Stability analysis of a seabed under combined action of waves and seismic loading

João Paulo C. Quiuqui[1], Samir Maghous [1]

[1]*Department of Civil Engineering, Federal University of Rio Grande do Sul,*
Av. Osvaldo Aranha, 99, Bairro Farroupilha, 90035-190, Porto Alegre-RS, Brazil,
jpquiuqui@hotmail.com, samir.maghous@ufrgs.br

Abstract. The stability analysis of a seabed under wave and seismic loading is conducted within the framework of yield design theory using the pseudo-static method. The strength properties of the constitutive material are modeled by means of a purely cohesive condition. The loading associated with water waves is modeled through the linear wave theory, whereas the inertial forces induced by the passage of seismic waves are addressed in the framework of pseudo-static method. The material cohesion that increases linearly with dept is adopted in the analysis. A sufficient stability condition is obtained through the static approach of limit analysis, while a necessary condition is determined from the kinematic approach. The effects of seafloor surface inclination, as well as seismic intensity, are analyzed through pseudo-static coefficients. The determination of the limit load obtained through the pseudo-static approach is based on existing literature, particularly when seismic coefficients are not considered.

Keywords: Water waves, Stability of submarine slopes, Pseudo static approach.

1 Introduction

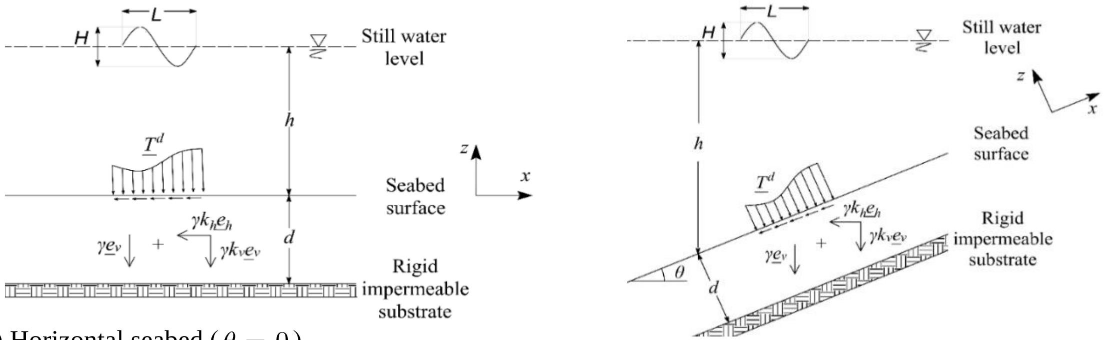
The wave pressure on the seabed surface undergoes changes due to wave propagation, which transfers to the seabed, inducing excess pore pressure (Zhang et al., [1]). The deterioration of marine structures is not caused exclusively by the action of marine waves; earthquakes are significant causes of instability. Additionally, according to Mangano et al. [2], about 90% of recorded earthquakes had their epicenters in coastal or offshore zones, indicating that the combination of these two forces can act simultaneously in seabed rupture events.

Several studies have been conducted in recent years to evaluate the response of the seabed to the propagation of marine waves. Yamamoto et al. [3] determined the effects on the seabed due to the propagation of linear surface waves considering an isotropic seabed through a poroelastic solution. Madsen [4] considered an anisotropic seabed, extending the study to the evaluation of seabed stability. However, these studies did not consider the evaluation of seabed stability under the combined action of waves and earthquakes.

In this context, the present work aims to analyze the stability of the seabed considering the action of waves and earthquakes. For stability analysis, limit analysis theory (Salençon, [5], [6]) will be applied, incorporating the pseudo-static method. The wave will be modeled using linear theory while the soil strength characteristics will be described by the non-homogeneous Tresca criterion. Sufficient stability conditions will be obtained through the static approach and the necessary condition will be obtained through the kinematic approach.

