Finite Element Analysis of a Steel Catenary Riser Transporting Gas-Liquid Slug Flow

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Abstract. Steel catenary risers (SCRs) are main offshore structures that transport produced oil and gas from the seabed to floating units such as floating production storage and offloading (FPSO) units. This work analyzes the dynamic behavior of a steel catenary riser transporting a gas-liquid slug flow. A finite element formulation for computing elastic large displacement in planar and spatial frames, with consistent numerical implementation, is presented using Updated and Total Lagrangian approaches. To account for geometric changes as external forces are applied, the nonlinear solution is linearized in steps, each representing a load or time step. Moreover, nonlinear terms are included to calculate the stiffness matrix due to the presence of large deformations. The Newmark algorithm is used for numerical integration, coupled with the Newton-Raphson method. The two-phase slug flow is modeled as a homogeneous mixture with varying density over time and space. Numerical results agree well with available experimental data. The effects of the slug flow parameters on the dynamic behavior of the steel catenary riser are investigated.

Keywords: Steel Catenary Riser, Riser Transporting Fluid, Slug Flow, Finite Element Method, Dynamic Analysis

1 Introduction

In oil and gas exploration and production operations, steel catenary risers are widely used to connect the christmas trees to a floater or fixed oil production platform, and during this production the gas-liquid flows occur frequently as a two-phase mixtures with a flow pattern known as slug flow, alternating the distribution of liquid and gas along the riser. Intermittent forces caused by slug flows can cause significant oscillations in the suspended riser, depending on the flow conditions and mechanical properties of the pipe structure. This phenomenon raises concerns about potential fatigue and damage, once structural failure in these pipelines may result in gas leaks, oil spills, premature failure of surface equipment, and operational disruptions, impacting both the economics of oil production and the environment.

The dynamic response of risers experiencing internal slug flow has received limited attention from riser mechanics researchers over the past decades, primarily due to the complexities involved in mathematically modeling internal slug flow. Civil or mechanical engineering problems, such as hydrology or hydraulics, deal with one-phase flow, being liquid or gas, as presented by Tentarelli and Brown [1] while petroleum risers usually transport two-phase flow of oil and gas mixture. In the nuclear one, by its way, a two-phase flow is considered, being liquid water and its vapor or steam with condensed water. Patel and Seyed [2] represented a simplified one-dimensional slug flow model based on the sinusoidal behavior of the internal fluid density along the riser, concluding that the effects of the dynamic slug flow can cause large tension variation on the riser. In the study of Wu and Lou [3] they concluded how the bending rigidity influences the dynamic response of riser when the internal fluid flows at high velocities. Monprapussorn et al. [4] used a parametric analysis to check the influence of the internal intermittent flow on the dynamic response of the riser, concluding that the fluctuation amplitude of the internal fluid velocity amplifies the top tension and the displacement in the upper region of the riser. In the Valdivia [5] work experiments were conducted to determine the influence of an internal two-phase flow composed of water and air on the riser dynamic response. Ortega et al. [6] and Gundersen et al. [7], analysed the combined effects of waves and slug flow. Bossio et al. [8] conducted their study to the analysis of a straight horizontal pipe submitted to effects of vortex induced vibration and slug flow, showing that, due the slug mass the system weight and inertia, the pipe oscillation induced by the VIV were modified. An and Su [9] presented a vertical straight riser subjected to crossflow sea current and internal slug flow using one dimensional beam simplification, concluding that the amplitude of oscillation of the riser depends on the flow rate of liquid and gas. Bordalo and Morooka [10] proposed a model to simulate the oscillations induced by the internal flow on pipelines conveying liquid and gas mixtures. Vásquez and Avila [11] presented a two-dimensional parametric analysis of the influence of an internal slug flow on the structural dynamic response of flexible risers. The study presented by Trapper [12] introduced a numerical approach for the structural analysis of the static configuration of the risers; his method employed nonlinear large deformation beam theory and executed it in an incremental and iterative way, allowing efficient handling of the interaction between the riser and the seabed. Ogbeifun et al. [13] presented a two dimensional tabular optimization method for the riser reducing a multidimensional problem to a two dimensional type providing a significant reduction in the required computational resources, combining the design variables in pair and assigns indices to the resulting design points for each combination.

