Structural analysis of beams on masonry: an analytical modeling

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Abstract. Structural masonry stands out in building projects as an efficient and versatile solution due to its function not only in sealing environments, but also in supporting and resisting different types of loads, in addition to presenting fast construction. The stiffness of masonry plays a crucial role in the global behavior of the structure, as it contributes to the safety and resistance of the set, thus obtaining a more robust and resilient system. Thus, this paper aims to do a structural analysis of a beam on masonry using elastic supports in the calculation of bond beams used in structural masonry. In the end, the analysis highlighted the influence of masonry stiffness on the elastic line and the distribution of sectional forces in the beam. The results provided a detailed visualization of the behavior of the beam and the masonry that supports it. In this context, the study in question constitutes an excellent application of concepts with an interesting use to the technical community and proved essential to deepen the structural analysis in question.

Keywords: analysis of bond beam on masonry, elastic supports, bond-beam.

1 Introduction

Structural masonry is a construction system in which the walls go beyond the function of isolating environments and take on also a structural function, supporting the vertical and lateral loads. Among the components, the bond-beam acts as a critical element, ensuring uniform distribution of the loads transmitted to the wall and contributing to the stability and integrity of the construction. Therefore, accurate dimensioning is essential to ensure that the beam performs its function effectively, preventing cracks, unevenness and structural collapses.

The calculation of bond-beams can be approached in some ways, one of them is using elastic supports. In this context, the masonry and the beams are treated as materials that obey the laws of elasticity, allowing the application of mathematical models to predict their behavior under different load conditions. In addition, an advantage of this method is the possibility of establishing the effects of the redistribution of forces in the beams, making the modeling more efficient and reliable.

This paper discusses a practical example of the calculation of a beam used in structural masonry by applying the classical theory of beams on elastic foundation. For this purpose, the wall highlighted in Fig. 1 and Fig. 2 was used, which is located on the top floor of a four-story residential building, with a length of 15.35 m and a floor-to-ceiling height of 2.70 m. The analyzed beam has a rectangular cross-section of 14 x 30 cm and two loads uniformly distributed along its length, due to the roof slab and the upper water reservoir, these being concentric loads of 1.40 tf/m and 3.10 tf/m acting on 15.35 m and 6.64 m of the beam, respectively. Finally, the properties of reinforced concrete considered were characteristic strength (f_{ck}) of 30 MPa and modulus of elasticity (E_c) calculated according to ABNT NBR 6118:2023 [1].

