Dynamic Analysis of Plates Subjected to Explosive Loading: Comparison Between SDOF and FEM Models

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Abstract. With the global increase in explosive events, arising from armed conflicts and/or accidental occurrences, a thorough analysis of the dynamic behavior of structures under this type of load becomes important. The explosive phenomenon causes an overpressure (positive) wave with a subsequent underpressure (negative) phase, which is often disregarded. Moreover, recent studies have highlighted that this negative phase can generate displacements and stresses as relevant as the ones from the positive phase only. This work investigates simply supported plates subjected to explosive loading considering the membrane effect (laterally immovable condition) which introduces nonlinearities in the model. Due to the problem's mathematical complexity, these nonlinear dynamic analyses are usually performed using complicated finite element models. In this way, a software called DYNAblast was developed using a simpler approach based on a Single Degree of Freedom (SDOF) analytical model which incorporates the von Karman plate theory. Dynamic analyses were conducted in DYNAblast to understand the influence of the positive and negative phases of the blast loads, as well as the contribution of the membrane energy of the plate. Numerical models implemented in ABAQUS software were used for the validation of DYNAblast results, with excellent agreement.

Keywords: Dynamic analysis; Finite Element Method; Von Karman Theory; Membrane effect; Blast loads.

1 Introduction

The study of explosive phenomena, from past decades to the present day, has become a topic of great relevance due to the increase in both accidental and intentional events. Thus, research into the influence of this type of loading on the behavior of structures has gained significant prominence.

Fernández [1] discussed the explosion in Beirut, Lebanon, which generated a shock wave so intense that it disrupted the ionosphere, comparable to the energy released by a volcanic eruption that erupted in Japan in 2004. Jithin [2] focused his studies on explosions, which are high-intensity dynamic loads applied to plates. This phenomenon refers to a sudden and intense release of energy, giving rise to shock waves, which can cause a structure to collapse partially or completely.

Reis et al. [3] developed a software for analyzing the behavior of thin plates, making it possible to obtain displacement, stress, and frequency results by varying parameters such as TNT mass and scaled distance. In addition, Reis [4] discusses the presence of two phases in an explosion. Until recently, one of them, known as the negative phase, was often dismissed. However, it has been proven that, due to geometric non-linearity and the membrane effect, its consideration becomes even more significant.

This study investigates the behavior of thin plates under explosive load, considering the condition of a laterally immovable membrane. To this end, a comparison was made between the single-degree-of-freedom (SDOF) model used in the DYNAblast 1.1 [12] software, an updated version of DYNAblast 1.0 [11], and the structure modeled in the finite element software ABAQUS. The aim was to validate the results of both static and dynamic analyses to assess the accuracy of the latest version of DYNAblast. In addition, the impact of geometric non-linearity when considering the membrane effect was investigated.

2 Explosive Phenomena

An explosion is defined as the sudden release of energy, caused by factors such as chemical reactions, pressure differences or nuclear processes. This event can happen naturally, or influenced by anthropogenic action, and is characterized by a rapid increase in pressure and temperature, causing damage to structures, people, and the environment [5]. Explosions can be categorized into two types, depending on their location in the environment: spherical and hemispherical. In the spherical type, the detonation occurs in the air and its energy is dissipated, transferring pressure to a bulkhead, while in the hemispherical the detonation occurs on a surface. The energy dissipated is caused by shock waves originating from pressure variations ([6], [7]).

2.1 Shockwave

The propagation of a shock wave can be seen in Figure 1, which shows pressure as a function of time, and the parameters involved. After detonation, the originating gases expand rapidly, reaching a peak of maximum overpressure, ($P_{m\acute{a}x}$). This overpressure drops rapidly until it reaches the ambient level, corresponding to the time of the positive phase, (t_d). After this process, a vacuum is produced in the center of the explosion and the negative phase begins, with a duration represented by (t_d^-) and maximum underpressure value, ($P_{m\acute{\imath}n}$). The amount of energy released in an explosion is calculated by the impulse of the shock wave, both in the positive phase (i_d) as in the negative phase, (i_d^-) [4]. In both phases, the parameters mentioned are related to the nature of the explosion, which can be spherical or hemispherical, the equivalent mass of TNT (W_{TNT}) and the distance on the Hopkinson scale (Z) [8], as shown in Eq. (1). Equations for calculating the explosion parameters $P_{m\acute{a}x}$, $P_{m\acute{\imath}n}$, t_d, i_d and i_d^- were obtained based on experimental studies by Rigby [8] and US Department of Defense abacuses [9].

