Implementation of a fatigue analysis program based on the dual formulation of the Boundary Element Method

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Abstract. The study and analysis of fatigue are extremely important, given the countless financial and social losses caused by this damage process. For this reason, a complete computational program is implemented for two-dimensional fatigue analysis in cracked components, coded in Fortran 95 language. The Dual Boundary Element Method (DBEM) is used to evaluate the elastic fields of displacement and traction, through the placement of Boundary Integral Equations (BIE) of different natures on each of the crack faces. Stresses and strains at boundary points are evaluated through the hypersingular BIE. Once the variables of interest are obtained in the boundary, the fatigue subcritical propagation of cracks is evaluated based on the concepts of Linear Elastic Fracture Mechanics (LEFM). J-Integral is used to obtain the stress intensity factors at the crack tip, due to its effectiveness, precision and excellent coupling with the dual formulation of the Boundary Element Method (BEM). The direction of crack propagation is defined by the Maximum Tangential Stress criterion. Finally, the lifespan of the cracked component is determined using an adapted Paris Equation, in order to correctly evaluate mixed mode fracture and non-zero mean stresses. Furthermore, using the Rainflow Cycle Counting algorithm, the program is capable of evaluating components under the action of cyclic loads of variable amplitude. In order to evaluate the implemented computational program, two examples are presented. The first considers a Compact Mixed Mode (CMM) specimen under mixed mode fracture. The second example evaluates the driven wheel of an iron ore pelletizing furnace under cyclic loading of variable amplitude. According to the results presented, it is proved the effectiveness and reliability of the developed computational program, which allows to accurately analyze real situations of components under mode I, II or mixed mode fatigue.

Keywords: Fatigue, Linear Elastic Fracture Mechanics, Dual Boundary Element Method

1 Introduction

Fatigue is a process of damage and failure caused by loads with cyclical variation, which may lead to the sudden failure of the component. For this reason, a computational program is implemented for the two-dimensional analysis of fatigue cracks. The program is based on the Dual Boundary Element Method (DBEM), which consists of applying boundary integral equations (BIEs) of different natures on opposite faces of the crack, allowing the resolution of general crack problems in a single region. This formulation is coupled with the concepts of Linear Elastic Fracture Mechanics (LEFM), a branch of Fracture Mechanics applicable to problems in which plastic deformations and nonlinearities are restricted to a small region at the crack tip. According to Lee et al. [1], the Fracture Mechanics approach assumes that the analyzed component already has preexisting cracks, and the LEFM concepts are used to study the propagation of these cracks. This subcritical fatigue crack propagation is analyzed by the Paris equation, modified to evaluate mixed mode fracture and nonzero stress ratios. The program is coded in Fortran 95 language and is applied in the two-dimensional analysis of fatigue cracks in a Compact Mixed Mode (CMM) specimen and in the driven wheel of an iron ore pelletizing furnace.

2 Numerical implementation

First, displacement and traction components are obtained at all boundary points using the Dual Boundary Element Method (DBEM), proposed by Portela et al. [2], which consists of applying the displacement boundary integral equation (BIE) to one of the crack faces, while the traction BIE is applied to the opposite face. The external boundary can be discretized using either of the two equations, with the displacement BIE being adopted throughout this work, as done in Portela et al. [3]. The boundary mesh is composed by quadratic elements, with semi-discontinuous elements at the corners and discontinuous elements at the cracks. The integrals present in the BIEs are solved by Gauss-Legendre quadrature, when regular, or by the method proposed by Gao [4], when singular. This method consists of expanding the non-singular part of the integrand into polynomials of the distance r between the source and field points and then analytically removing the singularities.

Once all boundary variables have been obtained, the crack behavior is evaluated. According to Broek [5], when analyzing fractured components using the Linear Elastic Fracture Mechanics approach, the most important parameter is the Stress Intensity Factor (SIF), which indicates the stress magnitude at the crack tip. This parameter depends on the fracture mode, which, according to Feng et al. [6], can be: mode I (opening), mode II (in-plane sliding) and mode III (out-of-plane sliding). Combinations between them give rise to mixed mode fracture. Stress Intensity Factors are calculated using the J-Integral, proposed by Rice [7], which is a function of the stress and strain fields near the crack tip.

