Determination of deflection in composite slabs considering concrete creep

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Abstract. The analysis of the behavior and resistance of composite steel and concrete slabs covers several parameters. Among these parameters, the consideration of concrete creep can be an important influence on checking the deflection. Generally, when checking the serviceability limit state, regarding deflections, technical standards recommend that the moment of inertia of the composite section be given by the simple average of the moments of inertia of the uncracked and cracked sections. However, experimental investigations have shown that this procedure inadequately characterizes the behavior of the composite slab, resulting in an underestimated effective moment of inertia and lower deflection. Therefore, using expressions of the effective moments of inertia in composite slabs suggested in the literature, this work aims to present the determination of the deflection by evaluating the consideration of concrete creep during the loading phase until collapse.

Keywords: Composite slab; Deflection; Concrete creep.

1. Introduction

The composite structure is a system that combines and takes advantage of the best characteristics and specific properties of each of the associated materials, both steel and concrete. While steel provides high strength and ductility, concrete provide resistance to compression. This combination optimizes structural performance and durability. Thus, the Brazilian literature presents ABNT NBR 8800:2008 [1] as the standard that regulates the design of composite slabs, including serviceability limit state analysis, for example the deflection, as one of the parameters. However, the standard does not specify the effective moment of inertia to be used when calculating displacement. International technical literature therefore recommends using the simple average of the moments of inertia of the uncracked section and the cracked section of the composite system for the calculation. However, experimental studies show that the proposed method inadequately characterizes the behavior of the composite structure, resulting in an overestimation of the effective moment of inertia and non-conservative deflections. Furthermore, it is noticeable that this method does not consider certain parameters, such as creep deformation (deformation as a function of time), which is addressed in ABNT NBR 6118:2023[2].

The aim of this study is therefore to evaluate the behavior and mid-span deflection of steel-concrete composite slabs, considering the effect of concrete creep. To this end, the results of a laboratory test program carried out by Costa [3] using the Deck-60 steel form were analyzed. The program involved tests on twelve specimens of two-sided composite slabs subjected to bending, in accordance with EN 1994-1-1:2004 [4].

Based on these results, this paper proposes to consider the effect of concrete creep in the equation used to calculate the displacement (deflection) of composite steel and concrete slabs. The equation used to determine the deflection considers different expressions for determining the effective moment of inertia in composite slabs, including the simple average of the uncracked and cracked section, the Branson formulation cited by Tenhovuori

[5], Souza Neto [6], ANSI/ASCE 3-91:1992 [7] and EN 1994-1-1:2004 [4], as well as three proposals developed by Costa [3]. Finally, it is hoped that by analyzing the load *versus* deflection relationship, theoretical results will be obtained that are more accurate and closer to the experimental results of this structural system.

2. Tests and specimen characterization

The test program, carried out by Costa [3], was conducted at the Laboratory for Experimental Analysis of Structures (LAEES) of the Department of Structural Engineering (DEES) at the Federal University of Minas Gerais (UFMG). The aim of the program was to reproduce, as accurately as possible, the practical conditions under which slabs are installed in buildings, allowing the composite system of steel and concrete and its parameters to be analyzed. To do this, a series of twelve single-span specimens were used, with the typical cross-section of the Deck-60 steel form (Fig. 1). The deck forms were made from ZAR 280 and ZAR 345 steel, with two thicknesses (Tab. 1), measuring 2600 mm in length and 860 mm in nominal width.

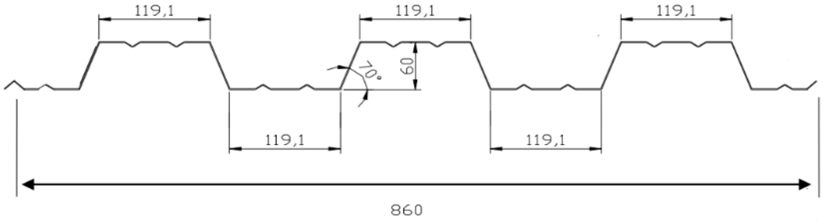

Figure 1. Typical deck-60 cross-section (dimensions in mm). Source: Costa [3]

Table 1. Dimensions and geometric properties of Deck-60.

