Global Sensitivity Analysis and Dimensionality Reduction in Relative Permeability Model in Core-Flooding Experiments

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Abstract.

Numerical simulation stands out as a fundamental method in the oil industry, allowing the prediction of fluid flow in porous media. Its primary objective is to analyze behavior and forecast oil production through fluid injection. However, the need for numerous simulations, each encompassing various multidimensional and compositional characteristics, presents a challenge. This leads to a significant accumulation of physical information, exacerbating the computational demands on the numerical model, especially regarding computational cost. To address this challenge, a potential solution is determining which input parameters are essential for accurate oil production prediction. Global sensitivity analysis emerges as a powerful tool for this purpose, aiming to identify which input parameters exert the most significant influence on the numerical model's response, thereby reducing computation time. Unlike traditional approaches that focus on sensitivity around a single operating point, this study adopts a comprehensive perspective, evaluating sensitivity across the entire input sample space. Specifically, this research investigates the impact of changes in parameters related to the relative permeability curves of a core plug during unsteady-state water injection experiments. The primary metric analyzed is accumulated oil production. Sobol' indices, a method for quantifying global sensitivity, are employed to assess the contribution of each input parameter's variance to the variance of the results. The mathematical framework used is based on the Buckley-Leverett equation for multiphase (water/oil), one-dimensional longitudinal flow, assuming incompressible fluids and constant water injection. The model is solved using an implicit finite difference methodology with time step control, with relative permeability parameterized by the LET model. The results of this analysis provide insights into reducing the number of varying parameters in the input for inverse problems on a global scale. Of the ten parameters analyzed, two were the most sensitive. This reduction significantly decreases computational costs, offering greater flexibility in constructing surrogate models for simulations of flows in porous media.

Keywords: Global sensitivity analysis; LET model; Relative permeability; Buckley Leverett.

1 Introduction

One of the main objectives of petroleum and gas engineering is to predict fluid flow through porous media in hydrocarbon reservoirs. These predictions are based on analyses obtained through bench studies, which utilize software simulations to analyze flow through rock samples. Numerical simulation is one of the computational methodologies employed in petroleum engineering to estimate reservoir characteristics and predict behavior [1].

This methodology involves inputting the physical properties of rocks and fluids present in the porous media into simulators to obtain responses closely resembling real flow conditions. One of these properties is the relative permeability, traditionally obtained by combining core-flooding experiment data such as net production and pressure drop, and computational methods for inverse modeling [2].

Core-flooding experiments usually lead to the acquisition of cumulative oil production curves differential pressure curves, and saturation profiles (depending on the exact type of experiment) that are matched by a numerical model by tuning the respective parameters of the flow model, such as the relative permeability k_r [2]. In this paper, water flow is injected into a rock plug, typically saturated with oil, and inserted into a core holder (to maintain controlled pressure). During this injection, sensors measure inlet and outlet pressure differentials, fluid volume, and flow rate at the rock outlet. This experiment can be conducted under steady-state or transient conditions. Transient state methods can maintain a constant flow rate (with pressure monitoring) or constant pressure (with flow rate monitoring). The oil produced during displacement, combined with flow rate or pressure differentials, provides data used to estimate relative permeability curves.

Equations representing this flow are based on hydraulic diffusivity equations, from which Darcy's law originated and has been further refined. The mathematical basis for interpreting test data can be summarized as follows: Leverett combined Darcy's law with a differential capillary pressure definition to obtain the water fraction in the output flow [3]. The Buckley-Leverett model is utilized in this study to analyze the water and oil saturation in the sample throughout water injection and within the rock plug space being studied. These equations describe the flow of water and oil fluids in a two-phase system. Additionally, it takes into account any porous solid material, except for geological characteristics.

Since these functions depend on numerous variables, it is important to know which ones are the most important. The important variables in one part of the input space may not be the same in another, and they can also fluctuate over time, which has motivated the development of global sensitivity measures.

Furthermore, sensitivity analysis is the study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input factors [4]. Global Sensitivity Analysis (GSA) can be defined by a variance decomposition method, which aims to decompose the output variance as the sum of the contributions of each input variable or their combinations. A GSA method recently explored in engineering is the Sobol' indices, which aim to determine the expansion of the computational model in terms of increasing dimensional sums relative to conditional variances. One of its primary advantages is the ability to handle nonlinear and non-parametric models and provide both quantitative and qualitative classification [5].

2 Methodology

2.1 Two-phase flow model

The Buckley-Leverett (BL) model can be used to approximately determine the water-oil displacement in a petroleum reservoir [6]. According to the model's fundamental assumption, the porous media is completely filled with immiscible and incompressible fluids, there is no chemical reaction occurring at the interface between the fluids, the flow is in a horizontal direction and the effects of capillary forces are not taken into consideration. The BL model is given by:

$$\phi \frac{\partial S_w}{\partial t} + v \frac{\mathrm{d} f_w}{\mathrm{d} S_w} \frac{\partial S_w}{\partial x} = 0, \tag{1}$$

where ϕ represents the porosity function, v the water flux velocity, S_w the water saturation, and $f_w = \frac{\lambda_w}{\lambda_w + \lambda_n}$.

