Multiobjective Optimization of Steel and Concrete Composite Slabs via NSGA algorithm

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Abstract. With the expansion of the civil construction sector, composite steel and concrete slabs with incorporated steel formwork have become increasingly common due to their ease of execution and the elimination of shoring. However, optimization studies focusing on this type of structural element are still limited. The objective of this study is to propose a formulation for the multi-objective optimization of composite steel and concrete slabs, considering the phase after concrete curing. Objective functions are established to minimize CO_2 emissions and slab costs, as well as to maximize the load-bearing capacity of the slab for a given span. To address this optimization problem, the Non-dominated Sorting Genetic Algorithm (NSGA) implemented in the MATLAB platform is used. Comparative analyses are conducted between examples from the literature utilizing single-objective optimization and those employing multi-objective optimization to validate the proposed formulation. Additionally, new problem instances are examined to identify the key factors that influence the final solution. The results indicate that the NSGA is effective in finding a solution and that for the same load situation, more than one solution was proposed for the problem.

Keywords: Composite Slab Multiobjective Problem, NSGA, CO_2 Emissions, Costs, Maximum Load

1 Introduction

Composite steel and concrete slabs have been used in civil construction as an advantageous structural solution concerning environmental and economic aspects, since they eliminate the need for shoring and formwork, thereby reducing material waste. Experimental and numerical studies have been conducted to investigate the contribution of steel sheeting and the addition of tensile reinforcement to the longitudinal shear strength of composite slabs, as observed in the works of Grossi et al. [1] and Celis-Imbajoa et al. [2]. Further research has been carried out using optimization techniques aimed at finding optimal solutions considering CO_2 emissions and cost criteria, as seen in the studies by Santoro and Kripka [3], Guimarães et al. [4], Teixeira et al. [5], and Silva et al. [6]. Afshari et al. [7], Liu et al. [8], and Santos et al. [9] have contributed to the topic using multi-objective formulations in the optimization of reinforced concrete structures. Mezzomo et al [10, 11, 12] developed a program using Genetic Algorithms (GA) to optimize trapezoidal steel decking to minimize displacement and simultaneously maximize critical buckling load and coverage area.

Among the multi-objective optimization algorithms cited in the literature, the Non-dominated Sorting Genetic Algorithm (NSGA) proposed by Deb and Agrawal [13] stands out. Applications of NSGA in structures can be found, for example, in the works of Madrigal et al. [14], He et al [15], and Ruiz-Velez et al. [16]. However, studies involving the multi-objective optimization of composite slabs to minimize costs, and CO_2 emissions, and maximize load capacity have not been found up to the present time. From this perspective, this study presents the application of multi-objective optimization in investigating solutions for composite slabs that offer minimal cost and CO_2 emissions simultaneously, along with maximum design loads. The problem is addressed using NSGA available in the MATrix LABoratory (MATLAB)[17] software library. The efficiency of the method will be verified by comparing the results obtained with the single-objective optimization implemented by Teixeira et al [5] using PSO and GWO for simply supported composite slabs.

2 Multi-Objective Optimization Problem Formulation

2.1 Objective Function

In this study, the application of the multi-objective formulation for composite steel and concrete slabs considered three objective functions described in eq. (1), eq. (2), and eq. (3). Equation (1) represents the total CO_2 emissions, while eq. (2) refers to the manufacturing cost of the slabs. Equation (3) is associated with the maximum design load that the composite slab can withstand.

$$Min\ CO_2 = CO_{2(formwork)} + CO_{2(concrete)} + CO_{2(add.pos.reinf)} + CO_{2(mesh)} \tag{1}$$

$$Min\ Cost = Cost_{(formwork)} + Cost(_{concrete)} + Cost_{(add.pos.reinf)} + Cost_{(mesh)} \tag{2}$$

$$Max\ Load = Q \tag{3}$$

Where $CO_{2(formwork)}$, , $CO_{2(concrete)}$, $CO_{2(add.pos.reinf)}$ and $CO_{2(mesh)}$ denote the CO_2 emission rates (in $kgCO_2$) from the steel formwork, concrete, additional positive reinforcement, and mesh, respectively. The costs of each material are represented by $Cost_{(formwork)}$, $Cost_{(concrete)}$, $Cost_{(add.pos.reinf)}$ e $Cost_{(mesh)}$, which respectively represent the cost of the steel formwork, concrete, additional reinforcement for positive bending moment, and mesh. For this study, the CO_2 emission rates of concrete and steel were extracted from Santoro and Kripka [3] and the Worldsteel Association [18], respectively. Regarding costs, SINAPI's [19] table provides values for concrete and reinforcements. The cost of the steel formwork was provided by national manufacturers. The unit emission rates and costs related to the components of the slabs are presented in Tab. 1 and Tab.2.