$t\ (mm)$	$te\ (mm)$	$b\ (mm)$	$h_F(mm)$	$A'_{F,ef}\ (mm^2)$	$y_{cg}\ (mm)$	$I'_{sf}\ (mm^4)$	$ppF\ (Kg/m^2)$
0,80	0,76	860	60	304	30	194.664,15	9,05
0,95	0,91	860	60	364	30	233.084,94	10,83

where, t is the nominal thickness of the steel sheeting; te is the thickness of the steel sheeting without the galvanizing layer; b is the width of the steel deck; h_F is the total height of the steel decking; $A'_{F,ef}$ is the effective area of the deck module; y_{cg} is the distance from the center of gravity to the lower external face of the deck; I'_{sf} is the moment of inertia of the deck module; and pp_F is the self-weight of the steel decking.

For a better analysis, the specimens were divided into two groups according to nominal thickness, with six specimens each. In this form, considering the mechanical properties of the steel used to make the decks, the average yield strength was 340 MPa for the 80 mm group and 390 MPa for the 95 mm group. The average tensile strength was 450 MPa for the 80 mm group and 490 MPa for the 95 mm group. For the longitudinal modulus of elasticity of the steel, the value adopted was 200 GPa.

The concrete was ordered from a plant with a characteristic compressive strength equal to or greater than 20 MPa, as specified by ABNT NBR 5739:2018 [8]. The secant modulus of elasticity of the concrete (E_{cs}), was determined in accordance with ABNT NBR 6118:2023 [2]. Table 2 shows the values of these strengths according to the effective age of the concrete tested.

Table 2. Nominal characteristics and mechanical properties of the test specimens.

Specimen	Deck thickness (t) (mm)	h_t (mm)	L_s (mm)	Effective age of concrete (T_∞) (days)	f_{ck} (MPa)	E_{cs} (MPa)
01A	0,80	110	800	56	25	23657
01B	0,80	110	800	38	17	19626
01C	0,80	110	800	65	19	21748
02A	0,80	140	450	81	21	21917
02B	0,80	140	450	113	25	23848
02C	0,80	140	450	77	20	21234
03A	0,95	110	800	31	18	20363
03B	0,95	110	800	58	21	21813
03C	0,95	110	800	52	19	20584
04A	0,95	140	450	71	20	21446
04B	0,95	140	450	87	21	21813
04C	0,95	140	450	120	25	23800

The test was conducted using a hydraulic actuator mounted on a reaction gantry. The total height of the slabs (h_t) and the shear span (L_s) were varied according to each specimen to identify the set of parameters that most influence the structural behavior and resistance of the composite system. The load was applied in a gradual and progressive form until the ultimate limit state (collapse) of each specimen was reached. The mid-span deflection was recorded using two displacement transducers (DTs), as shown in Fig. 2. The DTs were placed symmetrically 20 cm from the longitudinal edges of the slab, and the values recorded were taken as the simple average of the two readings.

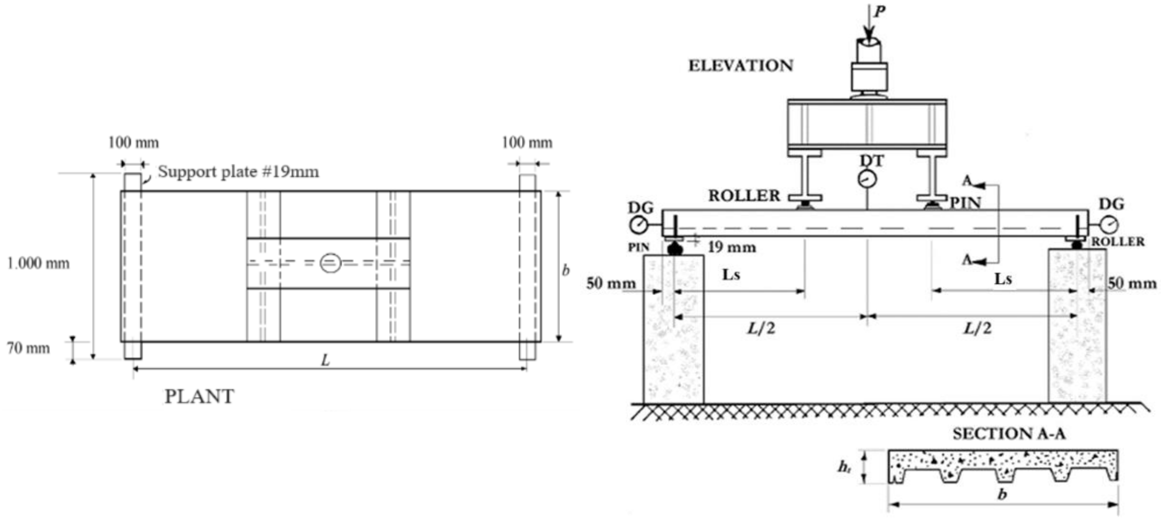

Figure 2 - Load application. Source: Costa [3]