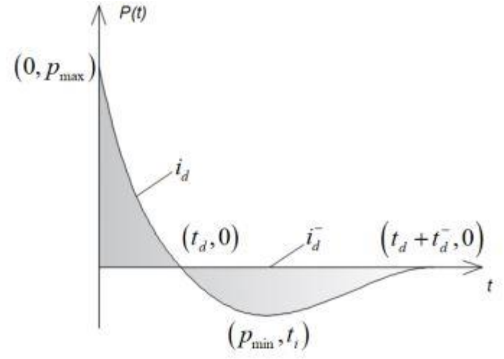


Figure 1. Pressure vs. time graph of shock wave (Adapted from [8])

$$Z = \frac{R}{W_{TNT}^{1/3}} \tag{1}$$

2.2 Dynamic load

Some studies use empirical and semi-empirical methods to formulate and calibrate pressure-time curves for both positive and negative phases. Figure 2 shows some approaches, including the linear approximation, Friedlander's extended model, Friedlander-Teich extended model and the cubic polynomial for the negative phase.

In this work, based on the analytical model of Reis [4], three types of loading were studied: only the positive phase and the extended positive phase, applying the Friedlander equation and the application of the negative phase using the cubic polynomial. The explosive loading *P(t)* is given by equation 2, where the determination of the positive phase includes the aforementioned parameters and the explosion decay coefficient (*a*'), obtained by solving a system of non-linear equations using the Newton-Raphson method [4]. The negative phase time is shown in equation 3 and is calculated by reformulating the Granstrom equation [10].

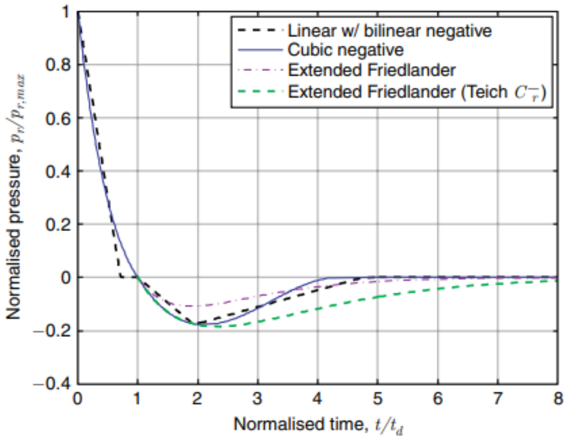


Figure 2. Curves for positive and negative phase using different models [8]

$$P(t) = \begin{cases} p_{máx}\left(1 - \frac{t}{t_d}\right) e^{-\frac{a' t}{t_d}} & , t \leq t_d \\ -p_{mín}\left(\frac{6.75(t - t_d)}{t_d^-}\right)\left(1 - \frac{(t - t_d)}{t_d^-}\right)^2 & , t_d \leq t \leq t_d + t_d^- \\ 0 & , t \geq t_d + t_d^- \end{cases} \tag{2}$$

$$t_d^- = \frac{16\, i_d^-}{9\, P_{min}} \tag{3}$$

3 DYNAblast 1.1

DYNAblast 1.1 [12] is a recent version of DYNAblast 1.0, developed by Reis et al. [11], which was designed using MATLAB. Its purpose is to investigate the behavior of slender plate structures under explosive loading, generating graphs and recording the values of stresses and strains, displacement, frequencies, and loads. DYNAblast 1.1 [12] has the same functions as 1.0, with the addition of the extended phase and the field for entering the distance (*R*) from the explosive to the bulkhead. The software has an interface with spaces for entering the parameters of the plate, the explosive, and the type of analysis desired, as shown in figure 3.

