Finite element analysis of double nanobeams having Pasternak foundation in between

Gabriella L. Henrique[1], Ana C. F. Siqueira[1], Arthur C. A. Pereira[1], Welky K. F. Brito[1], Angelo V. Mendonca[1], Paulo C. Nascimento Jr[2]

[1]*Dept. of Civil and Environmental Engineering, Federal University of Paraíba*
Campus I, 58059-900, João Pessoa/PB, Brazil
glh@academico.ufpb.br, claara_ferreira@hotmail.com, acppereira@gmail.com, klefsonbrito@hotmail.com, mendonca@ct.ufpb.br
[2]*University of Pernambuco*
Campus Garanhuns, Garanhuns/PE, Brazil
paulocavalcante_jr@hotmail.com

Abstract. Single/double nanobeams (nanowires) have attractive features including reduced sizes and high flexibility and conductivity. As a result, many engineering applications such nanoelectromechanical systems (NEMS) and biomedical devices have been developed in technology industries. Due to the extremely high surface area-to-volume ratio, the properties of nanobeams have size-dependent behavior. In this paper, elastically connected double nanobeams are represented on Eringen's nonlocal elasticity theory. Each nanobeam of double beam is modeled as Euler-Bernoulli beam and the interconnecting layer is represented by a Pasternak's elastic foundation model. A four-node double-beam finite element with eight degrees of freedom using approximate functions to interpolate transverse displacements and rotations is derived where both stiffness matrix and load vector are explicitly shown. The present FEM solution is validated by numerical examples where the influence of effects of dimensionless small-scale parameters, boundary conditions, and shear parameter of Pasternak foundation on displacements and stress resultants of double nanobeams are investigated.

Keywords: FEM, Two-Parameter Foundation, double nanobeams.

1 Introduction

Nanomaterials have intensely stimulated the interest of the scientific researcher's communities in physics, chemistry, and engineering. Due to nanoscale dimensions, their special properties need to be investigated and nonlocal differential elasticity has gained more popularity among researchers as compared to the nonlocal integral elasticity, as example Eringen's nonlocal theories [1]. Murmu and Adhikari [2] interoduced a nonlocal double-elastic beam model, and used it to Analyse size effects on the free vibration of double-nanobeam systems.

A notable class of engineering problems is concerned with the structures resting on an elastic type foundation, that had been initially simulated by an one-parameter linear model, introduced by Winkler (1867), consideredering the simplest and the most widely used, As an improvement, models of two and three elastic parameters had been established to deal with the basic disadvantage of Winkler's model, which is the displacements' discontinuity on the boundary being the example Pasternak (1954) and Kerr (1964).

The studies envolving the problems about the suitability of the generalized continuum theories for the simulation of the micro- and nano-materials, more specifically double or multiple nanobeams systems, are growing, approaching issues relating to, bending problems, transverse vibration of double systems [2], eletromechanical stability [3], the consideration of elastic médium by a Hamiltonian method [4], graded double-nanobeam system [5] analytical solutions of double-system with viscoelastic layer between [6], multiple-nanobeams system coupled by Winkler elastic layer [7] and a nonlocal finite element model [8] aiming about the complexity of carrying out experiments can be costly and technically demanding, making cientists resort to other models that satisfactorily address the problem.In the present work, finite element formulation for nonlocal elasticity approach of Euler–Bernoulli beam theory have been reported, where double-nanobeam system is interconnected by a two parameter elastic foundation by employing the Bubnov–Galerkin method.

2 Mathematical representations

2.1 Nonlocal elasticity theory

The theory of nonlocal elasticity, which has as one of its precursors Eringen, and the nonlocal stress and strain tensorm differential formo f the nonlocal characteristics can also be seen in Rahmani *et al.* [9]. Thus, the nonlocal constitutive relationship (to obtain the constitutive relation of the local theory, simply set the nonlocal parameter $e_0 a = 0$) is expressed as:

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\varepsilon_{xx} \tag{1}$$

where σ_{xx} is the axial normal stress, E is the elasticity modulus, ε_{xx} is the axial strain and $(e_0 a)^2$ can be simplified as the constant nonlocal parameter μ. The stress resultants can be written in terms of displacements as:

$$M_x = -EI \frac{\partial^2 w}{\partial x^2} - \mu q \tag{2}$$

where w and q are the transversal displacements and transversal load, respectively.

