# Reliability Analysis of the Collapse of Corroded Submarine Pipelines Subjected to Bending Moment

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**Abstract.** For offshore oil exploitation, there is a need for pipelines that operate at great depths, thus being subjected to intense external pressure conditions. Besides the external pressure, there are also bending moment loads due to the deformation between the platform and the exploitation point. These deformations result from the weight of the riser and the gravity, forming a concave region known as the "sag-bend region". Additionally, corrosion defects may form, reducing the pipe wall thickness and its load-bearing capacity. Empirical equations in the literature calculate the maximum of these loads supported by the pipes, and they were used, combined with the statistical variability of the pipe dimensions, through the Monte Carlo reliability method, to analyze the behavior of these structures under uncertainties. From these analyses, the equations demonstrated good statistical accuracy, showing a greater sensitivity of the external pressure to corrosion defects, which is the main loading for the case of ultradeep pipelines and can lead to collapse, thus highlighting the importance of a careful analysis to avoid serious environmental problems.

**Keywords:** Subsea pipelines, Reliability analysis, External pressure, Bending moment.

## 1 Introduction

With the exploitation of oil in deep-sea environments, the use of large pipeline arrays is necessary for transporting the materials extracted in this process. These pipelines are subjected to various loads, and in this work, the focus will be on external pressure and bending moment loads, stemming respectively from the water column and the concave region, resulting from the weight of the riser and the action of gravity. Therefore, operation within the required standards is of fundamental importance to prevent the severe environmental, economic, and social damages that the collapse of submarine pipelines can cause.

When subjected to external pressure, the pipeline deforms its cross-section, resulting in an accumulation of circumferential stresses, with maximum stresses occurring at the top/bottom and both sides of the flattened cross-section. The ratio between the diameter ($D$) and the wall thickness ($t$), $D/t$, is determinant to the structural behavior (Kirkwood and Stephenson [1]).

In the case of pure bending moment, the pipeline will fail due to increased ovalization of its cross-section and a reduction in the slope of the stress-strain curve, leading to a reduction in the section's moment of inertia, resulting in catastrophic collapse. In this load situation, the $D/t$ ratio also is determinant to the pipeline's behavior (Kirkwood and Stephenson [1]). The region of greatest concavity, previously mentioned, is named "sag-bend region" (Bjørset [2]), is of interest in terms of moment, as it exhibits the greatest magnitude. The presence of bending moments results in additional stresses, reducing the pipeline's resistance to external pressures, making their consideration essential.

Due to corrosive factors, pipelines present corrosion defects that reduce wall thickness and, consequently, their resistance capacity. Therefore, when analyzing corroded pipelines, it is necessary to consider the geometry of these defects, either in the mathematical process (simulation) (Motta et al. [3]).

The dimensions of corrosion defects and external pressure loads values present uncertainties, which may result from measurement errors (Motta et al. [3]) and tidal phenomena (Teixeira et al. [4]), respectively. Hence, for a safer analysis, it is necessary to incorporate these uncertainties using statistically distributed measures obtained experimentally and applying reliability analysis theory.

This work proposes to conduct a reliability analysis for external pressure and bending moment loads using consolidated equations from the literature, comparing the results of defect-free pipelines with those having defects, drawing conclusions about the influence of wall thickness reduction on the resistance of submarine pipelines.

## 2 Methodology

Next, the equations used in this work for the analysis of bending moment, external pressure, and the application of random variables for structural reliability analysis using the Monte Carlo method will be explained.

### 2.1 Bending Moment

Zheng et al. [5] developed an empirical equation for calculating the maximum resistant moment in corroded pipelines. To understand it, it is necessary first to understand the pipeline scheme presented in Fig. 1