3. Serviceability limit state

ABNT NBR 8800:2008 [1] addresses the service limit state as one of the parameters to be analyzed in structures. This parameter ensures adequate aesthetic conditions, good use of the structure and satisfactory comfort for users. The standard establishes two service limit states: the start of concrete cracking and vertical displacement (service deflection). In this context, Annex Q of the standard recommends that the serviceability limit state for deflection should not exceed a relation of de $L/350$, where de L is the theoretical span of the slab in the direction of the ribs.

Similarly, to obtain the deflection closest to the experimental result, for the simply supported composite slab, with two concentrated loads equidistant from the support, the calculation is made using the immediate deflection equation given by:

$$\delta = \frac{P_u \cdot L_s \cdot (3\ L^2 - 4\ L_s{}^2)}{2 \cdot 24 \cdot E_C \cdot I_{Im}} \tag{1}$$

where, P_u is the applied load; L_s shear span of the composite slab; L is the span between supports, E_C is the secant modulus of elasticity of the concrete; I_{Im} is the effective moment of inertia in composite slabs.

4. Expressions for calculating the effective moment of inertia

Authors such as Schuster [9] and Johnson [10], together with technical standards such as ANSI/ASCE 3-91 [7] and CSSBI S3 [11], recommend that the calculation of service deflections should consider the effective moment of inertia of the composite section (I_{Im}). This moment of inertia is determined by the simple average of the moments of inertia of the uncracked section, I_{cf}, and the cracked section, I_{II}.

$$I_{Im} = \frac{I_{cf} + I_{II}}{2} \tag{2}$$

The calculation of the moments of inertia of the uncracked and cracked section is based on the study by Costa [3], which considers the typical modulus of the cross-section of the composite slab, as illustrated in Fig. 3. The moment of inertia of the uncracked section (I_{cf}) is expressed by the following equation:

$$I_{cf} = n\left[\frac{b_n \cdot t_c{}^3}{12} + b_n \cdot t_c \cdot \left(y_{cf} - \frac{tc}{2}\right)^2 + I_T + A_T(h_t - y_{cf} - y_T)^2 + \alpha_e \cdot I'_{sf} + \alpha_e \cdot A'_{F,ef} \cdot (d_f - y_{cf})^2\right] \tag{3}$$

where n corresponds to the number of typical modules of the steel deck; b_n, t_c , h_t and d_f, are indicated in Fig. 3;

I_T is the moment of inertia of the trapezoidal section in relation to its center of gravity (GG_T); A_T is the area of the trapezoidal section of the core; α_e is the modular relationship between the modulus of elasticity of steel and concrete; I'_{sf} is the moment of inertia of the typical modulus of the steel decking and $A'_{F,ef}$ is the effective area of the typical modulus of the deck.

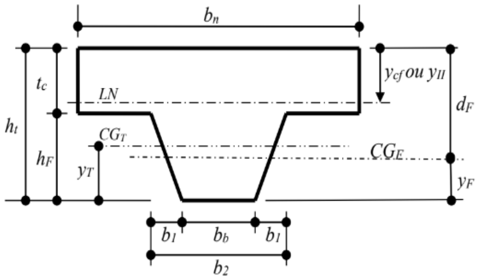

Figure 3: Typical cross-sectional modulus of the composite slab. Source: Costa [3].

The moment of inertia of the cracked section (I_{II}) is expressed by the following equation:

$$I_{II} = n \cdot \left[\frac{b_n \cdot y_{II}{}^3}{3} + \alpha_e \cdot I'_{sf} + \alpha_e \cdot A'_{F,ef} \cdot (d_f - y_{II})^2\right] \tag{4}$$

where, the LN (neutral axis) position of the cracked cross-section (Fig. 3); y_{II}, is the distance measured from the top end of the cross-section.

Another expression for calculating the effective moment of inertia was developed by Tenhovuori [5], who recommends D. E. Branson's formulation for structures made of reinforced concrete.

$$I_{lm} = I_{cf} \left(\frac{Mr}{Ma}\right)^3 + I_{II} \cdot \left[1 - \left(\frac{Mr}{Ma}\right)^3\right] \leq I_{cf} \tag{5}$$

where M_a is the acting bending moment that depends on the test performed and M_r is the cracking moment.