2.2 Double nanobeam system

Some hypotheses support the system of parallel nanobeams elastically connected by a connection layer. The main assumptions of the Euler-Bernoulli beam are: a) transverse normal stress is small compared to axial normal stress; b) straight lines that initially are normal to the mid-plane of the beams remain straight during the bending process; c) beam is made of a linear elastic, isotropic, homogeneous material (governed by Hooke's constitutive law); and, d) displacement, rotation, and strain are assumed to be smooth (small) fields.

The connecting layer is assumed to be a system of mutually independent and is adopted Pasternak's model to represent the nanobeam model, where linearly-elastic springs combined with shear layer

The representation of a model of a system of nanobeams with equal lengths L and subjected to distributed loads $g_1(x)$ and $g_2(x)$, can be seen Fig. 1.

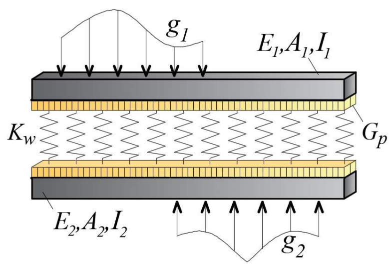


Figure 1. Double-nanobeam between Pasternak elastic layer under distributed load.

The force balance equations of the upper and lower nanobeam can be written as:

$$E_1 I_1 w'''' + K_w(w - v) - G_p(w'' - v'')$$
$$+\mu_1\left[-K_w(w'' - v'') + G_p(w'''' - v'''') + {g_1}''\right] = g_1 \quad (3)$$
$$E_2 I_2 w'''' - K_w(w - v) + G_p(w'' - v'')$$
$$+\mu_2\left[K_w(w'' - v'') - G_p(w'''' - v'''') + {g_2}''\right] = g_2 \quad (4)$$

where E_1, I_1, w and μ_1 are Young's modulus, moment of inertia, transverse displacement of the upper beam and nonlocal parameter respectively. E_2, I_2, v and μ_2 are the respective counterparts of the lower beam. L is the beam length. K_w is the spring coefficient and G_p is the coefficient of shear layer of the Pasternak's model.

In order to derive the double-nanobeam finite element, interpolation functions must be assumed to both displacements and loading. Using a nondimensional parameter $\xi = 2x/L$, the transverse displacements of upper and lower beams can be interpolated as follows

$$w(\xi) = [N]\{u_1\} = [N_1(\xi) \quad N_2(\xi) \quad N_3(\xi) \quad N_4(\xi)] \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix} \quad (5)$$

$$v(\xi) = [N]\{u_2\} = [N_1(\xi) \quad N_2(\xi) \quad N_3(\xi) \quad N_4(\xi)] \begin{Bmatrix} d_5 \\ d_6 \\ d_7 \\ d_8 \end{Bmatrix} \quad (6)$$

where $[N]$ denotes the row matrix containing N_i functions. In this paper, approximate interpolation functions for displacements of the Euler-Bernoulli double beam system are assumed to be

$$N_1(\xi) = \frac{(\xi - 1)^2(\xi + 2)}{4} \quad (7)$$

$$N_2(\xi) = \frac{L(\xi - 1)^2(\xi + 1)}{8} \quad (8)$$

$$N_3(\xi) = -\frac{(\xi + 1)^2(\xi - 2)}{4} \quad (9)$$

$$N_4(\xi) = \frac{L(\xi - 1)(\xi + 1)^2}{8} \quad (10)$$

and the nodal vectors associated with degrees of freedom d_i of upper and lower beams are $\{u_1\}$ and $\{u_2\}$, see Fig 2(a).