This paper proposes a model to simulate the effect of internal flow on pipe dynamics, aiming to address the issue of the riser oscillation during the conveyance of two-phase slug flow, presenting a methodology to model a riser system, considering SCR (steel catenary riser) configurations. In order to model the simple SCR and start the dynamic analsis, an analytical approach presented by Faltinsen [14] was considered. Once the riser behavior is governed by an ordinary differential equation, this work applies a numerical procedure using an iterative method adjusting the lengths of the laid-down section until the equilibrium solution be reached to a steel catenary riser, then its final condition is used as initial condition for the dynamic analysis, being solved by means of finite element method with total Lagrangian method with Newmark as integrator and Newton-Raphson as iterator.

2 Loads due the Internal Flow

The center of the proposed model of the internal flow load is the fluid mass distribution along the riser, and its geometry with two-phase flow depends on the flow rate of each phase, physical properties and riser geometry. Slug flow is characterized by its intermittency and studies indicate that a whipping motion can occur in risers solely due to the slug flow, even in the absence of significant surrounding currents, waves, or top riser movement, and this is the principal concern about this kind of flow.

In this work, the slug pattern is represented as a sequence of packages, each one containing certain amounts of liquid plugs and gas pockets flowing with a velocity v_c, as presented by Dukler and Hubbard [15]. To better represent the slug flow, a distribution function is used, presenting a discontinuity of the phases, which is close to a real pattern, exciting the riser in a similar way to the effect of a real flow.

To have the internal flow acting in the risers it is necessary to introduce the magnitude and direction in space of the flow induced loads. The most relevant internal flow forces are the force of gravity and the force due the curvature of the flow trajectory, neglecting the Coriolis effects. The gravity force is the weight of fluid at a riser section, depending on the density ρ, the area of the internal flow cross section A_i, and the intensity g of the gravity field, as presented by Johnson [16]. The vector that describes the gravitational load, in terms of force per unit length of the riser is as follows:

$$\overrightarrow{F}_g = \rho g A_i \overrightarrow{u}_g, \tag{1}$$

being $\overrightarrow{u}_g$ the unit vector of the direction of the gravity field.

In the case of slugs, the intermittent mass distribution $\rho(s,t)$ along the riser creates a variable weight load, resulting in an excitation frequency f_s. If this excitation frequency is near the natural frequency of the riser structure, the oscillation amplitude induced by the internal flow can become significantly severe.

The curvature of the flow trajectory, as seen in curved ducts or bent pipes like jumpers and suspended risers, generates a force due to changes in flow momentum as the direction shifts along the riser's path Johnson [16]. This force acts perpendicular to the local flow direction (which is tangent to the trajectory) and outward relative to the radius of curvature r. The curvature force depends on the density of the fluid ρ, the area of the internal flow cross section A_i and the centripetal acceleration, which can be calculated as $a_c = \frac{v^2}{r}$, being v the flow velocity. In this way, the vector which describes the curvature load in terms of force per unit length of the riser is given by:

$$\overrightarrow{F}_c = \rho \frac{v^2}{r} A_i \overrightarrow{u}_{cf}, \tag{2}$$

where $\overrightarrow{u}_{cf}$ is the unit vector of the centrifugal direction. As can be inferred, the variation of density with time and space $\rho(s,t)$ leads to a variable load. The flow velocity v can be calculated by dividing the constant mass flow rate by the density multiplied to the internal cross section area of the riser.

As presented by Bordalo and Morooka [10], the density of the fluid mixture at each point along the flow must be composed by both the density of the liquid phase (ρ_L) and the gas phase (ρ_G), given by,

$$\rho(s,t) = \phi\rho_L + (1-\phi)\rho_G, \tag{3}$$

where the mass distribution ϕ is given by an appropriate function F,

$$\phi = F\left(\frac{s - v_c t}{L_c}\right), \tag{4}$$

being ϕ a function of time (t), the position along the riser length (s), the velocity of the slug cell (v_c) and the length of the fluid cell L_c. In this work the ϕ function variation which describes the periodical approximation of the slug varies between 0 and 1, being 0 when the pipe section is full of gas and 1 when it is full of liquid, and there may be a film of liquid flowing in the gas pocket between two liquid plus, as depicted by Bordalo and Morooka [10].