Souza Neto [6], on the other hand, affirms that by using the Branson equation, the values of the stiffness of the effective moment of inertia remain high compared to the real values obtained experimentally. He therefore suggests another reformulation of the equation for slabs without end anchors:

$$I_{lm} = I_{cf} \left(\frac{Mr}{Ma}\right)^3 + \frac{I_{II}}{20} \cdot \left[1 - \left(\frac{Mr}{Ma}\right)^3\right] \leq I_{cf} \tag{6}$$

The ANSI/ASCE 3-91:1992 [7] adopts the method proposed by Lamport and Porter [12], for $h_F = 76mm$. When $M_a < M_r$, it is recommended to use the relation between the stiffness coefficient and the moment of inertia of the uncracked section.

$$I_{lm} = \alpha \cdot I_{cf} \cdot \left(\frac{Mr}{Ma}\right)^{1,3} + I_D \cdot \left[1 - \left(\frac{Mr}{Ma}\right)^{1,3}\right] \leq \alpha \cdot I_{cf} \tag{7}$$

where α is the stiffness coefficient and I_D is the moment of inertia of the steel sheeting.

The EN 1994-1-1:2004 [4] adopts that, for reinforced concrete structural elements subjected to bending, the effective moment of inertia can be obtained according to eq. (8). When $M_a < M_r$, it is recommended to use the moment of inertia of the uncracked section.

$$I_{lm} = I_{cf} \cdot \left(\frac{Mr}{Ma}\right)^2 + I_{II} \cdot \left[1 - \left(\frac{Mr}{Ma}\right)^2\right] \leq I_{cf} \tag{8}$$

In accordance with Costa [3, 13], he also affirms that the effective moment of inertia is still overestimated by all the equations presented, except for Souza Neto's proposal [6], which reduces the contribution of the cracked section. In this manner, he proposes three formulations to obtain the effective moment of inertia of the composite section, with the aim of bringing the theoretical values closer to the experimental results.

Proposal 1:
$$I_{lm} = I_{cf} \cdot \left(\frac{Mr}{Ma}\right)^2 \leq I_{cf} \tag{9}$$

Proposal 2:
$$I_{lm} = I_{cf} \cdot \left(\frac{Mr}{Ma}\right)^2 + \frac{I_{II}}{10} \cdot \left[1 - \left(\frac{Mr}{Ma}\right)^2\right] \leq I_{cf} \tag{10}$$

Proposal 3:
$$I_{lm} = I_{II} \cdot \left(\frac{Mr}{Ma}\right)^2 \leq I_{med} \tag{11}$$

In this context, the analysis of the behavior of the deflection will be carried out using the curves of the Load *versus* Deflection graph in the middle of the span (Fig. 4). Obtained using eq. (1), the curves were generated to determine the displacement as a function of the effective moment of inertia of each expression presented. The curves show the behavior of the specimens throughout the loading process until collapse, allowing a detailed analysis of the accuracy of the different approaches to calculating the deflection in relation to the results obtained experimentally. The load versus deflection curve at mid-span for specimen 02B is similar to the behavior of all twelve specimens tested, so for this article it was taken as an example for analysis.

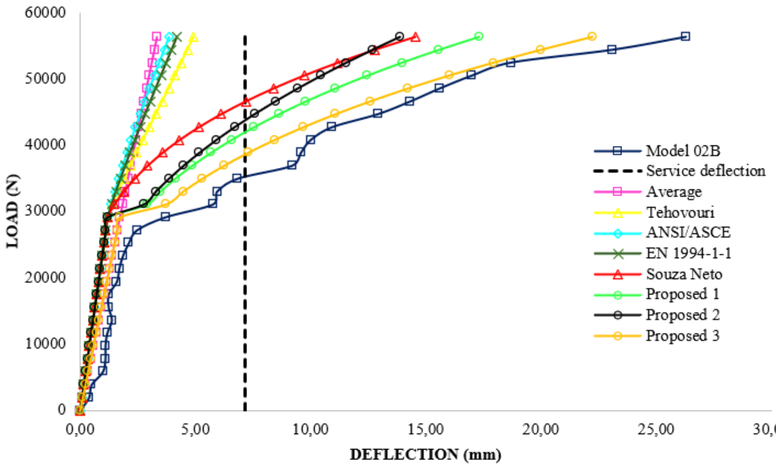

Figure 4: Load versus deflection curve at mid-span without considering creep of specimen 02B.