Once the SIFs have been obtained, the crack propagation direction is defined by the Maximum Tangential Stress (MTS) criterion, proposed by Erdogan and Sih [8]. The sign of the crack propagation angle is influenced by the sign of the mode II SIF, K_{II}. For this reason, the propagation direction is given by:

$$\theta = \begin{cases} 0 \quad \rightarrow \quad K_{II} = 0 \\ 2\arctan\left\{\dfrac{1}{2}\left[\dfrac{K_I}{K_{II}} + \sqrt{\dfrac{K_I^2}{K_{II}^2} + 8}\right]\right\} \quad \rightarrow \quad K_{II} < 0 \\ 2\arctan\left\{\dfrac{1}{2}\left[\dfrac{K_I}{K_{II}} - \sqrt{\dfrac{K_I^2}{K_{II}^2} + 8}\right]\right\} \quad \rightarrow \quad K_{II} > 0 \end{cases} \tag{1}$$

where θ is the crack propagation angle and K_I and K_{II} are mode I and mode II SIFs, respectively.

Fatigue crack growth occurs when the crack propagates due to cyclic loading. These cyclic loadings can have constant or variable amplitude. Constant amplitude cyclic loading is completely defined by the maximum stress, σ_{max}, and by one of two quantities: minimum stress, σ_{min}, or stress ratio, R. For variable amplitude cyclic loading, the maximum and minimum stresses are not well defined, due to the irregularity of the cycles. Therefore, to analyze fractured components under the action of variable amplitude cyclic loading, it is necessary to apply the Rainflow Cycle Counting algorithm. This algorithm transforms variable amplitude cyclic loading into an equivalent constant amplitude cyclic loading, with $R = 0$ and maximum stress defined by:

$$\Delta\sigma_q = \left\{\frac{1}{N_B}\sum_{j=1}^{N_B}[\sigma_{max}^j(1 - R_j)^\gamma]^m\right\}^{\frac{1}{m}} \tag{2}$$

where $\Delta\sigma_q$ is the maximum stress of the equivalent constant amplitude cyclic loading, N_B is the total number of cycles, j is the analyzed cycle and m is a material constant.

According to Broek [5], when analyzing the propagation of fatigue cracks in mixed mode fracture, it is necessary to define an equivalent stress intensity factor of mode I, obtained by the MTS criterion through the equation:

$$K_{Ieq} = K_I \cos^3\left(\frac{\theta}{2}\right) - 3K_{II}\cos^2\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) \tag{3}$$

Therefore, the subcritical propagation of fatigue cracks is then evaluated by:

$$\frac{da}{dN} = \frac{C_0}{(1 - R)^{m(1-\gamma)}}\left(\Delta K_{Ieq}\right)^m \tag{4}$$

where C_0 is a material constant for $R \approx 0$, ΔK_{Ieq} is the difference between the equivalent SIFs obtained for σ_{max} and σ_{min} and da/dN denotes the fatigue crack growth rate, where a is the crack length and N is the number of cycles. Equation (4) is a modification of the Paris equation, originally proposed by Paris et al. [9], in order to consider mixed mode fracture through eq. (3) and non-zero stress ratios by applying the Walker equation (see Dowling [10]). The lifespan of the cracked component is measured by the number of cycles to failure, obtained by numerical integration of eq. (4) via Simpson's rule. Figure 1 shows the schematic representation of the implemented program.