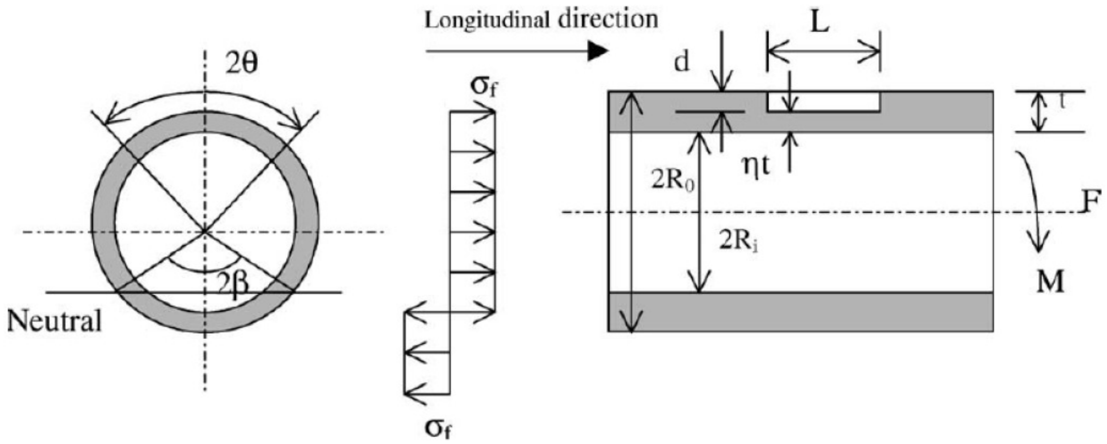


Figure 1. Pipeline geometry and corrosion defect, Zheng et al. [5]

The main variables of the pipeline geometry and the defect are described, which are: outer radius ($R_0$), inner radius ($R_i$), wall thickness ($t$), applied moment ($M$), defect depth ($d$), longitudinal length of the defect ($L$), circumferential angle of the defect ($\theta$), and neutral axis angle ($\beta$). eq. (1) is commonly used in the literature for calculating the resistant moment in intact pipelines, where we have the flow stress ($\sigma_0$) and the mean radius ($R_m$) of the pipeline.

$$M = 4\sigma_0 R_m^2 t \tag{1}$$

For the case of a pipeline with a corrosion defect, Zheng et al. [5] presents a factor that reduces the resistance of the intact pipeline to account for the corrosion defect ($M_c$), a factor that depends on the defect geometry. It is necessary to obtain the factor $\eta$, which is presented in eq. (2) and adjusts the total thickness that has been reduced due to corrosion defects.

$$\eta = \frac{1 - \frac{d}{t}}{1 - \frac{d}{t\sqrt{1+0.4\left(\frac{L}{\sqrt{Rt}}\right)^2}}} \tag{2}$$

For the calculation of the neutral axis angle, there are two possible cases that must be analyzed to apply the correct equations: if the sum of the angle with the neutral axis is less than $\pi$, eq. (3) is applied, and the resistant moment is obtained using eq. (4). If the sum is greater, then eq. (5) should be applied, and the resistant moment is obtained using eq. (6).

$$\beta = \frac{\pi}{2}\left(1 - \frac{1-\eta}{\pi}\theta\right) \quad \{\text{if } \theta + \beta \leq \pi\} \tag{3}$$

$$M_c = M\left(\sin\beta - \frac{1-\eta}{2}\sin\theta\right) \quad \{\text{if } \theta+\beta \leq \pi\} \tag{4}$$

$$\beta = \frac{\pi}{2\eta}\left(2\eta - \frac{1-\eta}{\pi}\theta - 1\right) \quad \{\text{if } \theta+\beta > \pi\} \tag{5}$$

$$M_c = M\left(\eta\sin\beta - \frac{1-\eta}{2}\sin\theta\right) \quad \{\text{if } \theta+\beta > \pi\} \tag{6}$$

### 2.2 External pressure

The equation widely cited in the literature for calculating the maximum external collapse pressure in intact pipelines is the DNV [6] equation, presented in eq. (7). To obtain the collapse pressure ($P_c$), the yield pressure ($P_y$) must be considered, as shown in eq. (8), which depends on the yield stress ($f_y$) and the factor $\alpha$ that accounts for the difference between the yield stress in the hoop and longitudinal directions (Motta et al. [3]). It is also necessary to provide the elastic buckling pressure ($P_{el}$), eq. (9), which depends on Young's modulus ($E$) and Poisson's ratio ($\nu$). The DNV equation also considers the initial ovalization of the pipeline ($O$), an imperfection not considered by Zheng et al. [5] equation.