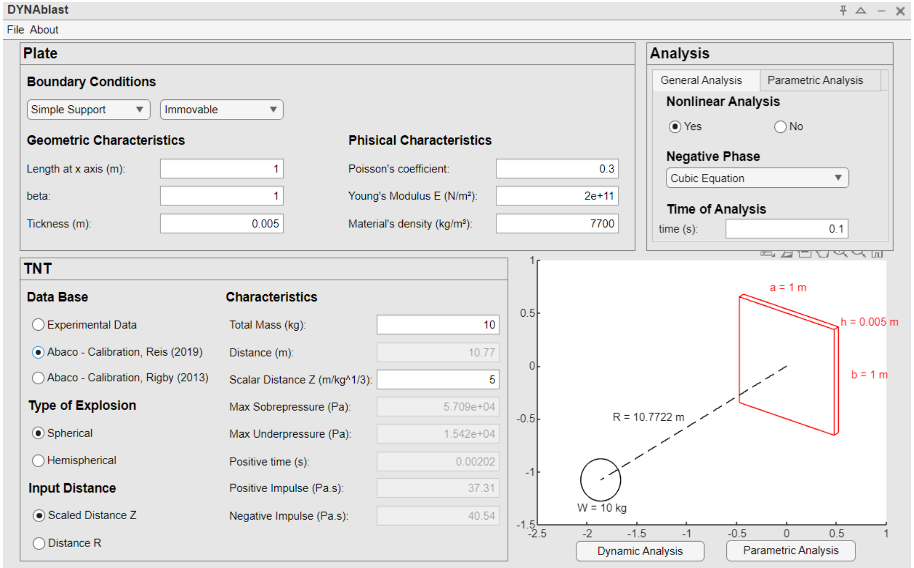
The results provided by DYNAblast are those obtained for the central part of the structure. The user inputs the geometric and physical parameters of the plate, chooses the database and the characteristics of the analysis. For the database, if it is experimental, it is necessary to add the input data such as maximum and minimum pressure, positive phase time, positive and negative impulse, or choose the abacus (Reis, 2019 or Rigby, 2013), only needing the explosive mass and the distance *Z* or *R*. The software uses the Friedlander equation to solve the positive and extended positive phase, and the cubic polynomial for the negative phase. DYNAblast provides an effective solution for plate analysis using the SDOF model, simplifying and speeding up analysis, allowing for more in-depth studies of the dynamic behavior of plate structures subjected to explosion. It is important to note that although explosions are high-frequency dynamic loads, what stands out is the fundamental mode of the structure [3]. The SDOF model is based on the following equations:

$$D\nabla^4 u_z + \rho h \frac{\partial^2 u_z}{\partial t^2} - h\left(\frac{\partial^2 \phi}{\partial y^2}\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 u_z}{\partial y^2} - 2\frac{\partial^2 \phi}{\partial x \partial y}\frac{\partial^2 u_z}{\partial x \partial y}\right) = P_z(x, y, t), \tag{4}$$

$$\nabla^4\phi = E\left[\left(\frac{\partial^2 u_z}{\partial x\partial y} - \frac{\partial^2 u_z \partial^2 u_z}{\partial x^2 \partial y^2}\right)^2\right] \tag{5}$$

$$u_z(x,y,t) = A(t)\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right) \tag{6}$$

The software developed by Reis [3] uses Von Karman plate theory, because due to the impact caused by the load on the structure, it is necessary to consider the possibility of large displacements, which requires the use of higher order terms in the strain tensor, as shown in equations (4) and (5), where D is the bending stiffness, u_z represents the displacement in z, h is the thickness, ϕ is the Airy function, ρ is the specific mass and E is Young's modulus. Thus, equation 6 shows how Navier's series is used to correlate the displacement in the case of a simply supported plate, where A is the amplitude resulting from the separation of the time and space variables.



Boundary Conditions

Simple Support ▼ Immovable ▼

Geometric Characteristics

Length at x axis (m): 1
beta: 1
Tickness (m): 0.005

Phisical Characteristics

Poisson's coefficient: 0.3
Young's Modulus E (N/m²): 2e+11
Material's density (kg/m³): 7700

Nonlinear Analysis

● Yes ○ No

Negative Phase

Cubic Equation ▼

Time of Analysis

time (s): 0.1

Data Base

○ Experimental Data
● Abaco - Calibration, Reis (2019)
○ Abaco - Calibration, Rigby (2013)

Type of Explosion

● Spherical
○ Hemispherical

Input Distance

● Scaled Distance Z
○ Distance R

Characteristics

Total Mass (kg): 10
Distance (m): 10.77
Scalar Distance Z (m/kg^1/3): 5
Max Sobrepressure (Pa): 5.709e+04
Max Underpressure (Pa): 1.542e+04
Positive time (s): 0.00202
Positive Impulse (Pa.s): 37.31
Negative Impulse (Pa.s): 40.54