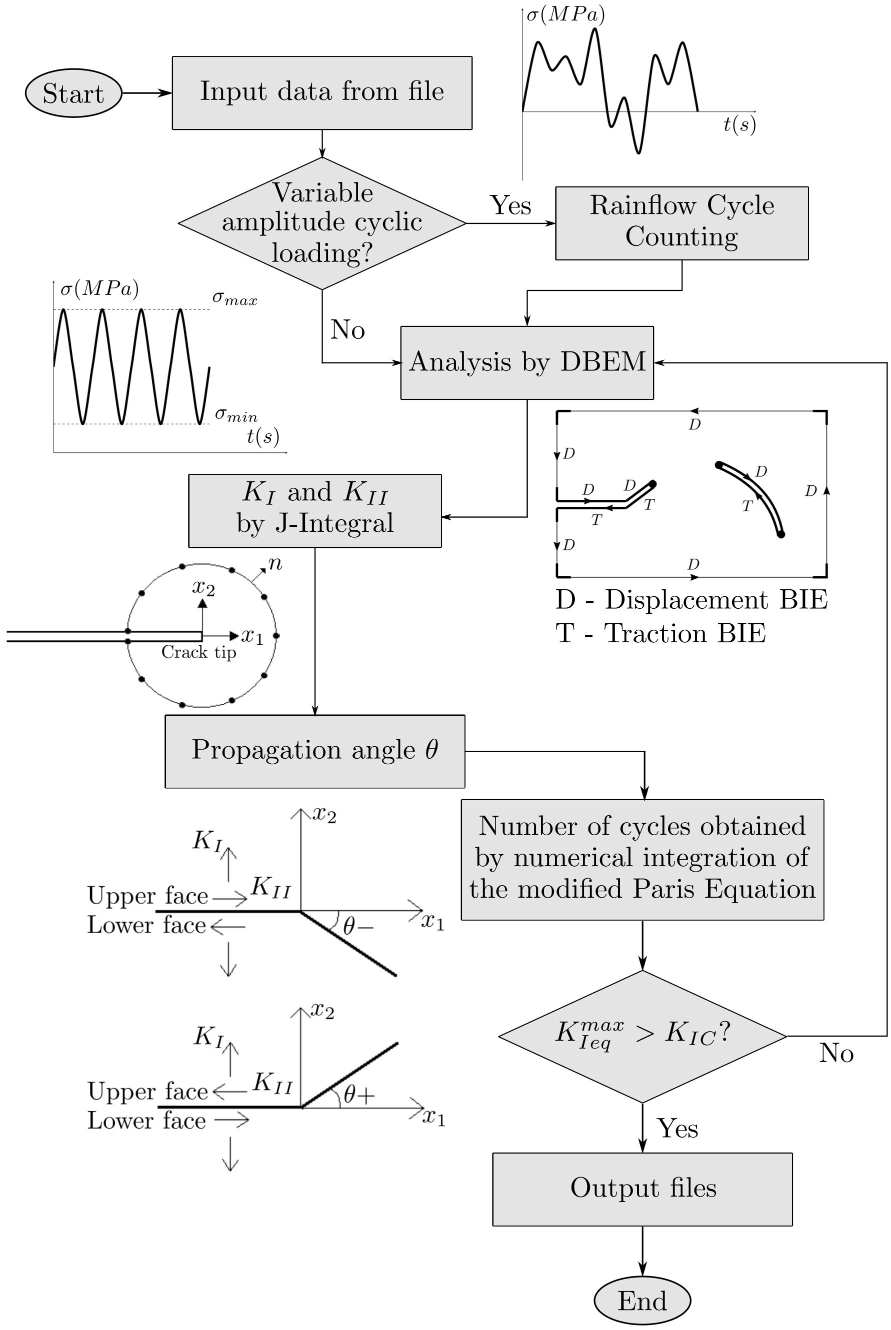


Figure 1. Schematic representation of the implemented program

3 Numerical Results

The implemented program is applied to crack propagation analysis in a Compact Mixed Mode (CMM) specimen, experimentally tested by Chambers et al. [11], and in the driven wheel of an iron ore pelletizing furnace. The results are shown in the following subsections.

3.1 Compact Mixed Mode (CMM) Specimen

The Compact Mixed Mode (CMM) specimen, whose material is Jethete M152 steel at a temperature of 550°C, is illustrated in Fig. 2.