Table 1. Unit Costs and Emissions - Concrete

Material		CO_2 Emissions ($kgCO_2/m^3$)	Source	Costs ($R\$/m^3$)	Source
Concrete (MPa)	20	129.85	Santoro e Kripka [3]	463.14	SINAPI [19]
	25	142.71		474.87	
	30	153.68		491.01	
	35	163.25		504.22	
	40	171.73		518.15	
	45	189.6		532.09	
	50	199.72		546.02	

Table 2. Units Cost and Emissions - Steel

Material			CO_2 Emissions ($kgCO_2/m^3$)	Source	Costs (R$/m²)	Source
Steeldeck formwork (280MPa)	MF50	esp.0.8mm	2.638	Worldsteel Association [18]	90.5	Local Supplier (2024)
		esp.0.95mm			107.64	
		esp.1.25mm			141.64	
	MF75	esp.0.8mm			99	
		esp.0.95mm			117	
		esp.1.25mm			154	
Reinforcement mesh (CA60)			1.924 ($kgCO_2/kg$)		10.48 R$/kg	SINAPI [19]
Wire mesh (CA60)						

2.2 Design Variables

For the multi-objective optimization problem, 6 decision variables were defined, as shown in Fig. 1. These variables are the concrete thickness above the steel formwork ($x_1 = t_c$), the characteristic compressive strength of concrete ($x_2 = f_{ck}$), the thickness of the steel formwork ($x_3 = t_f$), the additional reinforcement ratio for positive bending moment ($x_4 = \rho_R$), the type of steel formwork according to the manufacturer (x_5 = MF 50 ou MF 75) and the maximum design load of the slab (x_6).

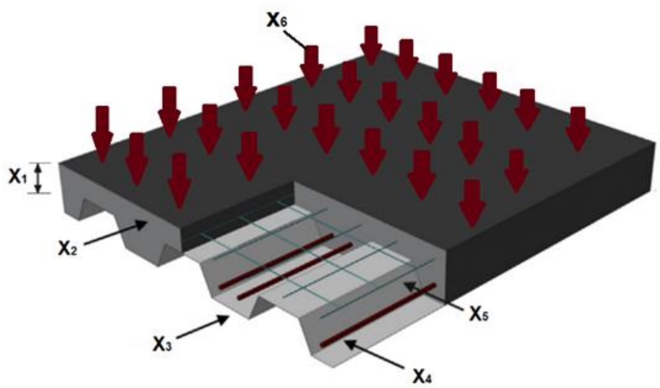

Figure 1. Design variables for composite steel and concrete slabs: Adapted from Teixeira et al [5].

2.3 Search space for the optimal solution

For this study, the decision variables used were discrete. The ranges defined for each variable are described in Tab. 3.

Table 3. Space of search design variables

Variables	Intervals
Thickness of concrete layer (mm)	50 to 125, increase from 5
Characteristic compressive strength of concrete (MPa)	20 to 50, an increase of 5
Thickness of steel formwork (mm)	0.8, 0.95, and 1.25
Additional reinforcement for positive bending moment rate (%)	0, 0.05, 0.1, 0.15, and 0.2
Steel formwork (2019)	MF50 and MF75
Maximum design load (kN/m²)	5 to 16, an increase of 0.1

2.4 Constraints

Equations (4) to (7) present the design constraints following the design criteria prescribed by ABNT NBR8800:2008 [20] for simply supported composite slabs. The verifications consider Ultimate Limit States (ULS) and Serviceability Limit States (SLS) for excessive deflection.

$$C(1) : \frac{M_{Sd}}{M_{Rd}} - 1 \leq 0 \tag{4}$$

$$C(2) : \frac{V_{Sd}}{V_{v,Rd}} - 1 \leq 0 \tag{5}$$

$$C(3) : \frac{V_{Sd}}{V_{l,Rd}} - 1 \leq 0 \tag{6}$$

$$C(4) : \frac{\delta}{\delta_{máx}} - 1 \leq 0 \tag{7}$$

Constraint C(1) concerns the verification of limit states associated with positive bending moment. C(2) and C(3) verify vertical and longitudinal shear, respectively. Finally, constraint C(4) relates to the serviceability limit state associated with excessive deflection.