The distributed loading of upper and lower beams can be interpolated by

$$g_1(\xi) = [P]\{g_1\} = [P_1(\xi) \quad P_2(\xi)] \begin{Bmatrix} g_{1i} \\ g_{1j} \end{Bmatrix} \quad (11)$$

$$g_2(\xi) = [P]\{g_2\} = [P_1(\xi) \quad P_2(\xi)] \begin{Bmatrix} g_{2i} \\ g_{2j} \end{Bmatrix} \quad (12)$$

where $[P]$ denotes the row matrix containing P_i functions. $\{g_1\}^T = (g_{1i} \quad g_{1j})$ and $\{g_2\}^T = (g_{2i} \quad g_{2j})$ are the nodal loading vectors of upper and lower nanobeams, see Fig 2(b).

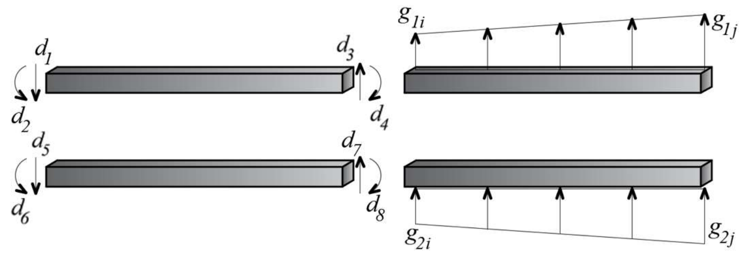


Figure 2. (a) Degrees of freedom of double nanobeam element; (b) Distributed loads.

Using principle of minimum total potential to form the functional in case of nonlocal elasticity theory, the weighted-integral statement of eq. (3) and eq. (4)can be written as:

$$\int_0^L \{E_1 I_1 w'''' + K_w(w - v) - G_p(w'' - v'') + \\ +\mu_1[-K_w(w'' - v'') + G_p(w'''' - v'''') + g_1'']\chi\}dx = 0 \tag{13}$$

$$\int_0^L \{E_2 I_2 w'''' - K_w(w - v) + G_p(w'' - v'') + \\ +\mu_2[K_w(w'' - v'') - G_p(w'''' - v'''') + g_2'']\lambda\}dx = 0 = g_2 \tag{14}$$

where χ and λ are upper and lower nanobeam weight functions respectively. In this paper, a Bubnov-Galerkin procedure is adopted.

Performing integration by parts on eq. (13)-(14), the weak form can be written as:

$$\int_0^L E_1 I_1 w''\chi'' + K_w(w - v)\chi + G_p(w' - v')\chi' - g_1\chi \\ \mu_1[-K_w(w' - v')\chi' + G_p(w'' - v'')\chi'' + g_1\chi'']dx = 0 \tag{15}$$

$$\int_0^L E_2 I_2 w''\lambda'' + K_w(w - v)\lambda - G_p(w' - v')\lambda' - g_2\lambda \\ \mu_2[-K_w(w' - v')\lambda' + G_p(w'' - v'')\lambda'' + g_2\lambda'']dx = 0 \tag{16}$$

Performing all integrals in Eq.(15)-(16), the FEM algebraic system can be shown as follows

$$[K]\{D\} = \{F\} \tag{17}$$

where $[K]$ is the stiffness matrix and $\{F\}$ is the equivalent load vector, writted in the alternative forms as:

$$[K] = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{12}] \end{bmatrix}, \{F\} = \begin{Bmatrix} \{F_1\} \\ \{F_2\} \end{Bmatrix}, \{D\} = \begin{Bmatrix} \{u_1\} \\ \{u_2\} \end{Bmatrix} \tag{18}$$

where

$$[K_{11}] = (E_1 I_1 + \mu_1 G_p)[K_b] + (G_p + \mu_1 K_w)[K_\gamma] + K_w[K_\beta]$$
$$[K_{12}] = -\mu_1 G_p[K_b] - (G_p + \mu_1 K_w)[K_\gamma] - K_w[K_\beta]$$
$$[K_{22}] = -\mu_2 G_p[K_b] - (G_p + \mu_2 K_w)[K_\gamma] - K_w[K_\beta] \tag{19}$$
$$[K_l] = (E_2 I_2 + \mu_2 G_p)[K_b] + (G_p + \mu_2 K_w)[K_\gamma] + K_w[K_\beta]$$