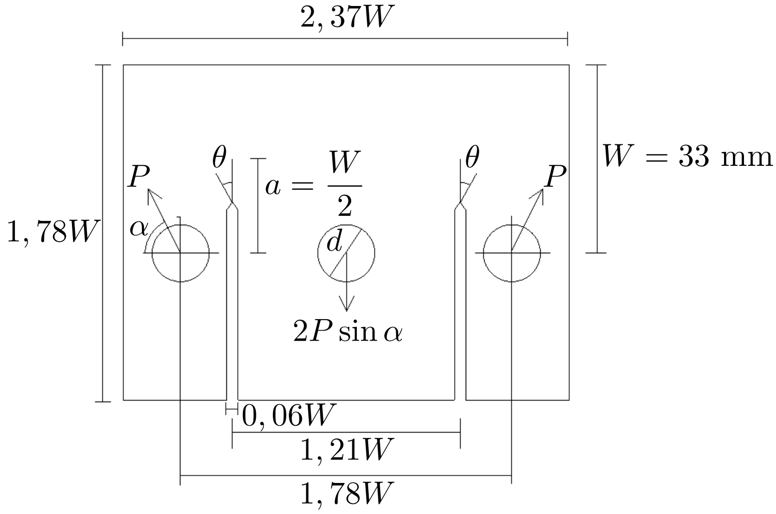
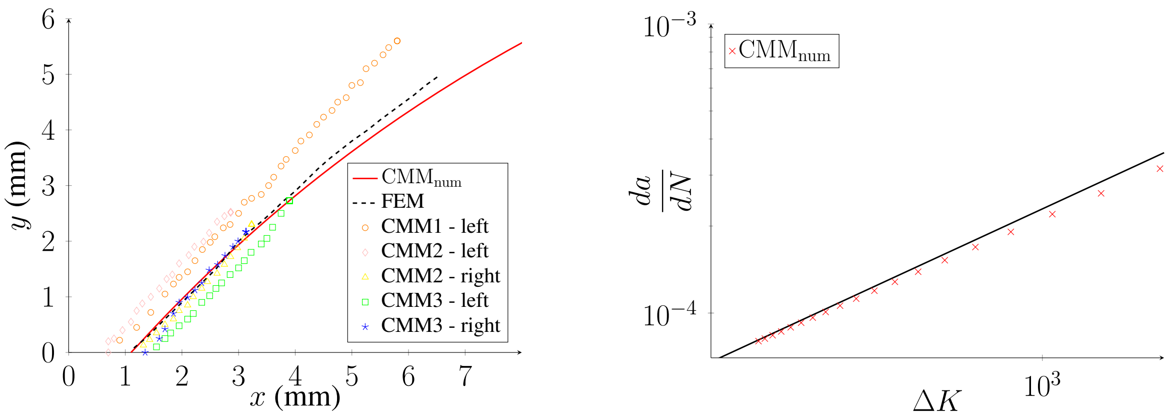


Figure 2. Compact Mixed Mode (CMM) Specimen

Due to symmetry, only the left half of the specimen is evaluated. Two different configurations are tested, varying the stress ratio (R), the load application angle (α) and the crack initial angle (θ), illustrated in Fig. 2. The crack path is compared with numerical and experimental results reported by Chambers et al. [11], while the fatigue crack growth rate is compared with the Paris equation provided in the reference. The results obtained with the computational program are identified by CMM_{num}, while the numerical result of the reference is identified by FEM and the experimental results by CMM1, CMM2, CMM3 and CMM4. The first analysis is performed with $R = 0,111$, $\alpha = 90°$ and $\theta = -3°$ and the results are shown in Fig. 3. The second analysis is performed with $R = 0,0625$, $\alpha = 110°$ and $\theta = -1.5°$, with the results presented in Fig. 4.