$$\left(\frac{P_c}{P_{el}} - 1\right)\left(\frac{P_c^2}{P_y^2} - 1\right) = O\frac{RP_c}{2tP_y} \tag{7}$$

$$P_y = f_y\alpha\left(\frac{t}{R}\right) \tag{8}$$

$$P_{el} = \frac{E}{4(1-\nu^2)}\left(\frac{t}{R}\right)^3 \tag{9}$$

With the collapse pressure obtained for the intact pipeline, it is necessary to apply a reduction factor that depends on the geometry of the corrosion defect, similar to the moment case. A factor abundantly used in the literature is the factor developed by Netto [7], ($R_{Netto}$), presented in eq. (10). The only difference compared to the moment case is that the circumferential geometry of the defect used is not the angle but the circumferential length ($w$), which is easily obtained from basic concepts of plane geometry. Once the reduction factor value is obtained, it is simply multiplied by the pressure obtained from the DNV equation to get the maximum pressure supported by the corroded pipeline.

$$R_{Netto} = \left[\frac{1-\frac{d}{t}}{1-\frac{d}{t}\left(1-\left(\frac{w}{\pi D}\right)^{0.4}\left(\frac{L}{10D}\right)^{0.4}\right)}\right]^{2.675} \tag{10}$$

### 2.3 Reliability Analysis and Deterministic Variables

For the reliability analysis, the Monte Carlo method was used, where a the first part involves involves generating samples according to the joint probability density function (Beck [8]). From the generated data, failure counts are conducted using a failure function, and finally, the probability of failure is calculated.

The failure function, eq. (11), in the case under study, involves comparing a limit state ($f_{lim}$), which are the equations of resistance already presented, with an applied service load ($f_{apl}$) on the pipeline. The random variables used ($X$), presented in Table 1, were the defect's depth and circumferential length dimensions, and the external pressure service load. These will be the samples generated by the method and tested in the failure function, will indicate failure when returning negative values.

$$f_{fail}(X) = f_{lim}(X) - f_{apl}(X) \tag{11}$$

Table 1. Random variables

| Random variable | Mean | Unit | COV (%) | Distribution | Reference |
|---|---|---|---|---|---|
| Defect depth | 0.97 | mm | 10 | Normal | DNV [9] |
| Circunferential lenght | 4.62 | mm | 11 | Normal | Teixeira et al. [4] |
| External pressure | 27.5 | MPa | 3 | Gumbel (parameters) | Teixeira et al. [4] |

The other variables considered in the process were treated as deterministic, meaning they do not have an associated probability distribution. These variables are described in Table 2, and were selected from the paper by Netto et al. [10], where an experimental study of pipelines subjected to external pressure until collapse is conducted.

Table 2. Deterministic variables

| Deterministic variable | Value | Unit |
|---|---|---|
| Young's modulus | 211 | MPa |
| Poisson's ratio | 0.3 | ad |
| Alpha coefficient | 0.85 | ad |
| Yield stress | 291 | MPa |
| Flow stress | 403 | MPa |
| Thickness | 2.77 | mm |
| Diameter | 42.05 | mm |
| Defect length | 21.025 | mm |
| Ovalization | 0.0773 | % |
| Bending Moment | 1.693 | kNm |

## 3 Results

To conduct the entire analysis, an algorithm was developed in Matlab; it generates random values, applies them to the empirical equations, and generates distribution graphs of the results. The Fig. 2 and Fig. 3, are plotted based on the product, effective area ($x$), between the dimensionless defect dimensions ($\frac{d}{t}$; $\frac{c}{\pi D}$), on the x-axis, and the loads (external pressure and bending moment) on the y-axis. The service loads, pressure, and resistant moment of the intact and corroded pipelines are described separately, allowing for better visualization and comparison of the results.

As we can observe, there is a difference in the load-bearing capacity in both cases, between the intact pipeline (red line) and the corroded pipelines (blue dots), as expected. The increase in the defect dimensions significantly impacts their resistance. The applied loads (in green) cause failure at the points with the most severe defects; in both cases, these failures begin in the effective area 0.015.