Figure 3. Interface of software DYNAblast 1.1 [12]

4 Numerical model

4.1 Physical and geometrical properties and membrane boundary condition

The model studied is a Kirchhoff plate with homogeneous, isotropic, and linear elastic material characteristics. The geometry was modeled in ABAQUS, adopting the three-dimensional, deformable SHELL element. The parameters used in the structure were as described: dimensions (a x b) 1 m x 1 m, thickness (h) equal to 0.005 m, Young's Modulus (E) equal to 200 GPa, Poisson coefficient equal to 0.3 and density equal to 7700 kg/m^3. In the "LOAD" module, the membrane condition is applied, imposing displacement constraints in the three directions (x, y, and z), thus removing the structure's lateral mobility.

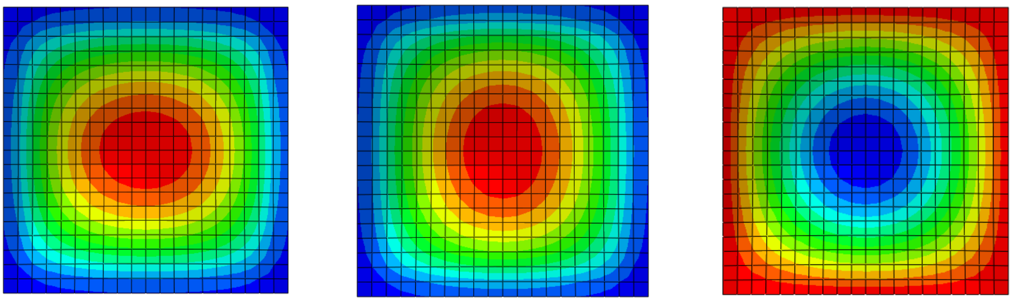
4.2 Mesh definition, types of analysis and applied loading

For static elements, the 0.05 m size was used with 20 elements per side, and for the dynamic analyses, a 0.0125 m size with 80 elements per side. These choices are justified because they provided the most accurate results in the mesh sensitivity study [13]. Dynamic analyses were performed using the "DYNAMIC, EXPLICIT" scheme. The load was applied as a uniform pressure distributed over the surface of the plate in all analyses. In the static analysis, the load remained constant, set at 0.03 MPa. In the dynamic analysis, an amplitude was set which determined the percentage of load applied over time. The explosion parameters were calculated using the Rigby equations [8] and, considering the scaled distance Z = 5 m/kg$^{1/3}$, total mass of TNT, W_{TNT} = 10 kg and a shock wave in a spherical form, based on Rigby [8], the parameters of blast wave are $P_{máx}$ = 73,124.44 Pa, $P_{mín}$ = 22,223.67 Pa, t_d = 0.006776 s, t_d^- = 0.016489 s, i_d = 177.51 Pa.s and i_d^- = 206.13 Pa.s.

5 Numerical Results

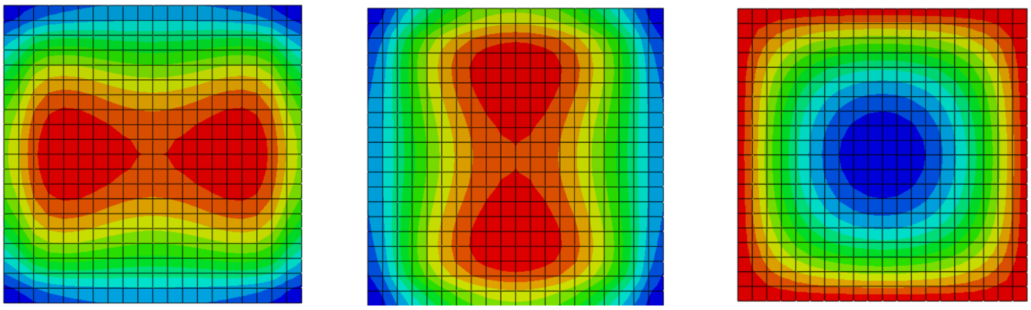
5.1 Static analysis

After modeling, the results obtained were correlated with the studies by Reis [4]. Figure 4 shows the distribution of stresses and displacements in the plate from the linear analysis, while Figure 5 presents the results of the nonlinear analysis. The results are shown in Table 2, together with those used to calibrate the structure.