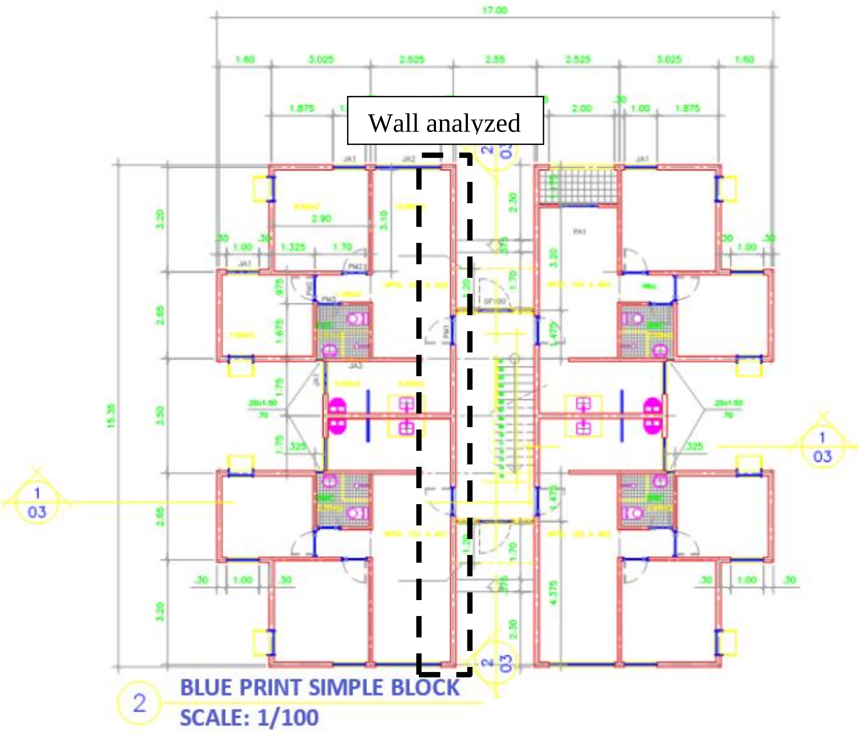

Figure 1. Analyzed element of the building
Source: Authors

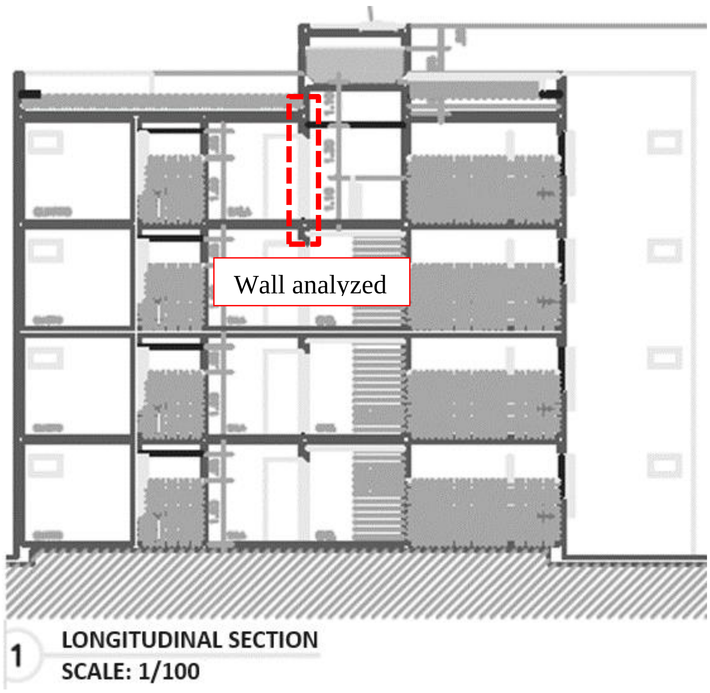

Figure 2. Longitudinal section of the analyzed building
Source: Authors

2 Beams on elastic supports

A load acting on a beam supported by structural masonry is transmitted to the masonry, which, under the action of the received load, will deform. It can then be said that the masonry is, for the beam, an elastic support, since it absorbs a reaction through a deformation in the direction of the absorbed force.

According to Hetényi [2] and Süssekind [3], for a beam with constant inertia, resting on an elastic support, subjected to a uniformly distributed load q acting on the section *AB*, shown in Figure 3a, the deflection line (y) and bending moment (M) of the sections under this load are given by the following equations:

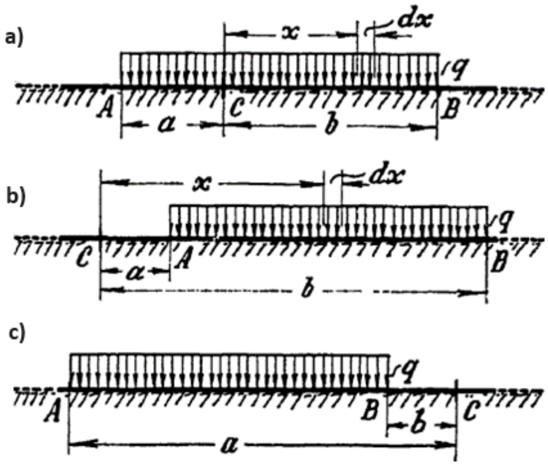

Figure 3. Scheme for calculating uniformly distributed loads
Source: Hetényi [2]

$$y = \frac{q}{2K}\left(2 - e^{-\beta a}\cos(\beta a) - e^{-\beta b}\cos(\beta b) \right) \quad (1)$$

$$M = \frac{q}{4\beta^2}\left(e^{-\beta a}\sin(\beta a) + e^{-\beta b}\sin(\beta b) \right) \quad (2)$$

Furthermore, β is obtained from eq. (3):

$$\beta = \sqrt[4]{\frac{K}{4EI}} \quad (3)$$

Where:

q – Distributed load applied to the beam
E – Modulus of elasticity of the material used
I – Moment of inertia of the beam cross section
a and b – Distances obtained as illustrated in Fig. 3
K – Spring constant

For the section located to the left of the loading section, as illustrated in Fig. 3b, eq (4) and eq. (5) are used.

