Dynamic amplification analysis in acoustic wave propagation problems subject to time-variable loads

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Abstract. *This work examines the physical vibrational behavior and the numerical phenomena resulting from applying periodic forcing in a continuous system governed by the Acoustic Wave equation. The simulations are carried out using the Boundary Element Method with Direct Interpolation. Situations in which the imposed frequencies are moderate and high are examined, as well as the effects of the modal contribution in the movement; the dynamic amplification is also discussed, and the participation of elastic and inertial contents in the response is investigated and the effect of the fictitious damping introduced by the time marching scheme is also evaluated. The numerical solutions are compared with the corresponding analytical solutions for a better accuracy evaluation.*

Keywords: *Acoustic Waves Problem, Boundary Element Method with Direct Interpolation, Numerical Simulation.*

1 Introduction

The best-known application of forced vibrational analysis consists of structures that support rotating equipment such as turbines, pumps and compressors, which must resist the effects of dynamic amplification [1]. In fact, the first studies of vibrations in mechanical engineering were motivated by the problem of balancing in engines, which causes damage to all other components of the machine, especially the bearings and housings, but also tremors on the ground and even environmental discomfort. The effects of vibration are diverse: it ranges from loosening screws to loss of precision in machining manufacturing processes. However, the most important effect consists of fatigue in equipment parts and accessories, resulting from the propagation of cracks under periodic loads [2].

Forced vibrational behavior can be very well evaluated through prototypes or devices that measure its effects on the equipment itself. However, computational modeling is more economical and versatile. Thanks to modern numerical simulation techniques, it is possible to evaluate and predict all the effects of dynamic loading that occur in machines, equipment and buildings subject to dynamic loading.

Among the main vibrational phenomena is also dynamic amplification, which is the factor relevant to structural analysis that indicates how much the static displacement of the equipment is increased due to the vibrational effect [1]. By changing the static magnitude, the amplification factor has enormous importance in the dimensional design of the component and acts significantly, although indirectly, in changing the resistive expectation of the component due to fatigue effects. Motor vehicles are the best example, as when traveling on bumpy roads you experience how much the suspension and other components are subjected to different movements due to irregularities in the ground.

This research aims precisely to examine the phenomenological aspects involved in the analysis of a forced and damped continuous system. The chosen structure is simple, but most of the physical phenomena to be examined and the conclusions that result can be generalized to other more complex components or structures. Considering the continuous system, instead of discrete models with few degrees of freedom, offers the possibility of greater conceptual extension. This analysis is carried out using a discrete numerical model, generated by the Boundary Element Method with Direct Interpolation (DIBEM), a computational tool whose accuracy and robustness have been proven by investigations carried out some years ago. The DIBEM has also been applied to problems pertinent to several related areas [3,4,5]. Due to the well-known difficulty of dynamic cases, the discrete model is relatively refined to be able to well represent the physical phenomena involved. Although this tool has already been studied in dynamic cases in other works [6,7], there is the challenge of simulating moderate to high frequency loads, with hundreds of degrees of freedom. In this sense, it is also of interest to investigate the robustness of the method in these more complex conditions and to evaluate in detail the numerical phenomena involved.

Therefore, this work presents the mathematical model of the problem, expressed by the scalar equation of the Acoustic Wave. Although the elastodynamic equation is the most general model for vibrational analysis, it can be divided into two distinct equations. One of them represents the scalar model, referring to elastic expansion waves, and a vector equation, related to shear waves [8], which travel at a lower speed. The first are more important because they describe acoustic problems, they apply to cases that examine the propagation of sound in environments, acoustic insulation in roads and barriers, and also prospecting seismic, which is of great importance today. In any case, the problem examined here consists of a bar with constant section, subject to axial load, which is basically a scalar problem, as there is no shear wave traffic. To results evaluation is deduced non-trivial analytical solution, obtained through the Variable Separation Method.

2 Differential Equation of Government

The simplest case of elastodynamics, which falls within scalar field theory, is the problem of propagating longitudinal plane waves in a bar. This action is produced by a uniformly imposed loading, so that the stresses are uniformly distributed in the section, so that the waves travel in the axial direction. The longitudinal vibration of a bar executes a movement that, at each instant, satisfies the wave equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

where u(x,t) is the displacement of the section of the bar where a force is applied in the direction of the abscissa and k is the propagation speed of the wave in the bar. In the Eq. (1) are considered small deformations, where Newton's Second Law and Hooke's Law [1] apply.