The shape of the proposed ϕ function, characterized by a series of pulses with squared edges, resembles actual slugs in risers. While the gas pocket may have a nose and tail, and small bubbles may exist in the liquid plug, the proposed ϕ function effectively represents the intermittency of the mass distribution of slugs, facilitating the induction of oscillations in the pipe.

Patel and Seyed [2] and Pollio and Mossa [17] used sinusoidal functions for ϕ due to their periodic nature, however, these functions do not capture the abrupt transitions between gas and liquid, that are characteristic of slugs. Additionally, sine and cosine functions inherently represent a hold-up of 0.5, i.e., a 50%-50% gas-liquid distribution, which restricts the range of simulations.

As can be presumed, the density variation presented by Eqs. (3, 4) effect directly on the Eqs. (1, 2). The length of the liquid and gas segments are determined using the volume fraction of the two phase, while the velocity of the slug cell can be obtained from the flow rates of the liquid and gas, as follows

$$v_c = \frac{Q_L + Q_G}{A_i}, \tag{5}$$

being Q_L and Q_G the liquid and gas flow rates, respectively.

As mentioned by Bordalo and Morooka [10], the slug frequency f_s or the length L_c of the slug cells are still an issue not fully resolved, but these two variables are made consistent by

$$f_s = \frac{v_c}{L_c}. \tag{6}$$

3 Finite Element Method

In this section the method used to model the riser is presented, along with the Newmark integrator coupled with Newton Raphson as an iterative method; and also the Lagrangian Total method responsible for updating the riser body geometry with a basis in what was displaced or strained, as highlighted by Bathe [18] and Casagrande et al. [19].

Given the large displacements of the riser, a geometric nonlinear element is employed, so, the 2d nonlinear dynamic problem of a beam is under consideration.

3.1 Newmark Method

The Newmark family is the most used group of direct methods to solve the equation of motion presented in Eq. (7) as shown by Hughes [20], this method consists of a numerical integration approach within the time domain

$$\boldsymbol{M\ddot{u}} + \boldsymbol{C\dot{u}} + \boldsymbol{Ku} = \boldsymbol{F}, \tag{7}$$

being **M** the mass matrix, **C** the viscous damping matrix, **K** the stiffness matrix, **F** the applied force vector, **u** the displacement vector, $\dot{\mathbf{u}}$ the velocity vector and $\ddot{\mathbf{u}}$ the acceleration vector.

The evolution of the approximate solution was described using the finite difference formulation. Once the non-linear problems are normally solved in *u-form* the equations were rewritten isolating the terms relative to the velocity and acceleration as function of the displacement which now can be solved for the displacement at time t+Δt after the own equation of motion, highlighted by Casagrande et al. [19].

A general non-linear dynamics analysis consists in finding the solution of the equation of motion according to the applied loads

$$\boldsymbol{Ma}^{(k)}_{t+\Delta t} + \boldsymbol{Cv}^{(k)}_{t+\Delta t} + \boldsymbol{K}^{(k-1)}_{t+\Delta t}\boldsymbol{\Delta u}^{k} = \boldsymbol{R}_{t+\Delta t} - \boldsymbol{F}^{(k-1)}_{t+\Delta t}, \tag{8}$$

being **u**, **v** and **a** the approximations of the displacement, velocity and acceleration vectors, respectively, at any instant of time t. $\mathbf{R_{t+\Delta t}}$ is the vector of externally applied nodal forces and $\mathbf{F_{t+\Delta t}}$ is the vector of internal forces. The superscript indicates that the equation is being evaluated at iteration k for an iterative solution, as presented by Bathe [18]. A huge explanation of this development can be found in Casagrande et al. [19].