Explicit forms for the matrices in eq. (19) can be obtained with the help of eq. (7)-(10) and (5a). resulting in

$$[K_b] = \int_{-1}^{1} \left[\frac{d^2N}{d\xi^2}\right]^T \left[\frac{d^2N}{d\xi^2}\right] J d\xi = \frac{1}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{20}$$

$$[K_\beta] = \int_{-1}^{1} [N]^T [N] J d\xi = \frac{L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{21}$$

$$[K_\gamma] = \int_{-1}^{1} \left[\frac{dN}{d\xi}\right]^T \left[\frac{dN}{d\xi}\right] J d\xi = \frac{1}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \tag{22}$$

In this paper was assumed a linear distribution for the loading, the functions, thus P_i are $P_1(\xi) = (1 - \xi)/2$ and $P_2(\xi) = (1 + \xi)/2$, so that the equivalent load vector can be written as follows

$$\{F_1\} = \int_{-1}^{1} [N]^T [P] J \, d\xi \{f_1\} \tag{23}$$

$$\{F_2\} = \int_{-1}^{1} [N]^T [P] J \, d\xi \{f_2\} \tag{24}$$

For the case of constant distributed loads, the equivalent load vector is explicitly written as:

$$\{F_1\} = \left[\frac{g_1 L}{2} \quad \left(\frac{L^2}{12} + \mu_1\right) g_1 \quad \frac{g_1 L}{2} \quad -\left(\frac{L^2}{12} + \mu_1\right) g_1\right] \tag{25}$$

$$\{F_2\} = \left[\frac{g_2 L}{2} \quad \left(\frac{L^2}{12} + \mu_2\right) g_2 \quad \frac{g_2 L}{2} \quad -\left(\frac{L^2}{12} + \mu_2\right) g_2\right] \tag{26}$$

3 Numerical results

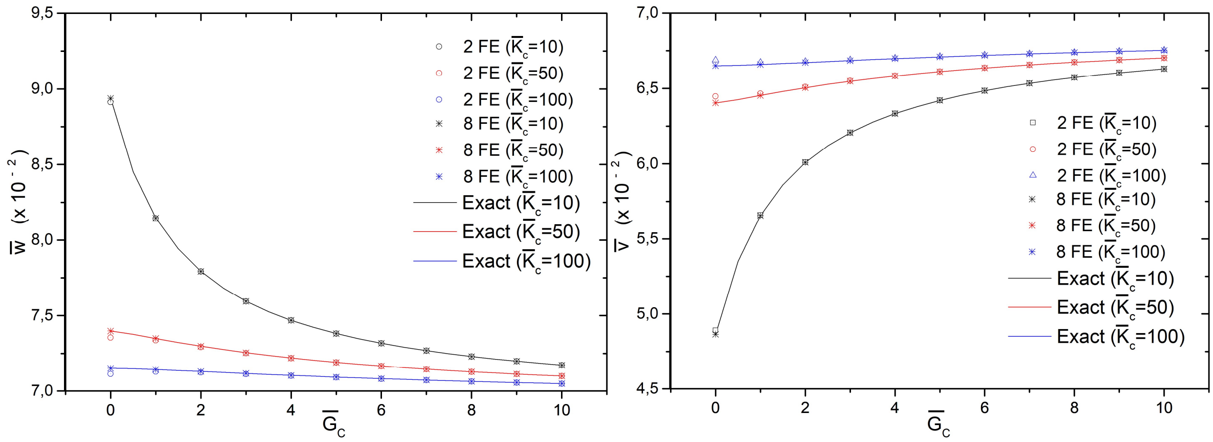
3.1 Simply support double-nanobeam system

Consider a system of simply supported double nanobeams of cross-section with dimensions b and h, length L, connected by an elastic layer of normalized stiffness ($\overline{K_c} = 100$) and normalized shear ($\overline{G_c}$=0.1). The nanobeams are identical and have a Young's modulus $E_1 = E_2$, $I_1 = I_2$. A uniform distributed load is applied to the upper beam g_1.