In Table 3, the probabilities of failure for both loading cases are described. It is the corroded pipes show failures of the same order of magnitude, with the bending moment case having more than three times the failure compared to the external pressure loading case.

In Table 4, we have the mean and standard deviation values for the limit state cases of the corroded pipelines. From the coefficient of variation, it is noticeable that both methods showed results with low variability, with a low coefficient of variation, especially in the bending moment case. In both cases, the low COV is due to the defects being relatively small, therefore resulting in low variability of the results.

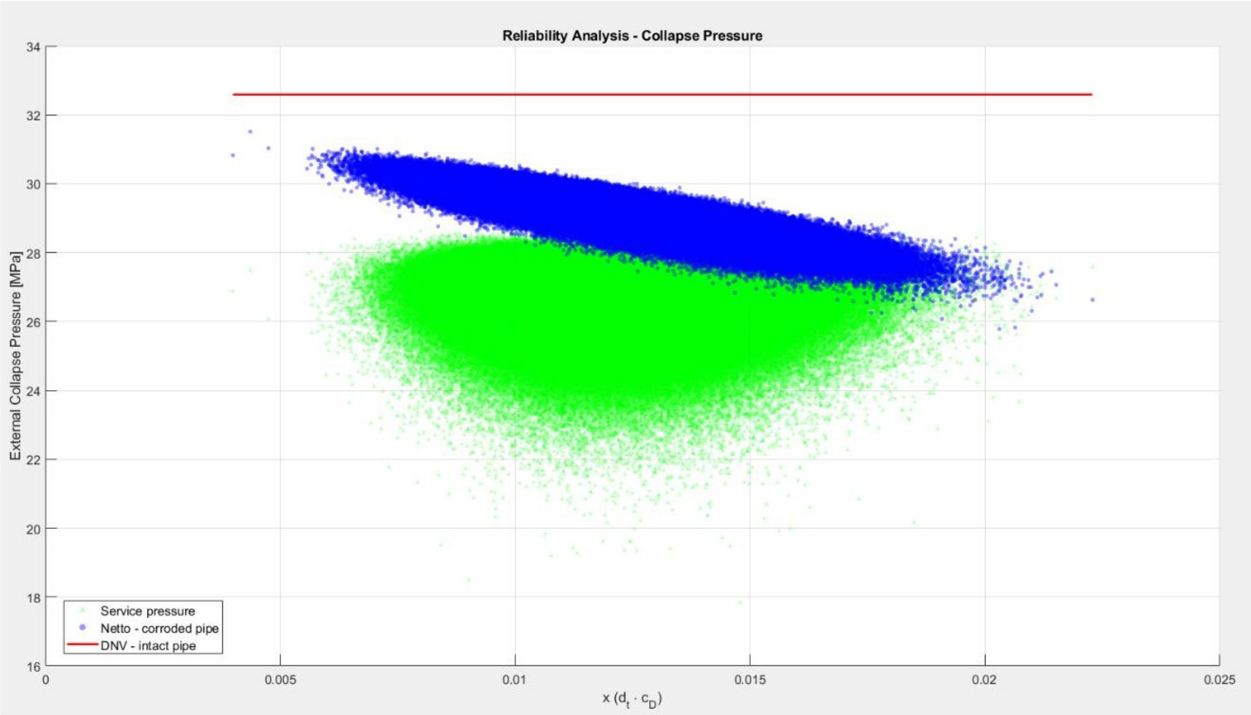


Figure 2. Reliability Analysis (Collapse Pressure)