To find the solution of Eq. (8), the displacement at the current iteration (k), $\mathbf{u^{(k)}_{t+\Delta t}}$, is decomposed into the displacement at the previous iteration (k-1), $\mathbf{u^{(k-1)}_{t+\Delta t}}$ and a variation of the displacement, $\mathbf{\Delta u^k}$. Then, the final equation can be written selecting the variation of the displacement as the main unknown in a reduced form as:

$$\hat{\boldsymbol{K}}^{(k-1)}_{t+\Delta t}\boldsymbol{\Delta u}^k = \boldsymbol{R}_{t+\Delta t} - \hat{\boldsymbol{F}}^{(k-1)}_{t+\Delta t}, \tag{9}$$

being $\mathbf{\hat{K}^{(k-1)}_{t+\Delta t}}$ the effective tangent stiffness matrix and $\mathbf{\hat{F}^{(k-1)}_{t+\Delta t}}$ the effective internal force vector.

After inverting the effective tangent stiffness matrix and solving Eq. (9) for the variation of the displacement, the solution of the system can be found, and the displacement, velocity and acceleration are updated to complete the cycle.

3.2 Total Lagrangian Method

Once production risers can present large rotation and small strains, a total Lagrangian method is strongly indicated to be used, and because of the large deflection, the force equilibrium equation must be done on the bases of the element's deformed configuration.

In order to reflect the effects of changes in geometry when external loads are applied, the nonlinear problems can be solved through a series of linear steps, where each step represents either a load or a time increment. Due to the presence of large deflections, the strain-displacement equations contain non-linear terms which have to be considered when calculating the stiffness matrix, as can be seen in Przemieniecki [21].

The element stiffness matrix $\mathbf{K^e}$ is modified by the nonlinear terms in the strain-displacement equation, as seen in Eq. (10) as presented by Casagrande et al. [19], where $\mathbf{K^e_E}$ is the linear elastic stiffness matrix calculated for the element at the initial configuration, giving the elastic properties of the material, while $\mathbf{K^e_G}$ is the geometric stiffness matrix which depends of the deformed configuration, deriving from the geometric changes of the structure. The sum of these both matrix build a total stiffness matrix, and this combination allows the analysis considering both material properties and changes in the geometry of the system as it deforms.

$$\boldsymbol{K}^e = \boldsymbol{K}^e_E + \boldsymbol{K}^e_G. \tag{10}$$

A residual stress can appear in the rigid body motion with finite elements due its large-displacement. A natural displacement vector $\mathbf{u^e_n}$ is defined to solve this problem in a context of the total Lagrangian method for dealing with large displacements and rotations in truss elements. The natural displacement vector is a representation of stress-related displacements in a local reference frame, essential for the accurate analysis of the internal forces and stiffness of the element. For each element, the global displacement vector is represented as:

$$\boldsymbol{u}^e = \boldsymbol{u_1 v_1 \theta_1 u_2 v_2 \theta_2}, \tag{11}$$

where $\mathbf{u_i}$ is the displacement in horizontal direction, $\mathbf{v}_i$ the displacement in vertical direction, and $\theta_\mathbf{i}$ the angular displacement, all of them related to the node i. The stiffness matrix of the element is defined in the local referential, being shifted to the global one using the rotation matrix $\mathbf{R^e}$. Then, the internal force vector $\mathbf{F^e}$ for an element is evaluated through the linear stiffness matrix $\mathbf{K^e_E}$ and natural displacement vector $\mathbf{u^e_n}$ previously calculated. The element mass matrix $\mathbf{M^e}$ is also defined.

$$\boldsymbol{F}^e = [\boldsymbol{R}^{e^T}\boldsymbol{K}^e_E\boldsymbol{R}^e]\boldsymbol{u}^e_n. \tag{12}$$

The sum of the internal forces of all elements results in the internal force at time $\mathbf{F^{k-1}_{t+\Delta t}}$. In a similar way, the tangent stiffness matrix $\mathbf{K^{k-1}_{t+\Delta t}}$ is obtained summing the stiffness matrices of each element rotated to the global coordinate system $[\mathbf{R^{e^T}K^e_E R^e}]$.

4 Results and Discussion

A straight horizontal pipe with both ends clamped and suspended in air is used in this paper to test the implementation of the slug gravity forces, using an internal diameter of 0.24 meters, external diameter of 0.28 m, length of 20 m and elasticity modulus of $2.07 \times 10^8 kN/m$. The curvature, mass and weight are neglected.