Through the relation Load versus Deflection (Fig. 4), which considers the effective moments of inertia by the simple average of the uncracked and cracked section, Tenhovouri [5], ANSI/ASCE 3-91:1992 [7] and EN 1994-1-1:2004 [4], resulted in rigid structures compared to the experimental results, presenting smaller deflections. On the other hand, the equations of Sousa Neto [6] and Costa [3, 13], especially proposals 1 and 3, more adequately represent the results obtained in tests, coming close to the projection of specimen 02B. However, they still project non-conservative deflections throughout the entire loading.

5. Concrete creep

In accordance with item 17.3 of ABNT NBR 6118:2023 [2], the verification of limit values for the deformation of concrete structures, especially linear structural elements, must include the analysis of deformations deferred over time. In this context, to determine the service deflection of composite slabs, it is important to consider the effect of creep in the concrete.

Thus, creep can be the slow and continuous deformation of a viscoelastic material, as concrete, under constant stress over time. According to Brazilian literature, the creep deformation of concrete (ε_{cc}) is composed of two parts: a rapid deformation (ε_{cca}) and a slow deformation. Rapid deformation is irreversible and occurs in the first 24 hours after the load is applied. Slow deformation, on the other hand, is composed of two other parts: irreversible slow deformation (ε_{ccf}) and reversible slow deformation (ε_{ccd}). Thus, creep deformation is expressed in eq. (12).

$$\varepsilon_{cc}\,(t,t_0) = \varepsilon_{cca} + \varepsilon_{ccd} + \varepsilon_{ccf} = \frac{\sigma_c}{E_{c,28}}\,\varphi(t,t_\infty) \tag{12}$$

where $\sigma_c$compressive stress in concrete, $E_{c,28}$ modulus of elasticity of concrete at 28 days, $\varphi(t,t_\infty)$ is the creep deformation coefficient, which will be analyzed in the next topics, is expressed in eq. (13).

$$\varphi(t,t_\infty) \;=\; \varphi_a \;+\; \varphi_\infty \cdot [\beta_f(t_\infty) - \beta_f(t)] \;+\; \varphi_d \cdot \beta_d \tag{13}$$

where φ_a is the rapid deformation coefficient [eq. (14)], being the relation of f_{ck} multiplied by the degree of hardening of the concrete at the time of loading (β_1).

$$\varphi_a \;=\; 0{,}8\;\cdot \beta_1 1(t_0)\text{ , para } f_{ck}\,\epsilon\,[20{,}45]\text{ MPa} \tag{14}$$

The variable φ_∞ is the coefficient of irreversible slow deformation, being the multiplication of φ_{1c} (Eq. 15) and φ_{2c} (Eq. 16), for concretes of classes C20 to C45.

$$\varphi_{1c} = 4{,}45 - 0{,}35 \cdot U, \text{ for the 5 to 9 cm slump test.} \tag{15}$$

$$\varphi_{2c} = \frac{42+h}{20+h} \tag{16}$$

The U represents the average humidity at the time of the test, obtained from INMET [14]; h is the weighted fictitious thickness ($\gamma \cdot h_{fic}$), expressed in centimeters; where, γ , is the coefficient adopted from Table A.1 from ABNT NBR 6118:2023 [2] ($\gamma = 1 + e^{(-7{,}8 + 0{,}1 \cdot U)}$); h_{fic}, fictitious thickness of the member ($h_{fic} = 2 \cdot A_c/U_{ar}$); A_c, is the cross-sectional area of the member, expressed in centimeters; U_{ar} , is the perimeter of the member in contact with the atmosphere.

The variables, $\beta_f(t_\infty)$ and $\beta_f(t_0)$, is the coefficient of irreversible slow deformation as a function of the age of the concrete, is given by:

$$\beta_f = \frac{t^2 + A \cdot t + B}{t^2 + C \cdot t + D} \tag{17}$$

where t_0 is the fictitious initial age of the concrete, expressed in days; t_∞ is the fictitious age of the concrete on the day of the test, expressed in days; and the other coefficients are values established in Annex A of ABNT NBR 6118:2023 [2].