(a) σ_{xx} (b) σ_{yy} (c) Displacement

Figure 4. Stress and displacement distribution from linear static analysis.



(a) σ_{xx} (b) σ_{yy} (c) Displacement

Figure 5. Stress and displacement distribution from non-linear static analysis.

Software	σ_{xx}e σ_{yy} (MPa)		Displacement (mm)	
	Linear	Non-linear	Linear	Non-linear
wxMáxima (Reis, [4])	384.35 (11.92%)	120.90 (28.91%)	53.52 (0.3939%)	9,49 (7.84%)
ABAQUS (Reis, [4])	343.41 (0%)	93.82 (0.042%)	53.31 (0%)	8,80 (0%)
ABAQUS (present)	343.41 (0%)	93.78 (0%)	53.31 (0%)	8,80 (0%)

Table 1. Obtained results and comparison with reference

Observing Table 1, stresses and displacements obtained by Reis [4] using an auxiliar algebraic software, wxMaxima, are compared with a model developed in the software ABAQUS. The comparison is based on the middle node located at the center of the plate and the stresses obtained corresponding to the maximum for each principal axis. Also, in this current research, the new model developed in ABAQUS showed similar stresses and displacements to the static analysis in both studies. Considering Table 1, the percentage corresponding the difference between the study of Reis [4] and the current study indicated that the discrepancy in displacements is 0.3939 %, while in stresses it is 11.92% for the linear analysis. In the non-linear analysis, there percentage difference between the results obtained by ABAQUS (present study) and the WxMáxima software used by Reis [4] for calibration, with a difference of 7.84% for displacements and 28.91% for stresses. The margin of error for displacements was considered satisfactory, as the variation of approximately 30% in the stresses can be attributed to the consideration of only the first harmonic in the analytical model. Another aspect is the concentration of stresses in the central region of the plate when comparing linear and non-linear analyses. This disparity can be attributed to the influence of geometric non-linearity in the structure, which induces high deformations and displacements, significantly affecting the location of the stresses. The model was thus validated and moved on to dynamic analysis to verify the accuracy of DYNAblast 1.1 [12].

5.2 Dynamic analysis

In the dynamic analysis in ABAQUS, the loading was calculated by determining the parameters using the equations proposed by Rigby [8], and then applied to the model. In DYNAblast 1.1 [12], the experimental method was used, inserting the data shown in item 4.2. The results obtained in both software packages were correlated. It is important to note that the total time determined for the analyses was 0.1 seconds and that the positive phase, extended positive phase and positive and negative phase were used, with the cubic polynomial. The results obtained in both software programs show the same number of cycles in both types of analysis and are shown in Figures 6 and 7, linear and nonlinear analysis, respectively. Table 2 shows the results of maximum displacement considering each curve presented in Figures 6 and 7.

Software/Analysis	Positive Phase (m)	Extended Positive Phase (m)	Positive and Negative Phases (m)
DYNAblast 1.1 – Linear Analysis	0.0479	0.0627	0.075
DYNAblast 1.1 – Nonlinear Analysis	0.01563	0.0156	0.0156
ABAQUS – Linear Analysis	0.0444	0.0602	0.0777
ABAQUS – Nonlinear Analysis	0.0166	0.0166	0.0166

Table 2. Maximum displacements in dynamic analysis

Considering the results using the positive phase in loading, the percentage difference between the software results was 7.31% for the linear analysis and 5.27% for the nonlinear analysis. For the case of extended positive phase, there was a discrepancy of 3.99% for the linear result and 6.41% for the nonlinear result. Finaly, the case of positive and negative phases, the percentage for linear analysis is 3.6%, while for nonlinear analysis it is 6.41%.