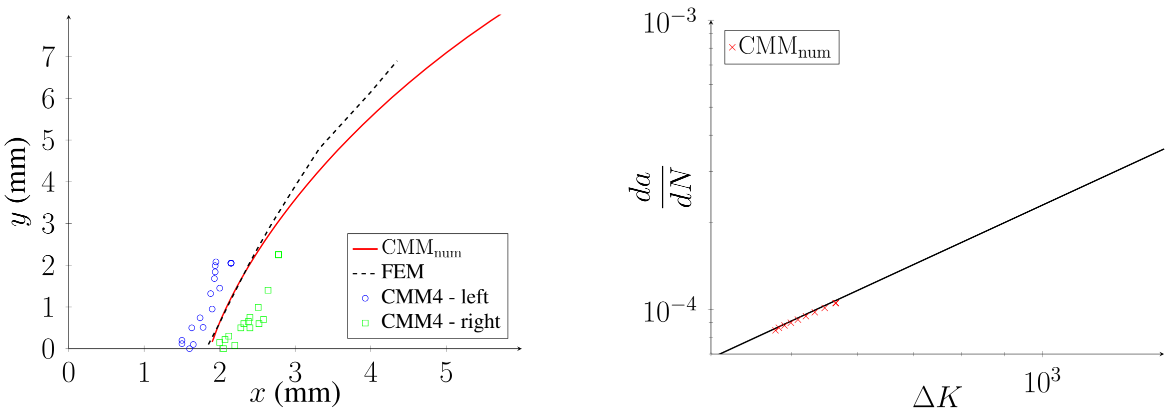


(a) Crack path

(b) Fatigue crack growth

Figure 3. Results for the CMM specimen with $\alpha = 90°$ and $\theta = -3°$

In both tests, the results obtained numerically by the implemented computational program show good agreement with the reference results.



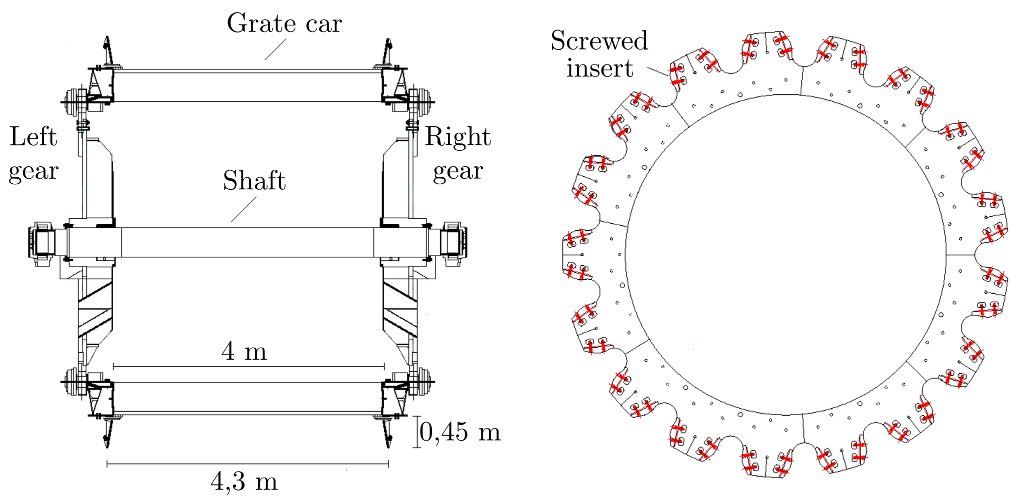
(a) Crack path

(b) Fatigue crack growth

Figure 4. Results for the CMM specimen with $\alpha = 110°$ and $\theta = -1,5°$

3.2 Driven wheel of an iron ore pelletizing furnace

The straight grate process is very common for drying and hardening iron ore pellets in mining industry, mainly in Brazil. This process consists of a continuous flow of pellet-loaded grate cars, which are not motorized and their movement occur exclusively due to electric motors coupled to the furnace's driven wheel. This wheel consists of two large AISI 4340 steel gears, interconnected by a single shaft. Each gear has 17 teeth and a pitch diameter of 4136 mm. The wheel and gear are illustrated in Fig. 5.



(a) Gear-type wheel that transports the grate cars

(b) Gear

Figure 5. Straight grate driven wheel

Due to repetitive contact, gear teeth often wear out. For this reason, screwed inserts are used in the contact region to facilitate maintenance. The placement of these inserts generates stress concentration points, which are the target of this study. To evaluate fatigue behavior, a tooth of this gear is analyzed. The insert is separated from the tooth body, being the fatigue load applied to the insert and the crack, with a length of 2 mm, starting at the meeting corner between the two parts. The models of the insert and the gear tooth are illustrated in Fig. 6 and Fig. 7(a) and the final deformed configuration in Fig. 7(b).