3 Method Application of the Boundary Element

For conciseness, the partial differential equation given by Eq. (1) is transformed into an integral equation in inverse form, typical of the Boundary Element Method (BEM) [9], resulting in the following equation:

$$c(\boldsymbol{\xi})u(\boldsymbol{\xi};t) + \int_{\Gamma} q^{*}(\boldsymbol{\xi};\mathbf{X})u(\mathbf{X},t)d\Gamma - \int_{\Gamma} q(\mathbf{X},t)u^{*}(\boldsymbol{\xi};\mathbf{X})d\Gamma = -\frac{1}{k^2}\int_{\Omega} \ddot{u}(\mathbf{X},t)u^{*}(\boldsymbol{\xi};\mathbf{X})d\Omega. \tag{2}$$

In Eq. (2), the term **X** represents the field point, any point related to the domain Ω(**X**), limited by the contour Γ(**X**). The base point of the integrations is called the source point ξ. The term c(ξ) is a coefficient related to the location of the source point ξ relative to Ω(**X**) and, considering that it can be located on the boundary Γ(**X**), inside or outside it. Smoothness also influences the term c(ξ) [9]. It should be noted that u(**X**,t) is the displacement and q(**X**,t) is the normal stress in the boundary; u* (ξ;**X**) is the fundamental solution correlated to the Laplace problem and q* (ξ;**X**) is its normal derivative:

$$u^*(\boldsymbol{\xi};\mathbf{X}) = -\frac{\ln[r(\boldsymbol{\xi};\mathbf{X})]}{2\pi}, \qquad q^*(\boldsymbol{\xi};\mathbf{X}) = -\frac{1}{2\ln[r(\boldsymbol{\xi};\mathbf{X})]} r_i(\boldsymbol{\xi};\mathbf{X}) n_i(\mathbf{X}). \tag{3}$$

In Eqs (3) the Euclidean distance $r(\xi,\mathbf{X})$ relates the source point $\boldsymbol{\xi}$ and any field point $\mathbf{X}$; and $n_i(\mathbf{X})$ is the external normal to the contour at point $\mathbf{X}$. To eliminate the domain integral referring to the inertia term, on the right side of Eq. (2) the DIBEM is used. All functions that make up the core of the domain integral on the right side of Eq. (2) are approximated by the sequence of radial functions. Thus, the fundamental solution is included, which depends on the distance between the field and source point. For this reason, the DIBEM presents singularity problems if the source points coincide with the field points. To avoid this, the regularization procedure is used as a strategy, adding two new integrals, as follows:

$$c(\boldsymbol{\xi})u(\boldsymbol{\xi};t) + \int_\Gamma u(\mathbf{X},t)q^*(\boldsymbol{\xi};\mathbf{X})d\Gamma - \int_\Gamma q(\mathbf{X},t)u^*(\boldsymbol{\xi};\mathbf{X})d\Gamma = \frac{1}{k^2}\left[\int_\Omega (\ddot{u}(\boldsymbol{\xi},t) - \ddot{u}(\mathbf{X},t))u^*(\boldsymbol{\xi};\mathbf{X})\, d\Omega\right] - \frac{1}{k^2}\left[\ddot{u}(\boldsymbol{\xi};t)\int_\Omega u^*(\boldsymbol{\xi};\mathbf{X})\, d\Omega\right]. \tag{4}$$

The singularity in the domain integral approximation is removed in Eq. (4), but another additional domain integral is created. However, it is easily transformed into a contour integral:

$$\ddot{u}(\boldsymbol{\xi},t)\int_\Omega u^*(\boldsymbol{\xi};\mathbf{X})\, d\Omega = \ddot{u}(\boldsymbol{\xi},t)\int_\Omega G^*_{,ii}(\boldsymbol{\xi};\mathbf{X}) = \ddot{u}(\boldsymbol{\xi},t)\int_\Gamma G^*_{,i}(\boldsymbol{\xi};\mathbf{X})n_i\, d\Gamma. \tag{5}$$