The fractional flow that indicates the wettable phase contribution. The correlation $\lambda_\alpha = \frac{k_\alpha}{\mu_\alpha}$ where k_α is the the relative permeability of the α phase (in our case, water and oil) and μ_α its respective viscosity.

The LET relative permeability correlations for water injection and oil production. The normalized water saturation is first defined as [7]:

$$S_{wn} = \frac{S_w - S_{wi}}{1 - S_{wi} - S_{orw}} \tag{2}$$

The correlations for oil relative permeability with water injection and water relative permeability with oil production are written as:

$$k_{ro} = k_{ro}^x \frac{(1 - S_{wn})^{L_o^w}}{(1 - S_{wn})^{L_o^w} + E_o^w {S_{wn}}^{T_o^w}}, \tag{3}$$

$$k_{rw} = k_{rw}^{o} \frac{S_{wn}^{L_w^o}}{S_{wn}^{L_w^o} + E_w^o (1 - S_{wn})^{T_w^o}}, \tag{4}$$

where k_{rw}^{o} and k_{ro}^{x} are, respectively, the relative permeability endpoints for the water relative permeability and oil relative permeability, while the parameters L_o^w, E_o^w, and T_o^w are called phenomenological parameters which define the shape of the relative permeability curves. S_{wi} stand for the water initial saturation and S_{orw} the residual oil saturation after water injection.

The open source 1D finite difference solver called Core2Relperm [3] is used to solve Eq. 1 and its corresponding boundary conditions for a base case (see Berg et al. [8] for additional details regarding grid, and boundary conditions). The petrophysical properties used in the simulations are as follow.

Property	**Units**	**Value**
Core length (L)	cm	5.0
Core diameter	cm	3.8
Absolute permeability	mD	50
Porosity	v/v	0.2
Water viscosity	cP	1.0
Oil viscosity	cP	3.0
Water density	kg/m³	1000.0
Oil density	kg/m³	850.0
Initial water saturation	v/v	0.2
Flow rate	cm³/min	0.1

Table 1. Summary of physical properties for the numerical simulations.

Subsequently, the water saturation values of the plug are determined as follow:

$$Q(t) = \int_0^L S_w(x,t)dx \tag{5}$$

2.2 Global sensitivity analysis

The global sensitivity analysis is used to assess how the parameters of Eq. 2, 3 and 4 and their interactions contribute to the aforementioned studied quantities, i.e., water saturation, pressure drop, and cumulative oil production. For this, we used the variance-based Sobol indices [9]. Given a mathematical model f that has k independent input parameters gathered into an input vector $\mathbf{X}$ and an output scalar $Y = f(\mathbf{X})$, where $\mathbf{X} = \{X_1, X_2, \ldots, X_k\}$. According to Sobol [9], the model can be decomposed into sums involving different dimensions of $\mathbf{X}$ to obtain Y.

Equation (6) has the property of orthogonality in terms of conditional means. In this way, it is possible to define the Sobol' decomposition in terms of conditional variances [4].

$$f = f_0 + \sum_i f_i + \sum_i \sum_{j>i} f_{ij} + ... + f_{12...k} \tag{6}$$

The first order Sobol' indices quantify the additive effect of each input separately in relation to the total variance, as can be seen in Equation (7).

$$S_i = \frac{\mathrm{V}[E(Y|(X_i)]}{\mathrm{V}(Y)} \tag{7}$$

The interaction effects of the inputs, as a second-order index, as we can see in the equation below,

$$S_{ij} = \frac{\mathrm{V}[E(Y|(X_{ij})]}{\mathrm{V}(Y)} \quad \text{for } i \neq j \tag{8}$$

and the total Sobol' indices, which are the sum of their primary parameter, plus the secondary ones:

$$S_{Ti} = \frac{E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i}))}{V(Y)} = 1 - \frac{V_{\mathbf{X}_{\sim i}}(E_{X_i}(Y|\mathbf{X}_{\sim i}))}{V(Y)}. \tag{9}$$

Sobol sampling was carried out for ten parameters with minimum and maximum intervals based on empirical values for LET parameterization [7], as depicted in Table 2.

Parameters	**Minimum**	**Maximum**
S_{wi}	0.05	0.35
S_{orw}	0.05	0.35
L_w^o	1	20
E_w^o	0.5	20
T_w^o	0.5	20
L_o^w	1	20
E_o^w	0.5	20
T_o^w	0.5	20
k_{rw}^o	0.05	1
k_{ro}^x	0.05	1

Table 2. Ranges used for the Sobol samples.