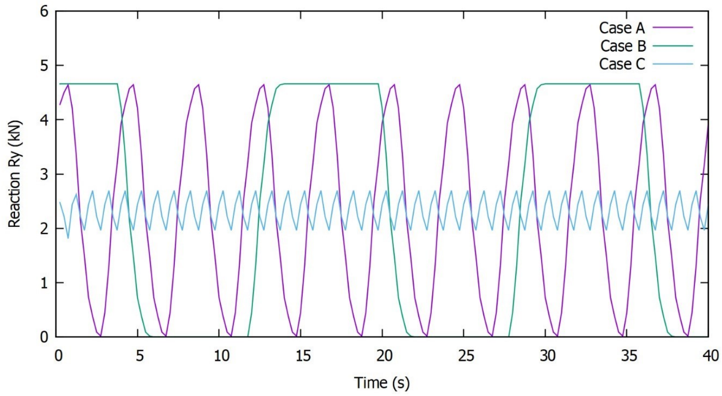


Figure 1. Right end reaction of a horizontal pipe submitted to a slug flow

To this first analysis, as presented in Bordalo and Morooka [10], three slug configuration are used, with the same holdup (volumetric fraction) and velocity, but with different lengths, which results in different frequencies, due the relation presented in eq. (6). The slug properties can be seen in Table 1. If the horizontal pipe be filled completely by liquid its weight is 8.8 kN.

Table 1. Slug properties for cases A, B and C.

Properties	Case A	Case B	Case C
Liq. spec. weight $[kN/m^2]$	9.81	9.81	9.81
Gas spec. weight $[kN/m^2]$	9.8e-5	9.8e-5	9.8e-5
Gas and Liquid Length [m]	20	80	5
Slug Length [m]	40	160	10
Velocity $[m/s]$	10	10	10
Slug Frequency $[Hz]$	0.25	0.0625	1
Slug Period $[s]$	4	16	1

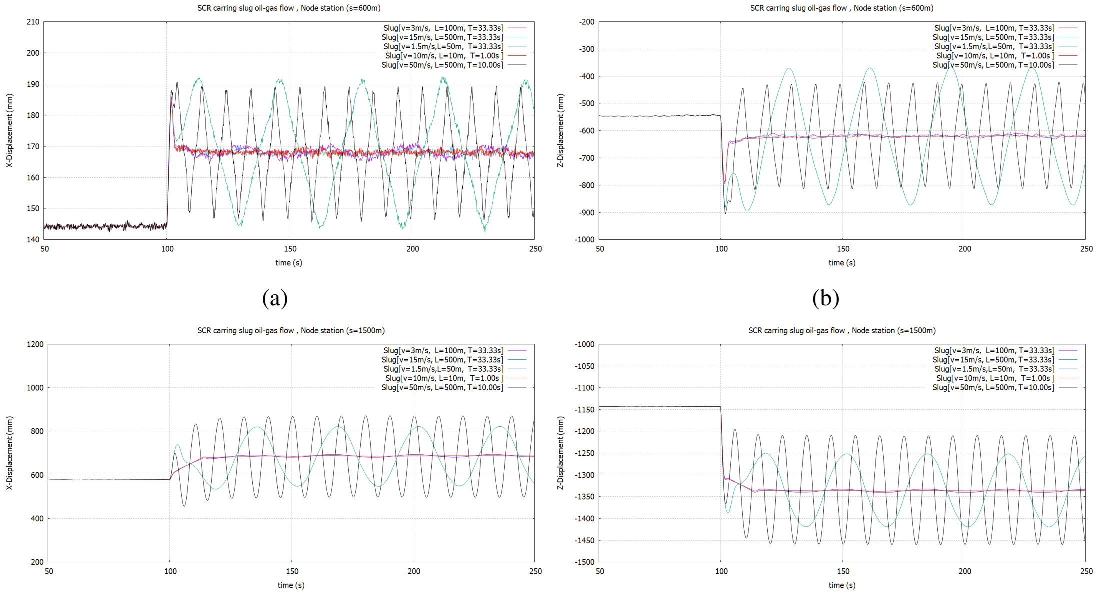
Table 2. SCR Case Properties [10]