Where:

$$G^*_{,i}(\boldsymbol{\xi};\mathbf{X})n_i = Z^{\xi}_{X}(\boldsymbol{\xi};\mathbf{X}) = \frac{1}{4\pi}\{0.5 - \ln[r(\boldsymbol{\xi};\mathbf{X})]\}r_i n_i. \tag{6}$$

Considering the additional term for regularization, the approximation radial basis functions consists:

$$[\,\ddot{u}(\mathbf{X},t) - \ddot{u}(\boldsymbol{\xi},t)]u^*(\boldsymbol{\xi};\mathbf{X}) \cong \ddot{\alpha}^i F^i(\mathbf{X^i};\mathbf{X}). \tag{7}$$

The function $F^i(\mathbf{X^i};\mathbf{X})$ is a radial basis function (RBF), there is no safe rule to determine the appropriate function, as it depends on many factors. In this work, the simple radial function was chosen:

$$F^i(\mathbf{X^i};\mathbf{X}) = r(\mathbf{X^i};\mathbf{X}). \tag{8}$$

When applying DIBEM, a primitive function is used, here called $\Psi^i(\mathbf{X^i};\mathbf{X})$ related to the interpolation function $F^i(\mathbf{X^i};\mathbf{X})$. Therefore:

$${}^{\xi}\alpha^i \int_\Omega F^i(\mathbf{X^i};\mathbf{X})d\Omega = {}^{\xi}\alpha^i \int_\Omega \Psi^i(\mathbf{X^i};\mathbf{X})_{,jj}\, d\Omega = {}^{\xi}\alpha^i \int_\Gamma \Psi^i(\mathbf{X^i};\mathbf{X})_{,j}\, n_i\,(\mathbf{X})d\Gamma = {}^{\xi}\alpha^i \int_\Gamma \eta^i\,(\mathbf{X^i};\mathbf{X})d\Gamma. \tag{9}$$

In Eq. (9), for each source point ξ, interpolation by radial functions implies sweeping all field points X in relation to points $\mathbf{X^i}$, which are weighted by the coefficients ${}^{\xi}\alpha^i$.

$$\begin{gathered}
\sum_{i=1}^{n} H_{1i}\, u_i - \sum_{i=1}^{n} G_{1i}\, q_i = -\frac{1}{k^2}[Z_1\ddot{u}_1 + {}^{1}\ddot{\alpha}^1 N_1 + {}^{1}\ddot{\alpha}^2 N_2 \ldots + {}^{1}\ddot{\alpha}^m N_m] \\
\sum_{i=1}^{n} H_{2i}\, u_i - \sum_{i=1}^{n} G_{2i}\, q_i = -\frac{1}{k^2}[Z_2\ddot{u}_2 + {}^{2}\ddot{\alpha}^1 N_1 + {}^{2}\ddot{\alpha}^2 N_2 \ldots + {}^{2}\ddot{\alpha}^m N_m] \\
\vdots \\
\sum_{i=1}^{n} H_{ni}\, u_i - \sum_{i=1}^{n} G_{ni}\, q_i = -\frac{1}{k^2}[Z_n\ddot{u}_n + {}^{n}\ddot{\alpha}^1 N_1 + {}^{n}\ddot{\alpha}^2 N_2 \ldots + {}^{n}\ddot{\alpha}^m N_m].
\end{gathered} \tag{10}$$

Matrix-wise, the surplus term generates a diagonal matrix that multiplies the accelerations:

$$\begin{bmatrix} \ddot{t}_1 \\ \vdots \\ \ddot{t}_n \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & Z_n \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \vdots \\ \ddot{u}_n \end{bmatrix}. \quad (11)$$

The terms generated by interpolation are a function of the coefficients $\ddot{\boldsymbol{\alpha}}$. It is necessary to rewrite such coefficients in terms of the time derivative of the potential, which is done through the following procedure. First, consider the basic interpolation sentence given by:

$$[\mathbf{F}]\{\ddot{\boldsymbol{\alpha}}\} = \{\ddot{\mathbf{u}}\} \quad (12)$$

After applying this procedure, it is possible to write the domain integral of the term related to inertia only in terms of an integral involving boundary variables. For brevity, the matrix treatment of this equation will not be discussed and can be found in other works [5]. Thus, one has:

$$[\boldsymbol{H}]\{\boldsymbol{u}\} - [\boldsymbol{G}]\{\boldsymbol{q}\} = [\boldsymbol{M}]\{\ddot{\boldsymbol{u}}\}. \quad (13)$$

In Eq. (13), the matrices $[\boldsymbol{H}]$ and $[\boldsymbol{G}]$ are typical matrices in the Boundary Element Method [9]. The matrix $[\boldsymbol{M}]$ corresponds to the System's inertia property, while $\{\boldsymbol{u}\}$ and $\{\boldsymbol{q}\}$ are vectors that contain the potential values and their derivative at the nodes.