Considering the linear velocity of the driven straight grate equal to 4.5 m/min, the gear rotates 413 cycles per day approximately. In each cycle, the gear tooth is subjected to two loads. The first load refers to half the weight of the grate car, equivalent to 50 kN, supported by the gear tooth when taking the car from the lower rail to the upper rail. Upon reaching the upper rail, this same gear tooth is responsible for propelling the row of 130 grate

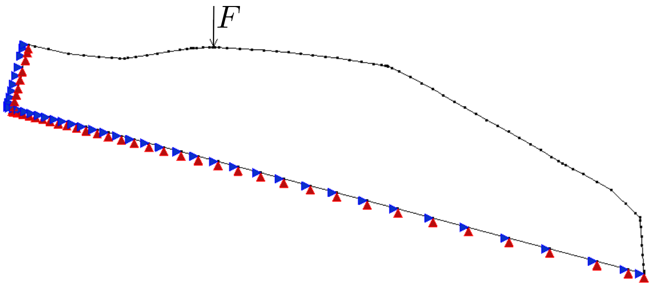
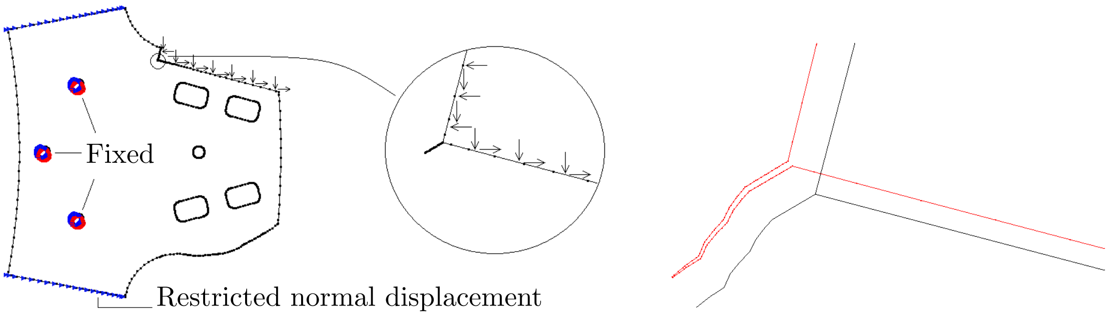