$$y = -\frac{q}{2K}\left(e^{-\beta a}\cos(\beta a) - e^{-\beta b}\cos(\beta b) \right) \quad (4)$$

$$M = \frac{q}{4\beta^2}\left(e^{-\beta a}\sin(\beta a) - e^{-\beta b}\sin(\beta b) \right) \quad (5)$$

For the section located to the right of the loading section, as shown in Fig. 3c, eq. (6) and eq. (7) are used.

$$y = -\frac{q}{2K}\left(e^{-\beta a}\cos(\beta a) - e^{-\beta b}\cos(\beta b) \right) \quad (6)$$

$$M = -\frac{q}{4\beta^2}\left(e^{-\beta a}\sin(\beta a) - e^{-\beta b}\sin(\beta b) \right) \quad (7)$$

Thus, the effect of several loads on the structure can be evaluated by means of the principle of superposition, adding the results obtained individually, so that for two uniformly distributed loads it is possible to calculate the displacements and bending moments generated individually and superimpose them.

Generally, the maximum values of deflection and bending moment are of great importance. According to Boresi and Schmidt [4], depending on the magnitude of βL, with L being the length of the beam, the following relationships are obtained:

i. $\beta L \leq \pi$: the maximum bending moment occurs at the center of segment L.

ii. $\beta L > \pi$: as βL increases, the bending moment tends to zero and the deflection tends to q/K everywhere except near the ends of segment L. Furthermore, the location of the maximum moment can be obtained with sufficient accuracy as being $\pi/(4\beta)$ from the ends of the uniformly distributed load within length L.

2.1 Spring constant for masonry

To consider the action of the masonry in the calculation of the beam, it was considered that the wall acts as an elastic support, thus presenting a spring coefficient along the length of the beam. Thus, to define the spring constant k, the guidelines of ABNT NBR 16868:2020 [5] were followed, starting with the calculation of the longitudinal deformation modulus of the masonry (E_{alv}), given by eq. (8) for ceramic structural masonry blocks.

$$E_{alv} = (600)\left(f_{pk}\right) \tag{8}$$

Where:

f_{pk} - characteristic compression strength of the prism.

Following the recommendation of ABNT NBR 16868:2020 [5] for the specification of structural masonry materials, for a solid ceramic wall block with dimensions of 14 x 29 cm, for the compressive strength of the block (f_{bk}) as 10.00 MPa, the characteristic compression strength of the prism (f_{pk}) is 6.00 MPa.

In this work, the masonry modulus of elasticity was related to the spring constant by multiplying E_{alv} by the masonry thickness, according to Sperotto [6], thus obtaining the spring constant per linear meter of masonry. For a thickness of 0.14 m, eq. (9) was used.

$$k = \left(E_{alv}\right)(0.14\,m) \tag{9}$$

2.2 Results and discussion

For the structure studied, for the length of 15.35 m, the diagrams obtained analytically for the deflection and bending moment due to the load of 1.40 tf/m are shown in Fig. 4 and Fig. 5. For the load of 3.10 tf/m, they are shown in Fig. 6 and Fig. 7.