For temporal discretization, it is proposed to use finite difference techniques. Among these, to deal with specific characteristics of the BEM, the most appropriate is the Houbolt time advance scheme:

$$\ddot{u}_{n+1} = \frac{2u_{n+1} - 5u_n + 4u_{n-1} - u_{n-2}}{\Delta t^2}. \quad (14)$$

Where Δt is the discretization time interval and are the time instants. Substituting into the equation for acoustic wave problems, we have:

$$(2\overline{M} + \Delta t^2\overline{H})u_{n+1} - (\Delta t^2\overline{G})q_{n+1} = (5\overline{M})u_n - (4\overline{M})u_{n-1} + (\overline{M})u_{n-2}. \quad (15)$$

The Houbolt scheme is considered unconditionally stable for discrete domain methods, but not for BEM.

4 Numerical Example

The solution is a bar subjected to periodic loading, in which the excitation frequency is an arbitrary variable and will be successively changed with a view to analyzing its physical and numerical behavior. For each simulation, only the displacement values at the right end of the member, shown with its geometric characteristics and boundary conditions in Fig. 1, are examined.

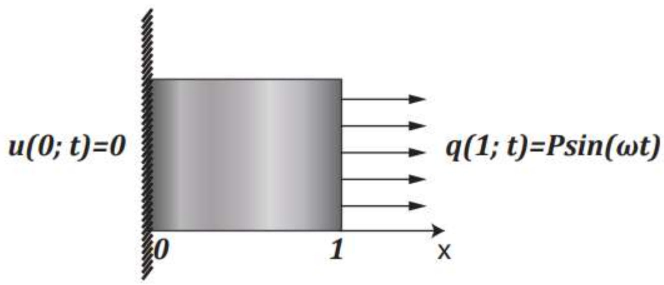

Figure 1. Bar subjected to time-varying load and boundary conditions.

The boundary and initial conditions are given by:

$$u(0,t) = 0; \quad q(1,t) = P\sin(\omega t). \quad (16)$$

The initial conditions are:

$$u(x,0) = 0; \quad u_t(x,0) = 0. \tag{17}$$

The analytical solution of the problem, obtained by the variable separation method, but following the appropriate mathematical procedure [8] for the case of time-varying loads, is given by:

$$u(x,t) = \sum_{n=1}^{\infty} 2(-1)^{n-1} \left(\frac{\omega_n \mathrm{sen}(\omega t) - \omega \mathrm{sen}(\omega_n t)}{\omega_n{}^2 - \omega^2} \right) \frac{\mathrm{sen}(\omega_n x)}{\omega_n}. \tag{18}$$

In Eq. (19), we have the natural frequency ω_n:

$$\omega_n = \frac{(2n-1)\pi}{2}. \tag{19}$$

Based on previous work, refined regular meshes were used, with 640 linear contour elements (BE), double nodes at the corners and 625 internal interpolating points (poles), so that more severe computational experiments could be carried out, with high frequencies, and also computational evaluations of vibrational phenomena for which high precision is necessary to represent them well. The integration step initially used is equal to 0.025s, but can be reduced if necessary. The simulations begin considering the angular velocity equal to 1rd/s. The results are shown in Fig. 2, which presents the numerical and analytical values together.

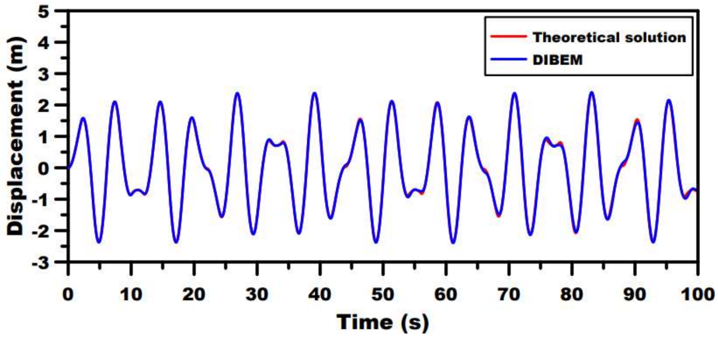

Figure 2. Displacement response with angular frequency of 1rd/s.