The multi-objective optimization was conducted using the Non-dominated Sorting Genetic Algorithm (NSGA-II), available in Matlab libraries. The parameters used in this method were 100 iterations and a population size of 100 individuals. Results were analyzed based on a compilation of 10 repetitions executed by the algorithm.

3 Multi-Objective Optimization Problem Formulation

The multi-objective formulation was applied based on the analysis of the problem proposed by Teixeira et al. [5], where a single-objective optimization using PSO and GWO algorithms was considered. In Teixeira et al.'s study [5], the objective function aimed to minimize the CO_2 emission rate of a simply supported composite slab with a span of 2.5 m and MF-50 steel formwork from manufacturer Metform [21]. The slab was subjected to a total load of 10.1 kN/m². Table 4 shows the optimal solution found by Teixeira et al. [5] and the solution obtained via multi-objective optimization concerning the total CO_2 emission of the composite slab.

From Tab. 4, it can be observed that the optimal solution for the 2.5 m span composite slab resulted in a CO_2 emission of 88.33 $kgCO_2$ using NSGA. In this case, the result was 44.5% lower than the manufacturer's result and 3.23% lower than that presented by Teixeira et al. [5]. The analysis also included results related to the final cost of the slab and the maximum design load the slab could withstand. Pareto frontier analyses were conducted to determine the best solutions based on CO_2 emissions, minimum costs, and maximum design load achieved.

Figure 2 presents the Pareto Fronts graph with CO2 Emission versus Load solutions, while Fig. 3 shows the Cost *versus* Load solutions.

Table 4. Comparison between manufacturer's properties and results of single-objective and multi-objective solutions

Solution	h_t (mm)	h_f	t_c (mm)	e (mm)	f_{ck} (MPa)	ρ_R(%)	CO_2 Emission ($kgCO_2$)
Manufacturer	130	MF50	80	1.25	20	0	127.66
PSO (Teixeira et al [5])	105		55	0.8	20	0.2	91.28
GWO (Teixeira et al [5])	105		55	0.8	20	0.2	91.28
Authors Multi-objective	100		50	0.8	20	0.15	88.33

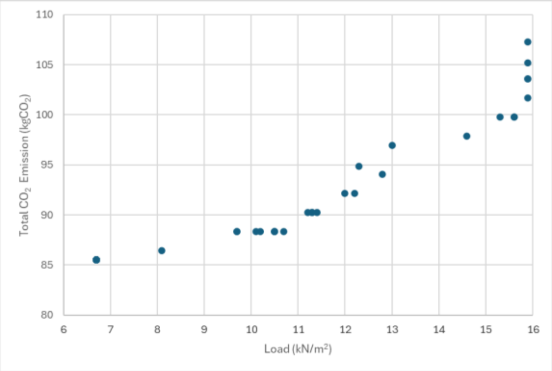

Figure 2. Pareto Frontier Plot – CO_2 Emission *versus* Load

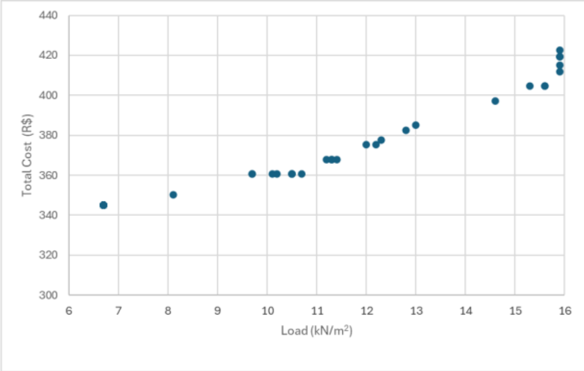

Figure 3. Pareto Frontier Plot – Cost *versus* Load

The analysis of the Pareto Fronts (Fig. 2 and Fig.3) showed that the multi-objective optimization provided more than one optimal solution for project loads close to the maximum limit of 16 kN/m². For instance, at a maximum design load of 15.9 kN/m², the algorithm generated four possible solutions with different CO_2 emission rates and costs, as shown in Table 5.