Figure 6. Screwed insert

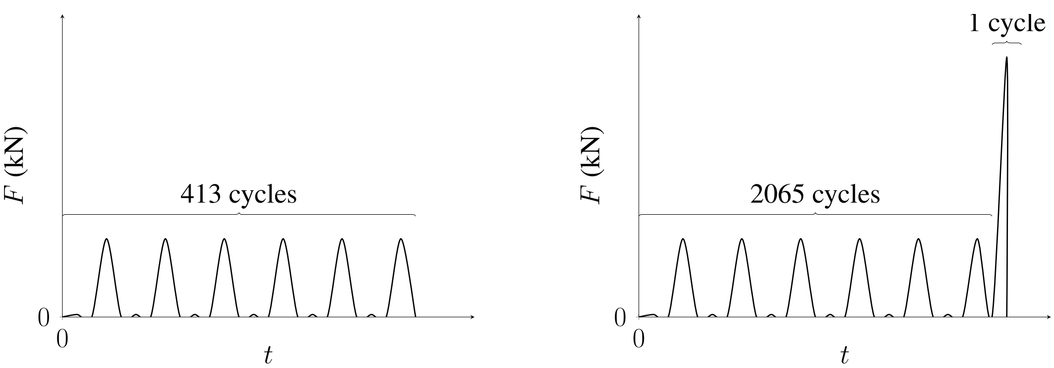


(a) Gear tooth body

(b) Deformed configuration

Figure 7. Gear tooth body and deformed configuration

cars loaded with pellets for the introduction of this new car. This boost load is directly influenced by the rolling resistance coefficient. In this work, three values are adopted for this coefficient: 0.075, 0.09 and 0.1, generating loads equal to 1550 kN, 1850 kN and 2057.5 kN, respectively. In addition to these fixed loads, the process is stopped every day to change a car in the row. At this point, the upper gear tooth is responsible for restarting the movement of all 130 loaded cars, generating an overload of 5143.75 kN in the system. This overload can be avoided if the cracked gear tooth is monitored, preventing the system from stopping with this tooth in the upper position. Therefore, two loading conditions will be considered: the tooth under study is monitored and is not subject to this third load or this tooth is not monitored and is subject to this overload once every five days, in a conservative manner. Both cyclic loadings are shown in Fig. 8.



(a) Without overload

(b) With overload every 5 days

Figure 8. Variable amplitude cyclic loads to which the gear is subjected

The results obtained for each loading condition are presented in Table 1, both for the number of cycles until rupture and for the component's lifespan, in days.

Table 1. Gear lifespan

	Without overload		With overload	
Rolling resistance coefficient	Cycles	Days	Cycles	Days
0.075	285090	691	258420	623
0.090	160729	389	147161	356
0.100	113890	276	104661	253

Depending on the rolling resistance coefficient, the gear's lifespan may be less than a year. As the maintenance plan for this equipment usually includes shutdowns every 12 months, there is a risk of brittle fracture before the scheduled intervention.

4 Conclusions

The implemented program is able to accurately evaluate the fatigue cracking behavior of cracked components subjected to mixed mode fracture and non-zero stress ratios, as demonstrated in the CMM specimen analysis. It demonstrates that the modifications made to the Paris equation were effective. Furthermore, the program is able to estimate the remaining service life of real components subjected to variable amplitude cyclic loading, by implementing the Rainflow Cycle Counting algorithm.

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References

[1] Y.-L. Lee, J. Pan, R. B. Hathaway, and M. E. Barkey. *Fatigue testing and analysis (theory and practice)*. Elsevier, 2005.

[2] A. Portela, M. H. Aliabadi, and D. P. Rooke. Dual boundary element analysis of cracked plates: singularity subtraction technique. *International Journal of Fracture*, vol. 55, pp. 17–28, 1992a.

[3] A. Portela, M. H. Aliabadi, and D. P. Rooke. The dual boundary element method: effective implementation for crack problems. *International Journal for Numerical Methods in Engineering*, vol. 33, pp. 1269–1287, 1992b.

[4] X.-W. Gao. Numerical evaluation of two-dimensional singular boundary integrals - theory and fortran code. *Journal of Computational and Applied Mathematics*, vol. 188, pp. 44–64, 2006.

[5] D. Broek. *Elementary Engineering Fracture Mechanics*. Martinus Nijhoff Publishers, 1986.

[6] G. Feng, Y. Kang, F. Chen, wei Y. Liu, and chuan X. Wang. The influence of temperatures of mixed-mode (i+ii) and mode-ii fracture toughness of sandstone. *Engineering Fracture Mechanics*, vol. 189, pp. 51–63, 2017.

[7] J. R. Rice. A path independent integral and the approximate analysis of strain concentration by notches and cracks. *J. Appl. Mech*, vol. 35, n. 2, pp. 379–386, 1968.

[8] F. Erdogan and G. C. Sih. On the crack extension in plates under plane loading and transverse shear. *Journal of Basic Engineering*, pp. 519–525, 1963.

[9] P. C. Paris, M. P. Gomez, and W. P. Anderson. A rational analytic theory of fatigue. *The Trend in Engineering*, vol. 13, pp. 9–14, 1961.

[10] N. E. Dowling. *Mechanical Behavior of Materials: Engineering Methods for Deformation Fracture, and Fatigue*. Pearson, fourth edition, 2013.

[11] A. C. Chambers, T. H. Hyde, and J. J. Webster. Mixed mode fatigue crack growth at 550°C under plane stress conditions in jethete m152. *Engineering Fracture Mechanics*, vol. 39, pp. 603–619, 1991.