It can be seen that DIBEM approximated the response for this low level of angular velocity with great precision, over a significant time. The period of the excitation load is 6.28 seconds, but unlike a system with one degree of freedom, it can be seen by inspection of Fig. 2 that the periodicity of the response corresponds to approximately 44 seconds. This results from the superposition of movements resulting from natural modes of vibration. Therefore, even when evaluating the response in detail for a time greater than 2400 seconds, it appears that the system's response frequency still does not correspond to the charging frequency.

For the angular velocity of 3 rd/s, the accuracy of the results remains quite good, as can be seen in Fig. 3, but the fictitious damping effects inherent to the Houbolt scheme can already be identified in the higher modes. This occurred because the excitation has a higher frequency and therefore the velocities of the particles making up the continuous system are greater. The damping acts, above all, on these particles with greater speed.

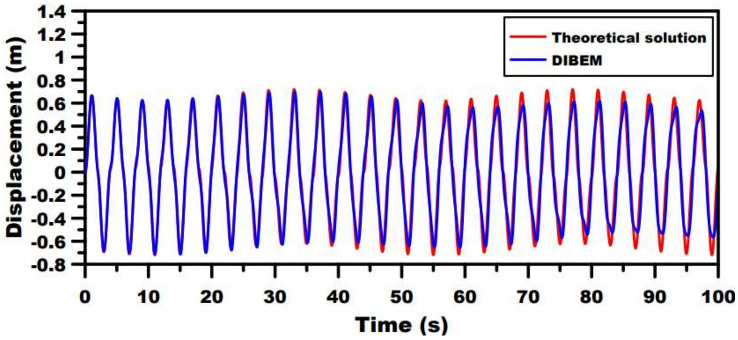

Figure 3. Displacement response with angular frequency of 3rd/s.

In Fig. 4 the behavior of the system is presented for an angular velocity equal to 5rd/s. However, in this condition, an interesting phenomenon emerged in vibrational analysis: beating. When two waves propagate in the same direction with slightly different frequencies, they interfere with each other, creating a frequency of repetition of the movement, called beat, which is proportional to the difference between the two original frequencies. The amplitude increases and decreases in a regular pattern. It can be seen that it is at a value very close to the periodicity shown by the analytical response. However, the numerical solution turns out to be something different from the analytical solution. The beat appears to be out of phase by a small amount, as if the natural frequency was slightly altered, see Fig. 4.

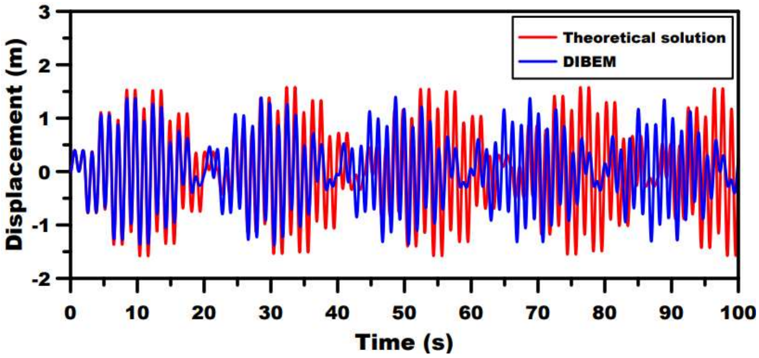

Figure 4. Displacement response with angular frequency of 5rd/s.

A clear lag can be seen, which could be explained by two factors: the first is the approximation of the numerical value of the natural frequency. Every numerical method introduces an approximation error into its calculations. However, the mesh used is very refined and is the third natural frequency, which is very well represented, which is not the reason for such behavior. The second factor refers to the fact that studies on damped free vibration show that the presence of damping changes the natural frequency, resulting in the so-called damped frequency, of lower intensity than the first. It is known that the presence of damping changes the wavelengths, lengthening the "periods" of damped vibration (strictly, in the presence of damping, the free system does not behave periodically). As shown in the Fig. 5, for an angular velocity of 8 rd/s. It should be noted that due to the occurrence of the beating phenomenon, with the proximity between the excitation and natural frequencies, the amplitudes tend to increase due to dynamic amplification. It is also clear that the step used has fictitious damping that is still not negligible, as the contribution of the high modes is being attenuated.