2 Statement of the stability problem

The seabed is modeled as a half-space material domain Ω whose upper boundary surface $z=0$ is inclined at an angle $\theta \neq 0$ with respect to the horizontal plane (Fig. 1b). The inclination angle is assumed to be notably small: $\theta \ll 1$. The soil layer of thickness d is delimited at the top by the water-soil interface located at $z=0$, where the positive axis points toward the sea surface, and at the bottom by a rigid and impermeable substrate located at $z=-d$. The particular case of a horizontal seabed where $\theta=0$ is also sketched in Fig. 1.a.



a) Horizontal seabed ($\theta=0$) b) Sloping seabed ($\theta\neq 0$)

Figure 1. Geometry of the seabed subjected to wave loading and gravity force

The first components of the loading are defined by the gravity forces $\gamma\underline{e}_v$ in which γ denote the specific weight of porous material, $\underline{e}_v$ and $\underline{e}_h$ are defined as:

$$\underline{e}_v=-(\sin\theta\underline{e}_x+\cos\theta\underline{e}_z) \qquad \underline{e}_h=-\cos\theta\underline{e}_x+\sin\theta\underline{e}_z . \tag{1}$$

The pseudo static forces are the horizontal $k_h\gamma\underline{e}_h$ and vertical $k_v\gamma\underline{e}_v$ components, where k_h and k_v denote, respectively, the average horizontal and vertical seismic coefficients that stand approximately for the intensity of the distributed earthquake-induced inertia forces. The surface forces $\underline{T}^d$ acting along the upper boundary are:

$$\underline{T}^d=\underline{T}_s^d+\underline{T}_a^d \qquad (z=0) \tag{2}$$

where $\underline{T}_s^d$ is the surface loading prevailing under static condition. The associated seismic component of the surface loading $\underline{T}_a^d$ is computed by applying the horizontal and vertical seismic coefficients directly on the expression of $\underline{T}_s^d$, however, this component can be disregarded. Within the general description of plane wave loading, the surface forces $\underline{T}^d$ acting on the boundary $z=0$ are defined by the hydrostatic pressure and the wave overpressure:

$$\underline{T}^d=\underline{T}_s^d=T_s^d\underline{e}_z=-(p_w(x,t)+\gamma_w h(x))\underline{e}_z \qquad (z=0) \tag{3}$$

where $h=h(x)$ denotes the local water depth calculated from the horizontal still sea water level. It may be observed that the value of h is constant for the horizontal plane seabed (Fig. 1a) whereas $h(x)=h_0-x\sin\theta$ decreases linearly with coordinate x in the case of sloping seabed (Fig. 1b), h_0 being the water depth at the origin of x -axis. In the above expression, the term $p_w(x,t)$ refers to the overpressure induced by the wave. It should be emphasized that, the water wave velocity is generally small when compared to the soil shear wave velocity, thus suggesting that inertia effects associated with the water wave can be neglected (Dormieux, [7]). The wave loading is modeled through an overpressure $p_w(x,t)$ acting on the planar sea floor $z=0$, with maximum amplitude p_0 . In the context of first order (linear) Stokes theory, which is usually used in engineering practice, the expression of cyclic overpressure is given as (Rahmah and Jaber, [8]):

$$p_w(x,t)=p_0\sin\psi=\frac{\gamma_w H/2}{\cosh\lambda h}\sin\psi \quad \text{with} \quad \psi=\lambda x-\omega t=\frac{2\pi}{L}x-\frac{2\pi}{T}t \tag{4}$$

where h is the local water depth at rest, γ_w is the unit weight of water, λ and ω is the wave number and wave frequency, respective, T is the wave period and $L=L(h)$ is the wavelength. Due to phenomenon of wave-breaking, the set $\mathscr{H}$ of the triples (H,L,h) which are physically allowable is bounded, more precisely, the wave steepness H / L cannot increase beyond a certain critical value at which the wave breaks (Rahmah and Jaber, [8]):

$$\mathscr{H} = \left\{ (H, L, h), \frac{H}{L} \leq \frac{1}{7} \tanh\left(\frac{2\pi h}{L} \right) \right\}. \tag{5}$$

In the following, we will also adopt the hypothesis that the pressure of the fluid is continuous through the interface at $z = 0$. A fundamental assumption of the loading model is that the value of fluid pressure at the sea floor acting along the seabed boundary $z = 0$ is not affected by the passage of earthquake waves. In particular, expression of p_w is still defined by eq. (4). In addition, continuity of the pore pressure through the sea-seabed interface $z = 0$ will be assumed throughout the subsequent analysis. Consequently, the difference in pore pressure in the porous medium with respect to its hydrostatic value, denoted by u, should comply with the following boundary condition:

$$u(x, z = 0, t) = p_w(x, t) = p_0 \sin \psi \, . \tag{6}$$

The formulation of the stability analysis problem within the framework of limit analysis theory requires a previously defined introduction to the strength capacity of the saturated porous material. The latter is expressed under a general yield condition form $f(\underline{\underline{\sigma}}, p) \leq 0$, where $\underline{\underline{\sigma}}$ and p denote respectively the stress and pore pressure fields in the porous seabed medium. For a given time t within the time interval considered for the study, the stability condition of the seabed structure under applied loading mode stems from compatibility between the equilibrium of the considered seabed structure and the resistance of its constituent porous material:

$$\text{Stabiliy} \quad \Leftrightarrow \quad \exists\, \underline{\underline{\sigma}} \ \text{ such that } \quad \begin{cases} \text{div}\underline{\underline{\sigma}} + \gamma \underline{e}_v + \gamma \underline{e}_a = 0 & \text{on } \Omega \\ \underline{T} = \underline{T}^d & \text{along } z = 0 \\ f(\underline{\underline{\sigma}}, p) \leq 0 & \text{on } \Omega \end{cases} \tag{7}$$

where $\underline{T} = \underline{\underline{\sigma}} \, . \, \underline{e}_z$ is the stress vector acting upon the upper surface of the seabed, while the prescribed surface loading is given by eq. (3).

3 Stability analysis of a purely cohesive seabed

In the context of total stress stability analysis, the strength capacities of the porous soil mass are classically described by a purely cohesive Tresca-like failure condition. In such an approach, the pore pressure distribution is assumed not to explicitly control the yield failure of the material, i.e., $f(\underline{\underline{\sigma}}, p) = f(\underline{\underline{\sigma}})$. In the present analysis, the material strength criterion is defined by a non-homogeneous Tresca criterion in which the cohesion $C(z)$ increases linearly with the distance to the boundary $z = 0$:

$$f(\underline{\underline{\sigma}}, z) = \sup(\sigma_i - \sigma_j) - 2C(z) \leq 0 \qquad \text{with} \quad C(z) = \eta_c \mid z \mid \tag{8}$$

where σ_i, with $i \in \{1, 2, 3\}$ stand for the principal stresses of total stress tensor $\underline{\underline{\sigma}}$. The scalar η_c, referred to as the cohesion gradient, is the single constitutive parameter required in this analysis, with reference values typically close to $\eta_c = 1.4kPa$ (Braun et. al., [9]). The convention of positive stress in tensile is adopted throughout the paper. One can notice no restrictions on the soil layer thickness in the formulation employed.

3.1 Lower bound static approach

This method directly applies stability definition eq. (7) by presenting a stress field that is statically valid under loading conditions and meets strength condition eq. (8). To achieve this, we divide the seabed loading into four basic components and develop a suitable stress field for each component. The two first components of the loading are the gravity forces $\gamma \underline{e}_v$ and the hydrostatic pressure $\gamma_w h(x)$ acting along the seabed surface. The stress fields associated by the equilibrium conditions are respectively defined as:

$$\underline{\underline{\sigma}}^{g} = \gamma z \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad \underline{\underline{\sigma}}^{hyd} = -\gamma_w \begin{bmatrix} h(x) & z\sin\theta \\ z\sin\theta & h(x) \end{bmatrix}. \tag{9}$$

The third loading component is the wave overpressure $p_w(x,t) = p_0 \sin\psi$ defined by eq. (4) in the context of linear theory. The latter stress solution, denoted by $\underline{\underline{\sigma}}^{p_w}$, has been formulated in Dormieux and Coussy [7], and the stress field considered for the lower static approach can therefore be defined as:

$$\underline{\underline{\sigma}}^{p_w} = p_0 \exp\lambda z \begin{bmatrix} -(1+\lambda z)\sin\lambda x & \lambda z\cos\lambda x \\ \lambda z\cos\lambda x & -(1-\lambda z)\sin\lambda x \end{bmatrix}. \tag{10}$$

Finally, the stress field associated with the seismic forces $\gamma \underline{e}_a$ is presented by:

$$\underline{\underline{\sigma}}_a^{g} = \gamma z \begin{bmatrix} -k_1 & k_2 \\ k_2 & k_1 \end{bmatrix} \tag{11}$$

where k_1 and k_2 are:

$$k_1 = -k_h \sin\theta + k_v \cos\theta \qquad k_2 = k_h \cos\theta + k_v \sin\theta. \tag{12}$$

Combining the above four elementary stress fields, the static approach is then formulated considering the following stress field:

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^{g} + \underline{\underline{\sigma}}^{hyd} + \underline{\underline{\sigma}}^{p_w} + \underline{\underline{\sigma}}_a^{g} \tag{13}$$

which is statically admissible in the loading mode. In order to be compatible with soil strength capacities, the stress field is thus defined by eq. (13) must satisfy the yield condition (8) at any point $(x,z) \in \mathbb{R} \times \mathbb{R}^-$, that is

$$\max_{\mathbb{R}\times\mathbb{R}^-} \sqrt{\left(\sigma_{xx} - \sigma_{zz}\right)^2 + 4\sigma_{xz}^2} - 2\,C(z) \le 0. \tag{14}$$

Substituting eq. (13) into eq. (14), and maximizing the equation with respect to x and z, one can determine the sufficient stability condition:

$$p_0 \le \frac{1}{\lambda}\left[\eta_c - (\gamma'\sin\theta + \gamma k_2)\sqrt{1 + \left(\frac{\gamma k_1}{\gamma'\sin\theta + \gamma k_2}\right)^2}\right] \tag{15}$$

where, in the case of gently sloping seabed, i.e., $\theta \ll 1$, the coefficient k_1, which depends on the seismic coefficients k_h, k_v and the slope θ, calculated from eq. (12), is close to zero ($k_1 \approx 0$), leading to the approximation of eq. (15) by:

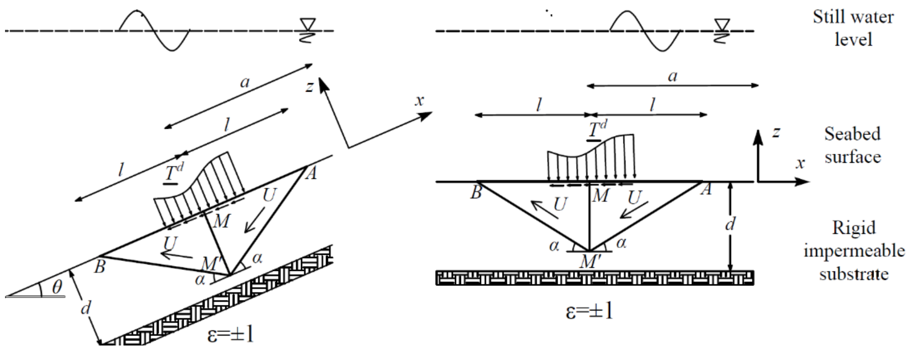
$$p_0 \le (\eta_c - \gamma'\sin\theta - \gamma k_2) / \lambda \tag{16}$$

3.2 Upper bound kinematic approach

For a specified point M located on the seabed, we consider the two velocity fields delineated in Fig. 2. It is observed that the two triangular blocks, $MM'A$ and $MM'B$, exhibit no deformation and undergo translation parallel to the directions of $M'A$ and $M'B$, respectively:

$$\underline{U}(MM'A) = -\varepsilon U(\cos\alpha \underline{e}_x + \sin\alpha \underline{e}_z) \qquad \underline{U}(MM'B) = \varepsilon U(-\cos\alpha \underline{e}_x + \sin\alpha \underline{e}_z) \tag{17}$$

where U is a positive scalar, and $\varepsilon = \pm 1$. The case of a horizontal seabed where $\theta = 0$ is also sketched in Fig. 2a. Throughout the soil mass, excluding the specified point M, the velocity field maintains a value of zero. The geometric attributes defining the velocity field include the abscissa a of point M, the length l represented by MA and MB ($MA = MB \ge 0$), and the angle α, which must satisfy the condition $\alpha \in]0, \pi/2[$.



a) Horizontal seabed $(\theta = 0)$ b) Sloping seabed $(\theta \neq 0)$

Figure 2. Piecewise rigid body motion velocity fields $\underline{U}$

The segments $M'A$ and $M'B$ represent the discontinuity lines of the velocity field under consideration. The velocity discontinuity W between the blocks $MM'A$ and $MM'B$ is oriented parallel to the line MM':

$$W = 2\varepsilon U \sin\alpha \underline{e}_x . \tag{18}$$

The kinematic approach relies on the upper bound theorem. In this framework, given a velocity field $\underline{U}$, the approach involves comparing the input power $P_e(\underline{U})$ with the maximum resisting power $P_r(\underline{U})$ that can be dissipated within the soil mass. Specifically, the upper bound theorem states the inequality:

$$P_e(\underline{U}) \leq P_r(\underline{U}) . \tag{19}$$

This relationship must be satisfied for all values of $\alpha \in]0, \pi/2[$, $l \geq 0$ and $\varepsilon = \pm 1$. The maximum resisting power consists of the contributions of each discontinuity line of the velocity field, i. e. $M'A$, $M'M$ and $M'B$:

$$P_r(\underline{U}) = U \int_{M'A} C(z)dS + \int_{M'B} C(z)dS + 2\sin\alpha \int_{M'M} C(z)dS = \eta_c l^2 U \sin\alpha \frac{1+\sin^2\alpha}{\cos^2\alpha} . \tag{20}$$

The input power comprises, now, four components: gravity forces P_g, hydrostatic pressure P_{hyd}, wave overpressure P_{p_w} and the pseudo-static forces $P_g{}^a$. The pseudo-static contribution of surface forces due to hydrostatic pressure P^a_{hyd} and wave overpressure $P^a_{p_w}$ are disregarded:

$$P_e(\underline{U}) = P_g + P_{hyd} + P_{p_w} + P_g{}^a = \varepsilon U l^2 \sin\alpha \left\{ \gamma' \sin\theta + \gamma k_2 + \frac{2p_0}{\lambda l^2} \left[\cos\lambda a (1 - \cos\lambda l) \right] \right\}. \tag{21}$$

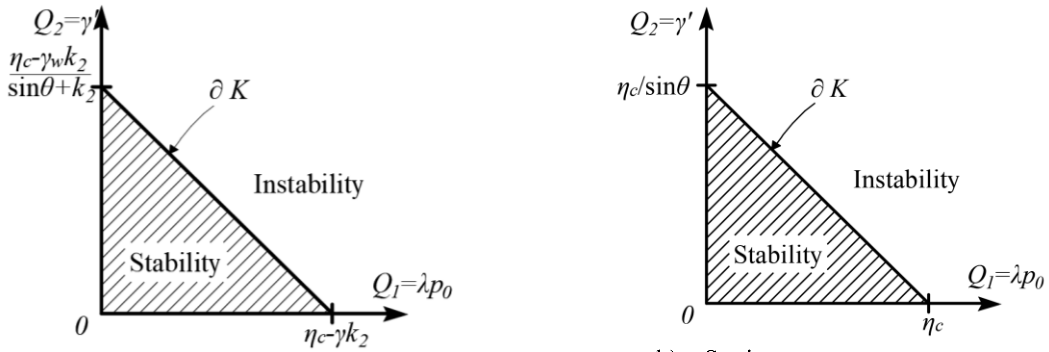
Using eq. (20) and eq. (21) in eq. (19), and maximizing the cumulative external powers on the left side while minimizing the maximum resisting power on the right side concerning the given parameters α, l, ε, and a, the necessary condition for seabed stability is obtained:

$$p_0 \leq (\eta_c - \gamma' \sin\theta - \gamma k_2) / \lambda . \tag{22}$$

3.3 Bound of limit loading

The domain K of sustainable loading values ($Q_1 = \lambda p_0, Q_2 = \gamma'$) is the triangular domain represented in Fig. 3a, where K is defined from the sufficient eq. (16) and necessary condition eq. (22), given by:

$$K = \{(Q_1, Q_2) ; \quad Q_1 \leq \eta_c - Q_2(\sin\theta + k_2) - \gamma_w k_2\} \tag{23}$$



a) Pseudo-static context b) Static context

Figure 3. Domain K of supportable loading values (Q_1, Q_2) in the pseudo-static and static context

Normally, the effects of the vertical component of seismic forces can be neglected ($k_v = 0$), however, the effects of the horizontal component k_h must be considered (Chen and Liu, [10]). Thus, the lower and upper bounds are given by:

$$p_0^{\lim} = (\eta_c - \gamma' \sin\theta - \gamma k_h \cos\theta) \,/\, \lambda \tag{24}$$

where $p_0^{\lim}$ is the supportable limit loading. When the seabed is considered horizontal (Fig. 2a), i.e. $\theta = 0$, the necessary and sufficient condition is obtained by $p_0^{\lim} = (\eta_c - \gamma k_h) \,/\, \lambda$. In the static case, where $k_h = k_2 = 0$, the domain $K = (Q_1, Q_2)\,;\quad Q_1 \leq \eta_c - Q_2 \sin\theta$ of supportable loading values ($Q_1 = \lambda p_0, Q_2 = \gamma'$) is the triangular domain represented by Fig. 3b. It is observed that the domain K expands with the cohesion gradient of the soil η_c. The limit loading in the static context, with the necessary and sufficient stability condition equal to that presented by Dormieux and Coussy [7], given by:

$$p_0^{\lim} = (\eta_c - \gamma' \sin\theta) \,/\, \lambda\,. \tag{25}$$

According to the results obtained for the wave maximum safe amplitude, which are independent of the soil layer thickness, it was found that in the case of a horizontal seabed, the maximum safe amplitude of the wave loading does not also depend on the specific soil weight γ', unlike the case of the inclined seabed. In general, comparing the inclined seabed case with the horizontal seabed case, the slope effect is equivalent to a reduction of the cohesion gradient value of $\gamma' \sin\theta$. In pseudo-static condition, the reduction of the cohesion gradient value is $\gamma' \sin\theta + \gamma k_h \cos\theta$. It is observed that Fig. 3b represents a specialization of Fig. 3a, in which, in a static scenario, the seismic coefficients k_v and k_h assume a null value, leading to $k_1 = k_2 = 0$.

For practical applications, it is convenient to characterize the set $\mathscr{K}$ of the triples (H, L, h) for which stability is ensured. In the pseudo-static context, $\mathscr{K}$ can be obtained according to eq. (4) and eq. (23):

$$\mathscr{K} = \left\{ (H, L, h), \frac{H}{L} \leq \;\; \eta_c - \gamma' \sin\theta - \gamma k_h \cos\theta \;\; \frac{\cosh(2\pi h \,/\, L)}{\pi \gamma_w} \right\}. \tag{26}$$

According to the Fig. 4, the intersection $\mathscr{H} \cap \mathscr{K}$ represents the set of (H, L, h) which are allowable concerning wave breaking and do not cause seabed instability. Noting $\mathscr{K}^c$ the complement of $\mathscr{K}$, the set $\mathscr{H} \cap \mathscr{K}^c$ represents the triples (H, L, h) which are allowable with respect to wave breaking and cause seabed instability. These two sets are represented graphically in the plane (H/L, h/L) in Fig. 4.

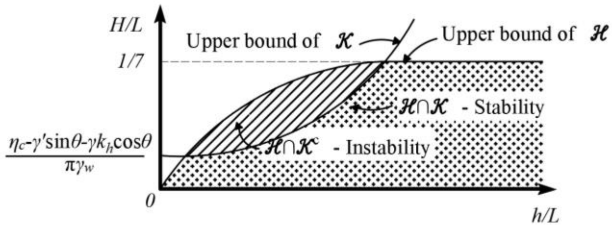


Figure 4. Boundary of the domain of safe loadings in the plane ($H / L, h / L$) in context pseudo-static

To illustrate the results, the following set of parameters was considered: the cohesion gradient of the soil $\eta_c = 1.4kPa$ and the specific weight of the material constituting the seabed $\gamma = 15.5kPa$ (Braun et. al., [9]). Consider the specific case where the vertical seismic coefficient is disregarded, and the seabed is assumed to be horizontal. This behavior is predicted by the stability condition presented earlier; in other words, for a horizontal pseudo-static coefficient $k_h \geq \eta_c / \gamma \cong 0.1$, instability should occur along the entire seabed. Conversely, for $k_h < \eta_c / \gamma \cong 0.1$, there will be an intersection area between eq. (5) and eq. (30), indicating seabed stability.

4 Conclusions

The application of limit analysis theory, was employed to examine the stability of the seabed, composed of cohesive material, under the combined action of marine waves and seismic forces. The main conclusions of the study indicate that, despite distinct necessary and sufficient conditions, due to the assumption of a gently sloping seabed and the insignificant vertical seismic coefficient compared to the horizontal, these conditions can be considered approximately equal. Furthermore, the necessary and sufficient condition in the pseudo-static context can be particularized to the static context, thereby recovering solutions from the literature, such as those by Dormieux and Coussy [7]. In the more general context, that is, in the pseudo-static analysis, instability results from the reduction of the soil cohesion gradient by a value of $\gamma' \sin\theta + \gamma k_h \cos\theta$, whereas in the static case, the reduction of η_c is equivalent to $\gamma' \sin\theta$.

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