Table 1. Dimensionless maximum deflections $\bar{w}$ and $\bar{v}$ and maximum rotation of double nanobeams subjected to uniform distributed load

$\overline{\mu_1} = \overline{\mu_2}$	$Response$	$2\ FE$	$4\ FE$	$8\ FE$	$Analytical$
1	$\bar{w}(x10^{-2})$	7.1180	7.1510	7.1510	7.1510
	$d\bar{w}/dx(x10^{-1})$	2.9728	3.0205	3.0306	3.0257
	$\bar{v}(x10^{-2})$	6.6840	6.6510	6.6510	6.6510
	$d\bar{v}/dx(x10^{-1})$	2.4439	2.3961	2.3860	2.3843
2	$\bar{w}(x10^{-1})$	1.3355	1.3400	1.3401	1.3401
	$d\bar{w}/dx(x10^{-1})$	5.5121	5.6011	5.6272	5.6196
	$\bar{v}(x10^{-1})$	1.2947	1.2902	1.2901	1.2901
	$d\bar{v}/dx(x10^{-1})$	4.9046	4.8156	4.7895	4.7836
3	$\bar{w}(x10^{-1})$	1.9599	1.9650	1.9651	1.9651
	$d\bar{w}/dx(x10^{-1})$	8.0292	8.1453	8.1855	8.1761
	$\bar{v}(x10^{-1})$	1.9203	1.9152	1.9151	1.9151
	$d\bar{v}/dx(x10^{-1})$	7.3874	7.2714	7.2312	7.2203
4	$\bar{w}(x10^{-1})$	2.5846	2.5900	2.5901	2.5901
	$d\bar{w}/dx$	1.0539	1.0674	1.0725	1.0714
	$\bar{v}(x10^{-1})$	2.5456	2.5402	2.5401	2.5401
	$d\bar{v}/dx(x10^{-1})$	9.8778	9.7431	9.6913	9.6752
5	$\bar{w}(x10^{-1})$	3.2093	3.2150	3.2151	3.2151
	$d\bar{w}/dx$	1.3045	1.3193	1.3255	1.3242
	$\bar{v}(x10^{-1})$	3.1709	3.1653	3.1651	3.1651
	$d\bar{v}/dx$	1.2372	1.2223	1.2162	1.2141

The maximum dimensionless deflections were computed for different values of the nonlocal parameter and nonlocal FEM. The parameters were normalized by: $\overline{K_c} = K_w L^4/E_1 I_1$, $\overline{G_c} = G_p L^2/E_1 I_1$, $\overline{\mu_1} = \mu_1/L^2$, $\overline{\mu_2} = \mu_2/L^2$, $\bar{w} = wE_1 I_1/g_1 L^4$ and $\bar{v} = vE_1 I_1/g_1 L^4$The analytical response could be obtained by the nonlocal differential equation substituting $w = \sum_{m=1}^{m1} A_m\, sin(m\pi x/L\,)$ and $v = \sum_{m=1}^{m1} B_m\, sin(m\pi x/L\,)$ in the equilibrium equation of the double nanobeam system and employing convenient mathematical procedures. For the convergence analysis, the nanobeam was discretized into 2, 4 and 8 finite elements (FE). Thus, the FEM responses to displacements using different discretizations and its analytical solution were compared in Table 1. As can be seen, the formulation is very attractive, since it presents satisfactory response and an excellent agreement between the FEM results and the analytical solutions.