Table 5. Comparison between solutions 1 to 4 for the maximum load of 15.9 kN/m²

Solution	h_f	t_c (mm)	e (mm)	f_{ck} (MPa)	ρ_R(%)	CO_2 Emission ($kgCO_2$)	Cost (R$)
1	MF50	90	0.8	20	0.15	103.58	419.25
2		85	0.8	20	0.15	101.68	411.91
3		90	0.8	25	0.15	107.28	422.62
4		85	0.8	25	0.15	105.21	415.14

It is important to highlight that the authors treated the optimal solutions presented by the algorithm in a way that selected only feasible solutions, i.e., those that meet all design constraints. As shown in Tab. 6, the four solutions for the maximum load of 15.9 kN/m² comply with regulatory and safety constraints, and therefore, they are suitable for adoption by the designer in the design of simply supported composite slabs. Additionally, for all solutions in Tab. 6, the longitudinal shear verification governs the design.

Table 6. Comparison between solutions 1 to 4 to maximum load 15.9kN/m²

Solution	Constraints			
	Positive bending moment	Vertical shear	Longitudinal shear	Excessive deflection
1	0.50	0.27	0.98	0.33
2	0.51	0.28	0.99	0.36
3	0.48	0.25	0.96	0.32
4	0.50	0.26	0.99	0.35

However, to address economic and environmental aspects, the solution with the lowest final emission and final cost was adopted, which were 101.68 $kgCO_2$ and R$411.91, respectively. It used an MF-50 formwork with a thickness of 0.8 mm, a concrete cover of 85 mm, a concrete characteristic compressive strength (f_{ck}) of 20 MPa, and an additional reinforcement for positive bending moment rate of 0.15%.

It is also noticeable that for composite slabs with maximum loads between 9.7 kN/m² and 10.7 kN/m², the algorithm found solutions with almost identical CO_2 emission values and costs. The same applies to solutions for composite slabs with maximum loads ranging from 11.3 kN/m² to 11.4 kN/m², and from 15.3 kN/m² to 15.6 kN/m². In other load intervals, there is variation in emission and cost, such as the load step from 12 kN/m² to 13 kN/m², where an increase of 4.8 $kgCO_2$ and R$ 9.975 was observed, corresponding to a ratio of R$ 2.078/$kgCO_2$.

Figure 4 illustrates the variation in total CO_2 emission versus cost as the load of the slab increases. The solutions plotted in the graph establish an almost linear increasing relationship, except for solutions near the emission values of 95 $kgCO_2$ and 105 $kgCO_2$. In these cases, corresponding to slab loads between 12.3 kN/m² and 12.8 kN/m², and the maximum limit found of 15.9 kN/m², respectively, there is a solution with lower emission and higher cost and another subsequent solution with higher emission and lower cost.

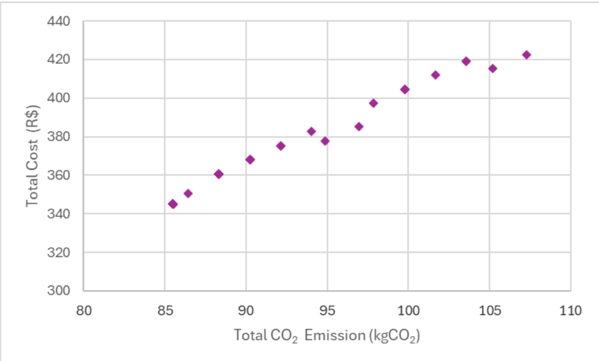

Figure 4. Graph of Pareto Frontier – Cost versus CO_2 Emission

4 Conclusions

After analyzing the results, it can be concluded that the proposed multi-objective formulation was efficient in finding a solution to the optimization problem of composite slabs, surpassing the solutions found by the single-objective optimization via PSO and GWO presented by Teixeira et al. [5]. The NSGA algorithm selected properties of the composite slab to achieve a 3.23% reduction in CO_2 emissions compared to the single-objective solution. Additionally, the multi-objective formulation successfully maximized the load-carrying capacity of the slab for the proposed span of 2.5m, achieving a maximum load solution of 15.9 kN/m².

Regarding the selection of the best solution, it was observed that the optimization program identified four possible solutions with a maximum load of 15.9 kN/m², all of which meet the design constraints and are suitable for adoption by the designer. However, the algorithm identified a single solution that outperformed the others in terms of lower CO_2 emissions and lower cost.

Examining the variation in cost relative to CO_2 emission, it was found that this variation is minimal for load intervals between 9.7 kN/m² and 10.7 kN/m², 11.3 kN/m² and 11.4 kN/m², and 15.3 kN/m² to 15.6 kN/m². In other intervals, there was a significant variation, such as in the load-interval from 12 kN/m² to 13 kN/m², where the algorithm selected solutions corresponding to a cost variation of R$ 2.078/kgCO_2.

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