The variable $\varphi_{d\infty}$, is the coefficient of reversible slow deformation, which is equal to 0.4 and β_d, is the coefficient of reversible slow deformation as a function of time after loading given by eq. (18):

$$\beta_d = \frac{t_\infty - t_0 + 20}{t_\infty - t_0 + 70} \tag{18}$$

Considering the creep deformation coefficient, it will be possible to apply the deformations deferred over time for each of the twelve specimens presented (Tab. 2), based on their effective age (T_∞). This will make it possible to obtain the equation for determining the deflection in this work.

6. Analysis of Concrete Creep in composite steel and concrete slabs

When it comes to verifying the serviceability limit state related to vertical displacement in composite steel and concrete slabs, ABNT NBR 8800:2008 [1] does not consider deformation over time. Therefore, this work aims to determine the deflection in composite slabs considering the effect of concrete creep. That said, item 17.3 of ABNT NBR 6118:2023 [2] states that to consider concrete creep, the relation of the permanent portion of the immediate deflection (eq. (1)) multiplied by the term 1+ $\varphi(t, t_\infty)$ (eq. (13)) must be obtained, according to eq (19).

$$\delta = \frac{Pu \cdot L_S \cdot (3L^2 - 4L_S{}^2)}{2 \cdot 24 \cdot E_C \cdot I_{Im}} \cdot (1 + \varphi(t, t_\infty)) \tag{19}$$

From eq. (19), the vertical displacements over time are obtained for the various moment of inertia expressions presented in section 4 of this paper, as shown in Fig. 5. Fig. 5 shows that taking concrete creep into account shows a reduction in the structure's bending stiffness (EI), in relation to the results presented in Fig. 4, causing an increase in the mid-span deflection.

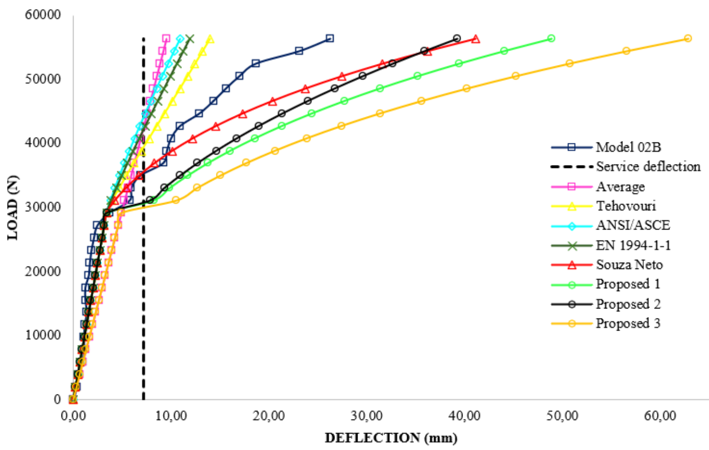

Figure 5 - Load *versus* deflection curve at mid-span considering creep of specimen 02B.

It can also be seen that using the effective moment of inertia as the simple average of the uncracked and cracked sections, Tenhovouri [5], ANSI/ASCE 3-91:1992 [7] and EN 1994-1-1:2004 [4], resulted in rigid structures compared to the experimental results, with smaller deflections. Checking the equations of Sousa Neto [6] and Costa [3, 13], they more adequately represent the results obtained in the test, approaching the projection of specimen 02B in a conservative manner throughout the loading process.

7. Conclusion

When analyzing the results obtained when applying eq. (19) of this work considering concrete creep (Fig. 5), in relation to the results of the deflections shown in Fig. 4, it can be seen that considering the effective moments of inertia of the simple average, Tenhovouri [5], ANSI/ASCE 3-91:1992 [7] and EN 1994-1-1:2004 [4], better approximate the projection of the experimental results. However, close to the serviceability limit state (L/350), these expressions show non-conservative deflections, because the effective moment of inertia equations are not appropriate, even considering concrete creep. From another perspective, the equations of Sousa Neto [6] and Costa [3] more adequately represent the projection of the results obtained in tests, with proposal 3 being the most conservative, while proposals 1 and 2 are the most appropriate for representing the composite system of steel and concrete slabs using steel deck.

Acknowledgements. The authors acknowledge the National Council for Scientific and Technological Development (CNPq), the Institutional Scholarship Program for Scientific Initiation at the Federal University of Bahia (PIBIC-UFBA) and the Dean of Research and Graduate Studies at UFBA for their support in carrying out this research.

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