3 Results and Discussion

As aforementioned, the main objective of the GSA is to generate a ranking regarding parameter importance throughout the whole time of the experiment. For each timestep was generated Sobol' indices using the SALib library [10, 11], which can be visualized in Fig. 1. It was observed that out of the ten parameters analyzed, two are the most sensitive to input variability, namely L_o^w and S_{orw}, followed by T_o and S_{wi}. Parameters related to the water flow, such as L_w^o, E_w^o, T_w^o, showed an insensitive behavior. Notice that in Fig. 1, the Sobol indices' are stacked, and the sum of their total values is expected to be equal to 1.

It is important to note that the sensitivity of the parameters varies over time. Therefore, depending on the specific moment being analyzed, certain parameters may exhibit higher sensitivity compared to others within the same sampling. This phenomenon is especially evident in small time intervals, where parameters such as T_w^o, L_w^o, and S_{orw} exhibit a peak followed by a decline and stabilization. Additionally, the parameter L_o^w experiences continuous growth and maintains the highest index.

Figure 2 shows the total Sobol' indices for all ten parameters analyzed. Notably, there is no change in the ranking of the most sensitive parameters when compared to the first-order index, particularly at the end of the experiment. This indicates that the individual response of each parameter is more crucial than their variation in pairs with the other analyzed parameters, i.e., from $S_{1.2}$ until $S_{1.10}$ indices.

The sensitivity of the parameters can be quantitatively observed in Table 3 over time. More precisely, it emphasizes four distinct time intervals, specifically T1, T2, T3, and T4, which correspond to 10, 40, 200, and 600 minutes, respectively. Highlighting the values of S1 and ST for S_{wi} at Time 1: The first-order index (S1) has a much lower value than the total index (ST), indicating that at this point, S_{wi} has greater variability with the other parameters than on its own. However, this same parameter at Time 4 is no longer as sensitive, either individually or in interactions with the other parameters. There is a noticeable trend where the interactions between parameters and their pairs are substantially greater at the start of the experiment, specifically during small times of the experiment. However, as the experiment progresses, the primary Sobol indices and the total Sobol indices show little deviations.

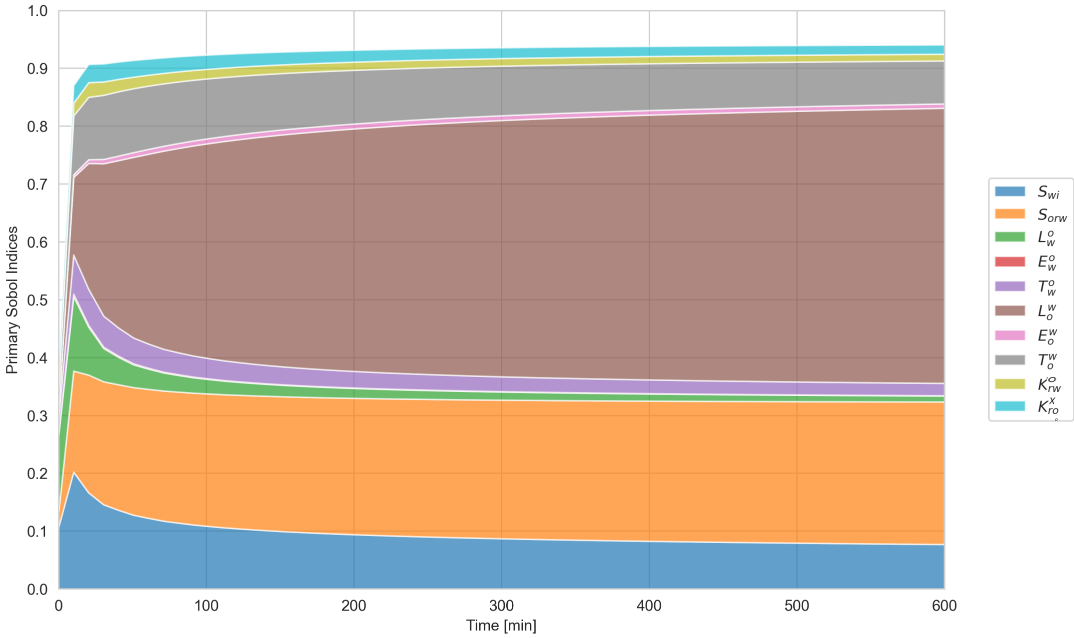

Figure 1. Sobol first-order index of accumulated oil production volume.