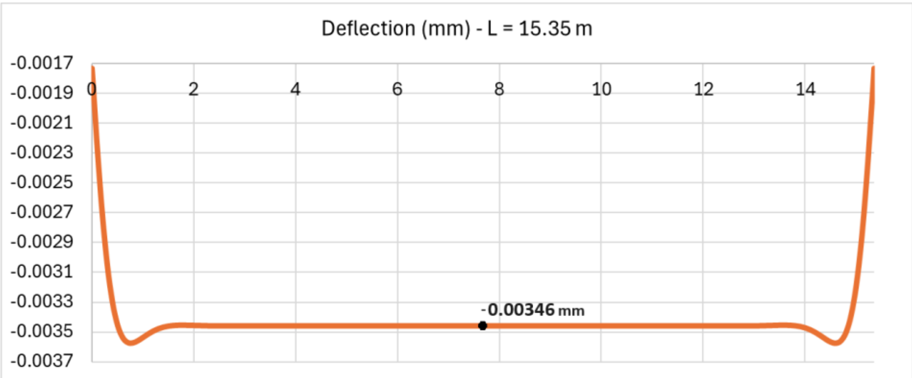

Figure 4. Displacements (in mm) in the beam due to the load of 1.4 tf/m

Source: Authors

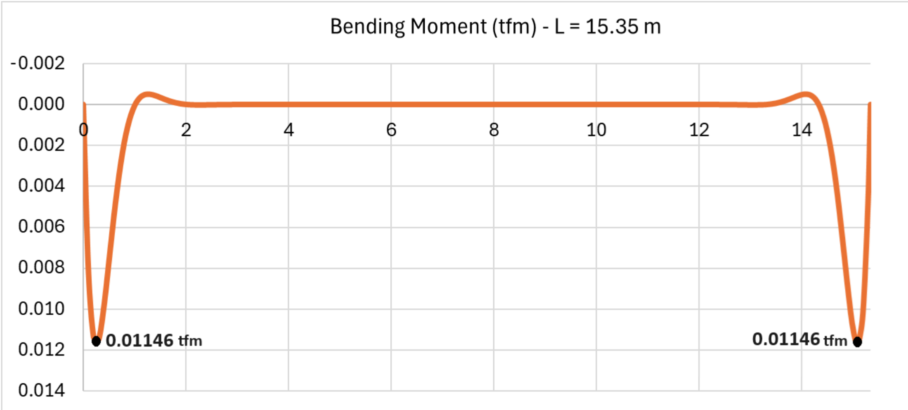

Figure 5. Bending moments (in tfm) in the beam due to the load of 1.4 tf/m
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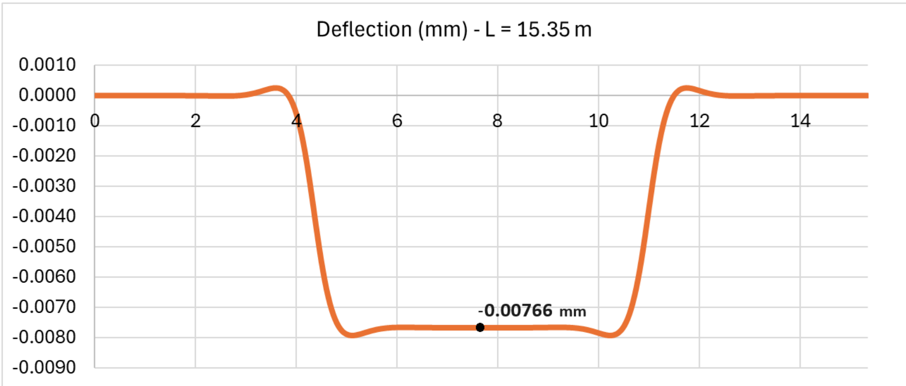

Figure 6. Displacements (in mm) in the beam due to the load of 3.1 tf/m
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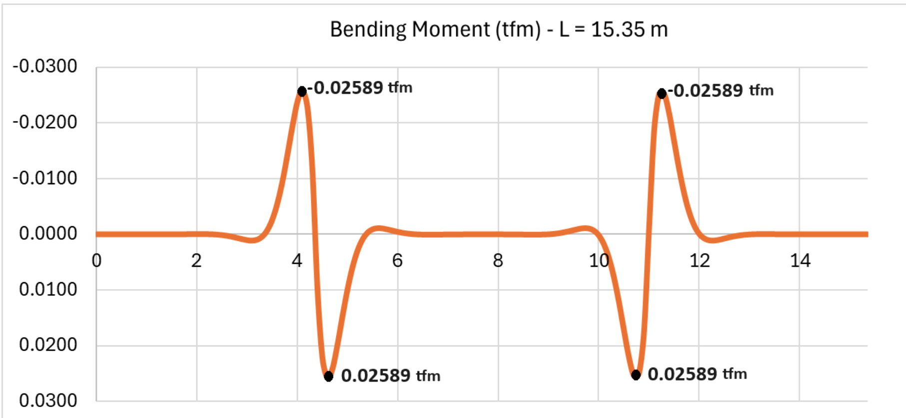

Figure 7. Bending moments (in tfm) in the beam due to the load of 3.1 tf/m
Source: Authors

The values of the total deflections found, adding each effect individually, are shown in Fig. 8, while the total bending moments along the beam are shown in Fig. 9.