Properties	Value
Riser length [m]	3100
Horizontal projection [m]	1918
Water depth [m]	2000
Suspended Length [m]	2594
External diameter [m]	0.212
Internal diameter [m]	0.165
Elasticity modulus [kN/m²]	207e6
Pipe spec. weight [kN/m³]	77.7
Liq. spec. weight [kN/m³]	6.76
Gas spec. weight [kN/m³]	4.63

Fig. 1 shows the results of reaction force at the right end of the pipe for each case with its respective slug traveling in 10 m/s. In regards of the case A, in each cycle the pipe is filled once with 20 m of liquid and then filled with 20 m of gas. The reaction force reaches, once every cycle, a maximum value of 4.4 kN, when the pipe is filled with liquid, and a minimum value of 0 kN, when filled with gas. In the case B, the pipe is kept filled up with each phase for 6 s, being necessary 2 s to fill up and empty out. The force reactions in this second case keep their maximum of 4.4 kN and its minimum of 0 kN for 6 seconds. The case C is quite different, the pipe is never filled up by a single phase, but for two slug inside the pipe at any given moment. The liquid inside the pipe is 50% of a filled up situation at every instant, and once the weight distribution varies along the time as the slugs pass through the pipe, the fluid weight resultant moves from one end to the other one, and the reaction of each end varies accordingly, reaching their peak every 1 second.

Proceeding with the study considering a steel catenary riser with the properties listed on Tab (2), it is analysed the vertical and horizontal displacement on two points, the first one at the length of 600 m of the riser from the X-tree, near the TDP, and the other one of 1500 m at the length of the riser approximately at the middle of it. Results are shown on Figs. (2d), respectively. Five cases were presented, the first one considering a slug with velocity of 3 m/s, length of 100 m and period of 33.33 seconds, the second one with velocity of 15 m/s, length of 500 m and period of 33.33 seconds, the third one with velocity of 1.5 m/s, length of 50 m and period of 33.33

seconds, the fourth one with 10 m/s of velocity, 10 meters of length and 1 second of period, finally the last one with velocity of 50 m/s, length of 500 m and period of 10 seconds.



(a) (b) (c) (d)

Figure 2. Node 600 - (a) horizontal displacement, (b) vertical displacement. Node 1500 - (c) horizontal displacement, (d) vertical displacement

As can be noted by the four figures which represent the displacement in horizontal and vertical directions, the riser oscillates even without external environment forces, only with the excitation from the internal flow. Another point which must be highlighted is about the node 600, which is close to the touchdown zone, the region where the pipe touches the seabed, it can be noted that the vertical displacement is higher than the horizontal one when looking for the slugs with higher length, once there is more freedom to move in its direction, which is almost perpendicular to the riser body, while the node 1500, that is almost the midway of the riser presents a higher displacement than the node 600 but with a similar magnitude in both directions, a media of approximately 13% of the external diameter of the riser.

In order to analyse the influence of the parameters on the dynamic response of the riser, different slug conditions are presented as mentioned before. It is possible to observe that when the slug length is keep the same but the velocity increases, for the node near the TDP there is no much change while for the node at the mid-length of the riser both the vertical and the horizontal displacement get higher. It is also observed that the amplitude for case 2 and 5 are amplified due its proximity to the natural frequency of the pipeline.

5 Conclusions

This study analysed the dynamic behavior of steel catenary risers subjected to internal gas-liquid slug flow. Using the finite element method with the Newmark integrator and Newton-Raphson method, it was possible to modeled the complex interactions between the riser's structural dynamics and the internal flow characteristics. The analysis showed that the slug flow induces significant oscillatory behavior in the riser even without any other load, which can lead to potential fatigue and structural failures. Numerical results showed that the density variation and the velocity of the slug flow are critical factors influencing the riser's response.The study also highlighted the necessity of accounting for large displacements and nonlinearities in the modeling process, as these factors are crucial for accurately predicting the riser's dynamic response. This research contributes to a deeper understanding of the challenges faced in the operation of offshore risers, particularly about slug flow-induced vibrations. Future work will focus on exploring different flow patterns and their impact on riser behavior with different configuration such as steel lazy wave riser, as well as the addition of current load and wave movement.

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