I [4] and M-II [1]), but accelerations presented different values when the dynamic coefficients mathematical formulation [8,9] was modified, see Fig. 3a.

On the other hand, when analysing the FB-II (Harmonic resonant: 1st; see Fig. 2b) the determined $a_{95\%}$ values considering the M-II [1] were greater than those obtained based on the use of M-I [4], to the both dynamic coefficients mathematical formulation [8,9]. These observations can be verified in Figs. 3a and 3b, looking at design spectra behaviour, when assessing the first and second harmonic.

Having in mind the structural damping variation related to the investigated pedestrian footbridges (see Figs. 4a and 4b), when the reference footbridge FB-I (Harmonic resonant: 2nd; see Fig. 2a) was investigated it is possible to observe that the $a_{95\%}$ values are higher than the experimental accelerations (a_{exp} = 0.24 m/s²), when the damping rates of 0.5% and 1.0% (ξ = 0.5% and 1.0%) are considered. On the other hand, all the $a_{95\%}$ values are lower than the design standard limit acceleration (External footbridges: a_{lim} = 0.49m/s² [3]) to all investigated damping rates (ξ = 0.5% to 2.0%), as illustrated in Fig. 4a.

Considering the design response spectrum of the reference footbridge FB-II (Harmonic resonant: 1st; see Figure 2b) illustrated in Fig. 4b, it is noted that the $a_{95\%}$ values are lower than the experimental accelerations (a_{exp} = 0.65 m/s²) to all investigated structural damping rates (ξ = 0.23% to 2.0%). It was verified that the $a_{95\%}$ values slightly surpass the design standard limit acceleration (a_{lim} = 0.49 m/s² [3]) to the damping rate ξ = 0.23% (damping experimental value [11]). However, all the $a_{95\%}$ values are lower the recommended design limit to external footbridges when the structural damping rates are equal to 0.5%, 1.0%, 1.5% and 2.0% (ξ = 0.5%, 1.0%, 1.5% and 2.0%), as presented in Fig. 4b.

7 Conclusions

In this research work, design spectra response are proposed, based on development of a probabilistic analysis methodology, aiming to assess the dynamic structural behaviour of pedestrian footbridges when subjected to human induced loadings considering the stochastic nature of the walking. This way, the following conclusions can be drawn from the results presented in this investigation:

1. The proposed probabilistic design response spectra demonstrated that the dynamic response of simply supported footbridges [95% percentile ($a_{95\%}$)], when subjected to pedestrian walking, can be modified based on the effect of the step intravariability, as well as the randomness of the other variables used in the probabilistic analysis methodology.

2. Considering a quantitative analysis, based on the 2,000 pedestrians crossing the investigated pedestrian footbridges, the results calculated through the proposed probabilistic analysis methodology indicates that the first studied footbridge (FB-I) attends the human comfort for all investigated damping rates (ξ = 0.5%, to 2.0%). On the other hand, when the second footbridge (FB-II) was analysed, it was concluded that the $a_{95\%}$ value slightly exceeds the design standard recommended limit when the experimental damping rate of ξ = 0.23% is considered, and is lower than this limit when the structural damping rates are equal to 0.5% to 2.0% (ξ = 0.5%, to 2.0%).

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