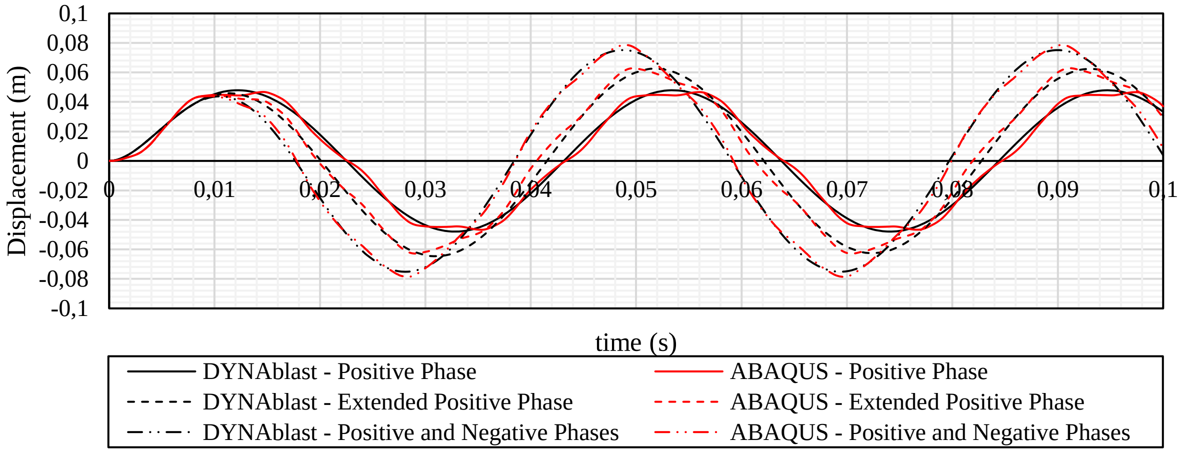


Figure 6. Results of linear case in dynamic analysis (Reference of [12])

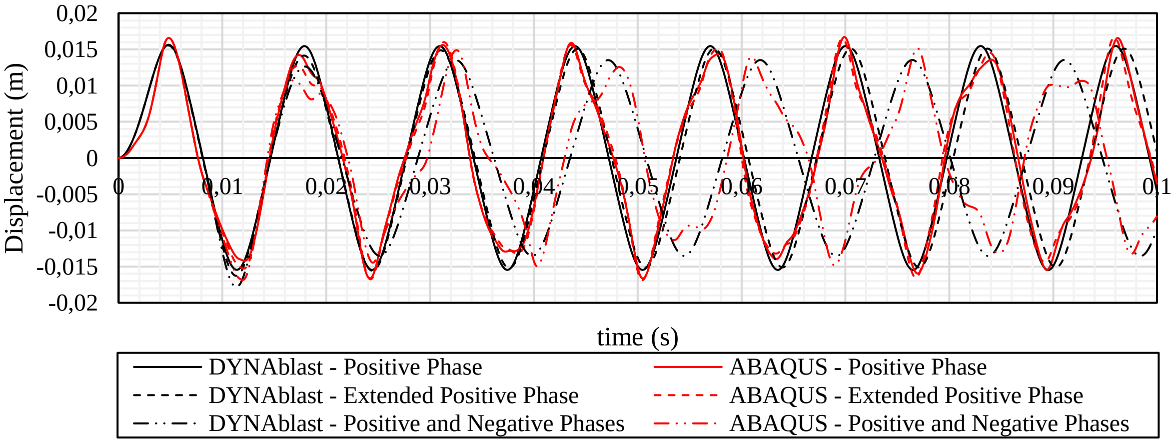


Figure 7. Results of non-linear case in dynamic analysis

The linear dynamic analysis graphs show lower frequencies than the non-linear ones. In addition, linearity does not consider some factors, such as large displacements and high strains, significantly influence the dynamic response. The higher frequencies of the non-linear analysis can also be evidenced by the membrane condition.

6 Conclusion

The main objective of this work was to correlate the results of Reis's analytical model [4] with a FEM model in ABAQUS to validate the software DYNAblast 1.1 [12].

In the static analysis, the values found were similar to those of Reis [4], with a satisfactory difference for the study. The influence of the membrane condition was also observed when geometric non-linearity was considered, since the displacement restrictions led to the redistribution of stresses. In the linear analysis the stresses were concentrated in the central region of the plate, while in the non-linear analysis the location was different.

In the dynamic analysis, it was observed that the frequencies between the linear and non-linear models are different, showing once again how geometric non-linearity influences structural behavior. Thus, we conclude that results are more accurate when this condition is applied, as this is when large displacements and strains occur.

In addition, the results of DYNAblast 1.1 [12] and ABAQUS obtained the same frequency, thus showing the consistency and reliability of the analytical software. Another factor observed that provides confidence in the results was the percentage difference with those obtained in ABAQUS, with a margin of less than 10%. This small disparity indicates that DYNAblast 1.1 [12] is satisfactory and accurate in its analysis.

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