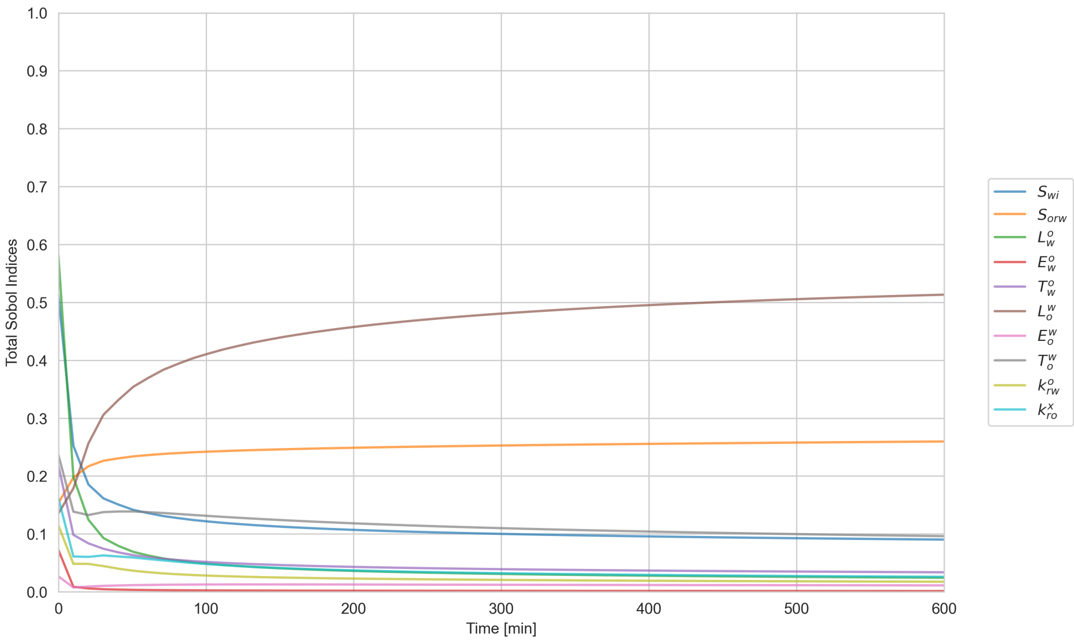

Figure 2. Sobol total index of accumulated oil production volume.

It's noteworthy that the number of samples directly influences the confidence interval of the data; hence, a convergence analysis of the Sobol method was conducted. In this study, the required number of simulated samples was 28,835,84. The total simulation time was 437.5 hours, using a computer with the following specifications: Processor 12th Gen Intel(R) Core(TM) i7-12700 2.10 GHz with 16,0 GB RAM Memory.

	T1		T2		T3		T4	
Parameters	**S1**	**ST**	**S1**	**ST**	**S1**	**ST**	**S1**	**ST**
S_{wi}	0.109	0.513	0.145	0.161	0.094	0.108	0.077	0.090
S_{orw}	0.027	0.155	0.213	0.226	0.235	0.248	0.246	0.260
L_w^o	0.138	0.578	0.058	0.093	0.017	0.36	0.098	0.024
E_w^o	0.003	0.071	0.002	0.004	0.001	0.002	0.001	0.001
T_w^o	0.026	0.215	0.054	0.075	0.029	0.043	0.021	0.034
L_o^w	0.012	0.134	0.264	0.306	0.417	0.455	0.474	0.512
E_o^w	0.002	0.027	0.007	0.010	0.008	0.013	0.007	0.011
T_o^w	0.044	0.236	0.109	0.138	0.093	0.118	0.074	0.096
k_{rw}^o	0.009	0.115	0.023	0.446	0.014	0.023	0.012	0.017
k_{ro}^x	0.022	0.163	0.031	0.062	0.020	0.037	0.016	0.026

Table 3. Sobol indices.

4 Conclusions

The global sensitivity analysis using the Sobol indices provides a very robust response to the analyzed model once it is possible to quantify the importance of a parameter, attributing to it a value in the range of 0 to 1 and analyzing its response alone and in pairs. It is particularly beneficial when used in core-flooding experiments, as it relies on properties that are highly parameterized. The optimization of relative permeability curves benefits greatly from this approach, as it allows for the fixation of insensitive parameters and the prioritization of sensitive parameters.

Out of the ten parameters examined, only two, L_o^w and S_{orw}, were found to have a significant impact on net production volume. S_{wi} and T_o^w showed some sensitivity, while E_o^w and the other water parameters (L_w^o, E_w^o and T_w^o) were found to be insensitive. Additional discoveries indicate that the sensitivity fluctuates over the course of the experiment and that the interactions between pairs of parameters are more significant at the start of the experiment. This preliminary analysis is important because it allows for reduced computational costs and better predictions for future optimizations. To enhance future analyses, incorporating the impacts of capillary pressure and devising strategies to address the less responsive parameters will yield valuable insights for a more comprehensive understanding of the core-flooding experiment.

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