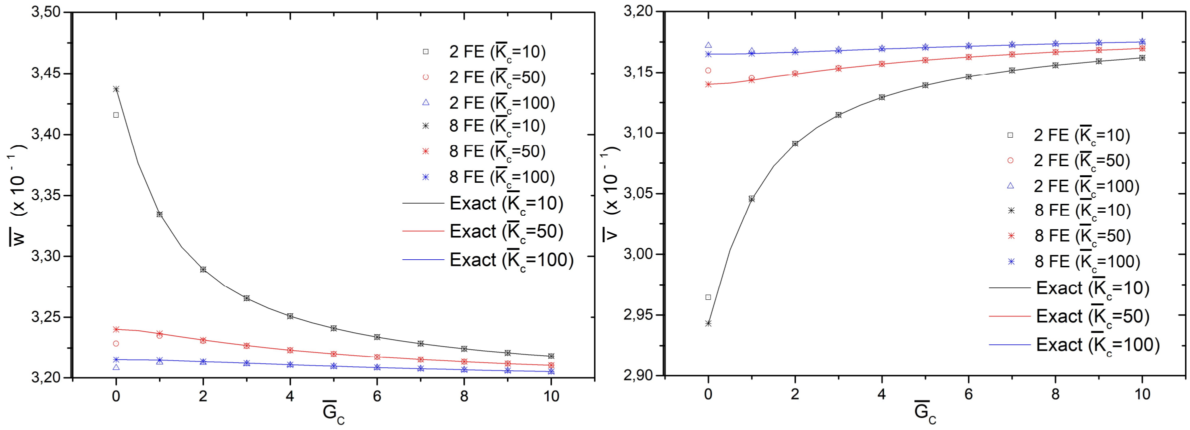


Figure 3. (a-b) Displacement of upper and lower nanobeam ($\overline{\mu_1} = \overline{\mu_1} = 1$); (c-d) Displacement of upper and lower nanobeam ($\overline{\mu_1} = \overline{\mu_1} = 5$).

4 Conclusions

In this present work, the flexural response of a double nanobeam system elastically connected by Pasternak layer is obtained using the Finite Element Method. Using the Euler-Bernoulli beam theory, a finite element with eight degrees of freedom is derived and the both stiffness matrix and load vector are explicitly obtained.

The formulation is very attractive, showing a satisfatory response when comparing wiht exact solutions, since, it can to solve any problems considering the more various boundary conditions that the analytical solutions cannot solve.

Acknowledgements. The authors acknowledge the Coordination for the Improvement of Higher Education Personnel (CAPES) for the financial support of this research.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] A. Rahmani, S. Faroughi, M. I. Friswell, and A. Babaei, "Eringen's nonlocal and modified couple stress theories applied to vibrating rotating nanobeams with temperature effects,"*Mech. Adv. Mater. Struct.*, pp. 1–26, 2021.
[2] T. Murmu and S. Adhikari, "Nonlocal transverse vibration of double-nanobeam-systems,"*J. Appl. Phys.*, vol. 108, pp. 083514–083519, 2010.
[3] R. Bahaadini, M. Hosseini, and Z. Khalili-Parizi, "Electromechanical stability analysis of smart double-nanobeam systems,"*Eur. Phys. J. Plus*, vol. 134, 2019.
[4] Z. Zhou, Y. Li, J. Fan, D. Rong, G. Sui, and C. Xu, "Exact vibration analysis of a double-nanobeam-systems embedded in an elastic medium by a Hamiltonian-based method,"*Phys. E Low-dimens. Syst. Nanostruct.*, vol. 99, pp. 220–235, 2018.
[5] S. Sari Me, G. W. Al-Kouz, and A M. Atieh, "Transverse vibration of functionally graded tapered double nanobeams resting on elastic foundation,"*Appl. Sci.*, vol. 10, p. 493, 2020.
[6] Chen, Bo et al. Analytical solutions of nonlocal forced vibration of a functionally graded double-nanobeam system interconnected by a viscoelastic layer. Zeitschrift für Naturforschung A, v. 77, n. 9, p. 851-873, 2022.
[7] Karličić, Danilo; Kozić, Predrag; Pavlović, Ratko. Nonlocal vibration and stability of a multiple-nanobeam system coupled by the Winkler elastic medium. Applied Mathematical Modelling, v. 40, n. 2, p. 1599-1614, 2016.
[8] Aria AI, Friswell MI (2019) A nonlocal finite element model for buckling and vibration of functionally graded nanobeams. Compos Part B Eng 166:233–246
[9] Rahmani, O.; Refaeinejad, V.; Hosseini, S. A. H. Assessment of various nonlocal higher order theories for the bending and buckling behavior of functionally graded nanobeams. Steel Compos. Struct, v. 23, n. 3, p. 339-350, 2017.