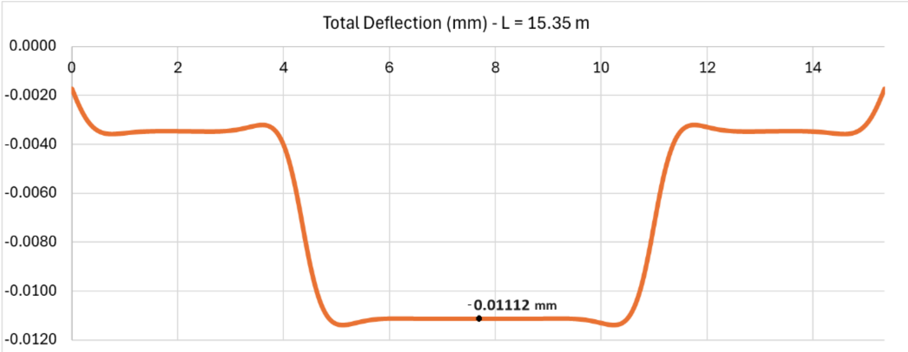

Figure 8. Total displacements (in mm) acting in the beam
Source: Authors

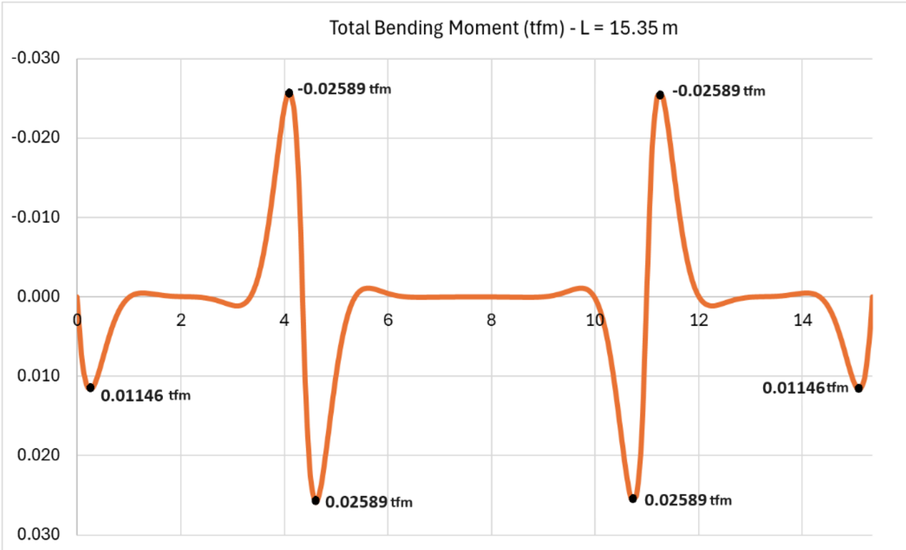

Figure 9. Total bending moments (in tfm) acting in the beam
Source: Authors

The previous figures show the effect of βL on the redistribution of forces along the beam, in which, for the length analyzed, a value of βL greater than π was obtained. Fig. 4 and Fig. 5 show the bending moment tended to zero and the deflection tended to q/K except near the ends. Fig. 6 and Fig. 7, which show the effects of the 3.1 tf/m load only, show the displacements and moments were more intense in the section under the load, with the same behavior mentioned above. However, there are also forces in the segments to the left and right of the applied load, which decrease as they move away from it. Fig. 8 and Fig. 9 show the contribution of the two loads to the structure, obtained by superimposing the individual results. Therefore, the results showed the influence of the masonry stiffness on the elastic line and on the distribution of sectional forces in the beam.

3 Conclusions

In this study, the effect of different loads acting on a structural masonry wall was evaluated using the theory of beams on elastic supports. The results obtained showed that the analysis agrees with the behaviors found in literature. With the beam of great length, it was possible to observe for each load separately the effect of using elastic supports on the elastic line and moments of the beam, displacing the points of maximum moments and deflections to the ends of the segments under the loads.

Therefore, the analytical calculation showed that the presence of an elastic support significantly modifies the structural response of the beam. Due to the continuous support provided by the elastic consideration, the deflection and bending moments did not behave in the classical manner expected for beams on point supports. By adding the effects of each load, peak moments and deflections were obtained at specific locations, varying according to the distributions of the loads along the length.

Acknowledgements. The first author wishes to express their gratitude to the “Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES)” for the financial support.

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