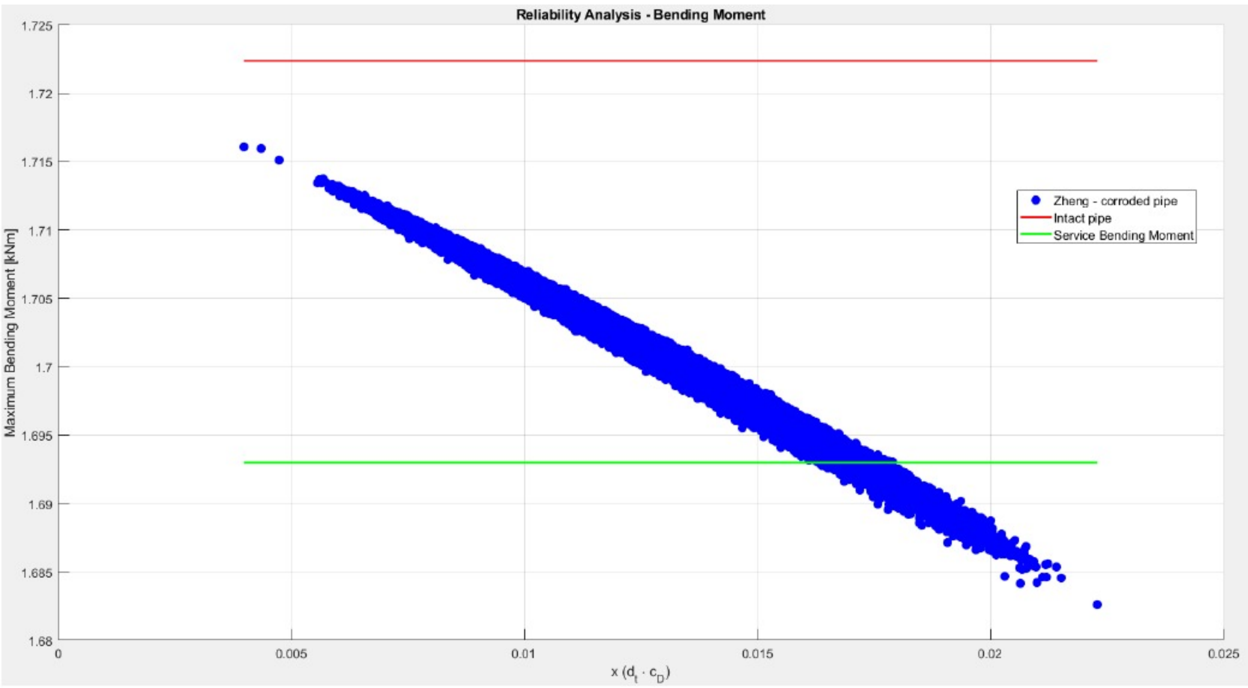


Figure 3. Reliability Analysis (Bending Moment)

Table 3. Probability of failure

| Case | Probability of failure ($10^{-3}$) |
|---|---|
| External pressure | 1.9 |
| Bending moment | 6.9 |

Table 4. Statistical Parameters

| Case | Mean | Standard Deviation | COV (%) |
|---|---|---|---|
| Netto's equation | 29.14 MPa | 0.51 MPa | 1.75 |
| Zheng's equation | 1.7 kNm | 0.0033 kNm | 0.17 |

To analyze which of the two cases showed greater sensitivity to corrosion, it is useful to compare the results for the lowest and highest effective area ($x$). This comparison is presented in Table 5, compared with the intact case, along with the percentage difference between the values. It is noted that the load-bearing capacity under external pressure is more sensitive to the defect, as well as compared to the intact case, where the loss of resistance is more pronounced in relation to the bending moment case. Since external pressure is a predominant load in subsea pipelines, this is an important conclusion in the safety analysis of pipelines.

Table 5. Comparative Analysis

| Case | Maximum | Minimum | Diff (%) | Intact case |
|---|---|---|---|---|
| External pressure | 31.51 MPa | 25.79 MPa | 18 | 32.59 MPa |
| Bending moment | 1.71 kNm | 1.68 kNm | 1.95 | 1.72 kNm |

## 4 Conclusions

- The increase in the defect tends to reduce the load-bearing capacity of the pipelines, and this behavior is represented by the Netto's equation and Zheng's factor;
- The equations led to statistical measures with significant accuracy;
- The bending moment equation resulted in a lower coefficient of variation compared to the external pressure case, hence more concise results. This conclusion is due to the relatively small defects;
- The loss of resistance under external pressure was shown to be greater compared to the bending moment;
- For ultradeep pipelines, the primary load is the external pressure that leads to collapse. Therefore, as it is the case most sensitive to the presence of defects, thorough analysis is essential to prevent environmental risks resulting from oil spills in the seas.

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**Authorship statement.** The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material included as part of the present paper is either the property (and authorship) of the authors.

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