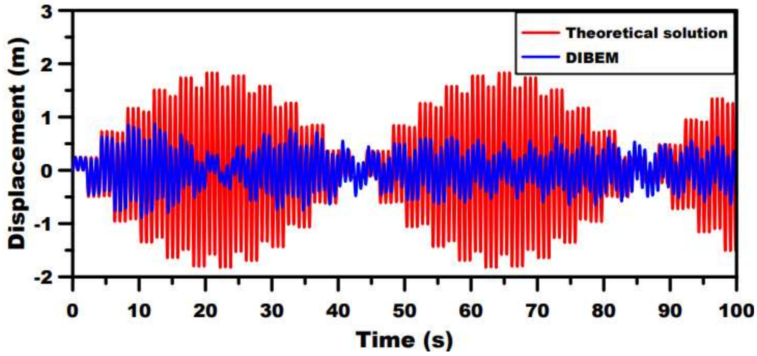

Figure 5. Displacement response with angular frequency of 8rd/s.

Physically, when the excitation frequency is much higher than the natural frequency, the force changes its value so quickly that the mass does not have time to follow it, causing a small amplitude. Thus, in the behavior of the system subjected to loads that have high frequencies, the predominant term opposing the external force is the inertia force [10]. The amplitudes are also very small, as their inertia prevents large movements.

When the excitation frequencies are medium, the low natural frequencies induce vibrational periods relative to free movement that are very long in comparison to the excitation period and are not felt in the first moments of the response, for the excitation angular velocity equal to 20rd/ see Fig. 6. They also show reduced amplitudes, which do not stand out. Thus, the oscillations are clearly governed by the frequency of the excitation.

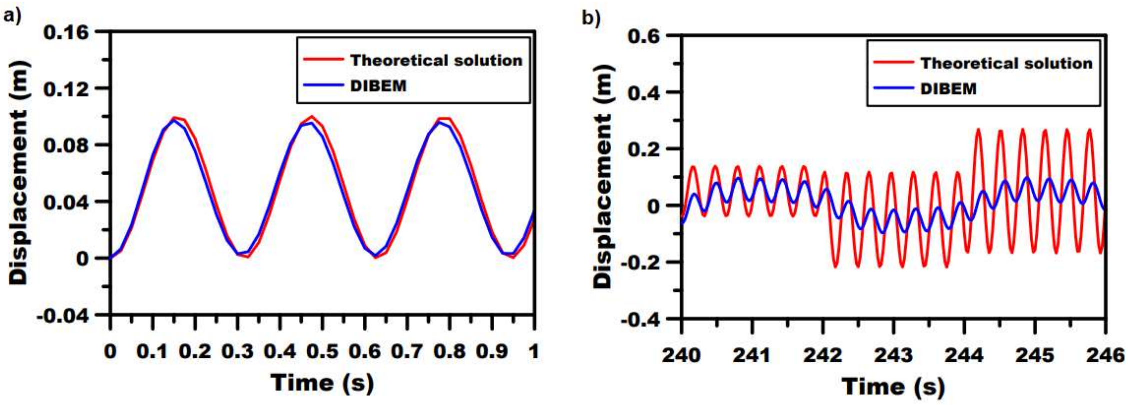

Figure 6. Displacement response with angular frequency of 20rd/s. (a) Starting times (b) Advanced times.

The response period is 0.314s. For more advanced times, shown in Fig. 6, the fictitious damping eliminated the high modes, but the fundamental period persists and appears clearly in the numerical response, slightly dilated. In the analytical response, the higher modes act. The prediction of vibration periods being dictated by the forced motion.

Conclusions

Due to the good performance of the Direct Interpolation formulation in problems of great interest in engineering, in this work the performance of the model was evaluated in cases of forced vibration, with the imposition of low, moderate and also high frequencies. The results confirmed the good performance of the DIBEM, considering the good agreement with the analytical values and also the adequate reproduction of the physical behavior expected by the continuous system in special conditions imposed by the forcing load.

Acknowledgements. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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