Numerical Analysis and Validation of Fracture Toughness Factors using BEMCRACKER2D

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Abstract. Fracture Mechanics (MF) is a complex field focused on the behavior of materials and structures with cracks. Over the past century, significant advancements have been made in MF, particularly in structural engineering applications, such as the naval and aerospace sectors, where crack prediction is critical. Due to the complexity of these problems, numerical methods like the Boundary Element Method (BEM) have become essential for analyzing crack propagation, based on Linear Elastic Fracture Mechanics (LEFM). This study analyzes fracture toughness factors, k1 and k2, comparing numerical curves from the literature with those obtained using BEM-based predictions. The BEMLAB2D and BEMCRACKER2D software were used for modeling and analysis. The results confirm the effectiveness of numerical methods, with strong convergence between literature curves and the ones generated, particularly highlighting the Maximum Circumferential Stress method for its alignment with experimental dat**Keywords:** Elastoplastics models; crack growth; DRM; DBEM.

1 Introduction

Fracture mechanics has become a fundamental discipline in engineering over the last two decades of the 20th century, driven by both research advancements and the broad application of these technologies to practical problems, Anderson [1].

The development of numerical methods for crack analysis has advanced in recent decades, [2–5]. A notable approach is the Dual Boundary Element Method (DBEM), [6, 7], which has emerged as a solution for crack problems involving distinct faces that could not be resolved by the conventional Boundary Element Method (BEM), Brebbia and Dominguez [8]. This method has significantly contributed to improving the accuracy of analyses conducted.

This study focuses on the application of DBEM to two-dimensional models of crack propagation under Mode I and Mixed Mode conditions in an elastoplastic analysis. Non-homogeneous terms in the domain integrals of the Boundary Integral Equations (BIE) are addressed through the Dual Reciprocity Method (DRM), enabling analysis of elastoplastic behavior in the vicinity of the crack tip. Plastic stress evaluation is conducted using the von Mises yield criterion.

2 Development of theory

2.1 Linear Elastic Fracture Mechanics

The principles of fracture mechanics formulated before 1960 applied exclusively to materials that followed Hooke's law, with analyses limited to linear elastic behavior. After this period, theories were developed to include nonlinear behaviors, such as plasticity and viscoplasticity, as well as dynamic effects. Recent theories are expansions of linear elastic fracture mechanics (LEFM), highlighting the importance of a fundamental understanding of LEFM for grasping more advanced concepts. This field was significantly influenced by the pioneering works of Inglis [9] and Griffith [10], who established the foundations for understanding energy release and stress intensity factors [1].

2.2 Stress analysis at the crack tip

Stress analysis at the crack tip is a fundamental aspect of fracture mechanics, given that stress concentrations in this proximity can be significant, sometimes exceeding the elastic limit and causing plastic deformation. The stress intensity factor (SIT) quantifies the stress state near the crack tip under various loading conditions. The singularity of the stress field, characterized by the inverse square root of the distance from the crack tip, was first described and published by Westergaard [11], Irwin [12, 13], Sneddon [14], and Williams [15]. The stress field in the elastic regime is given by:

$$\sigma_{ij} = \left(\frac{k}{\sqrt{r}}\right) f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{\frac{m}{2}} g_{ij^{(m)}}(\theta). \tag{1}$$

Where σ_{ij} is the stress tensor; r and θ are polar coordinates of a point defined in Figure 1; k is a constant; f_{ij} is a dimensionless function of θ for the principal term. For higher-order terms, we have: A_m is the amplitude; $g_{ij^{(m)}}$ is a dimensionless function of θ for the $m-th$ term, Anderson [1].

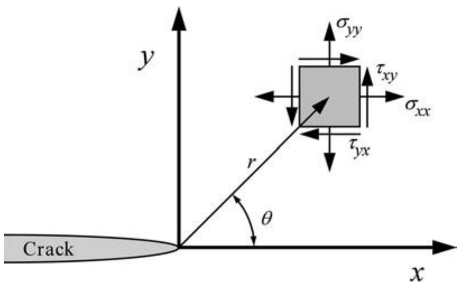


Figure 1. Coordinate axis ahead of a crack tip (Anderson, [1]).

Stress Intensity Factor (SIF). The analysis of stress states near a crack is defined by the parameter K, known as the Stress Intensity Factor. It is associated with the different modes of crack opening, including Mode I (K_I), Mode II (K_{II}), and Mode III (K_{III}), as illustrated in Figure 2.

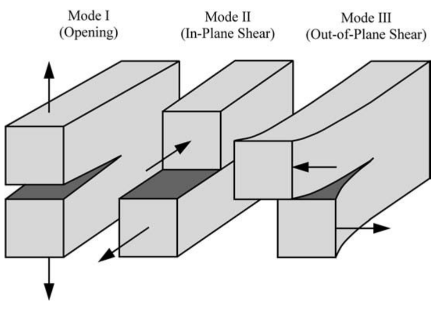


Figure 2. The three types of loading that can be imposed on a crack (Anderson, [1]).

2.3 J Integral

Rice [16] extended plastic deformation to analyze cracks in nonlinear materials. He demonstrated that the J integral could be formulated as a path-independent integral. Hutchison [17] showed that J can also characterize stresses and strains at the crack tip in nonlinear materials. Thus, the J integral can be viewed both as an energy parameter and a stress intensity parameter. Rice [16] further illustrated that the value of the J integral is equivalent to the rate of energy dissipation in a nonlinear elastic material.

Consider an arbitrary path (Γ) encircling the crack tip counterclockwise, as depicted in Figure 3. The J integral is defined as:

$$J = \int_{\Gamma} \left(w dy - T_i \frac{\partial u_i}{\partial x} ds \right) \tag{2}$$

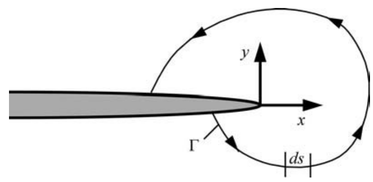


Figure 3. Arbitrary path encircling the crack tip, Anderson [1].

Where: strain energy density w, components of the traction vector T_i, displacement vector components u_i, and length increment along the contour Γ, denoted as ds.

2.4 Crack propagation direction

In this work, the Maximum Circumferential Stress criterion is used to determine the direction in which a crack will propagate.

Maximum Circumferential Stress criterion (MCS).

The Maximum Circumferential Stress criterion, proposed by Erdogan and Sih [18], is based on the idea that a crack propagates in the direction normal to the plane where the circumferential stress around the crack tip is maximal. According to this criterion, the direction of crack propagation is determined by the condition that the tangential stress reaches its maximum value at a specific point along the crack. This approach is useful for predicting propagation in brittle materials, where the crack tends to move in the direction where the stress is highest. Barsom and Rolfe [19] also contributed to the validation and application of this criterion. The propagation angle at which the crack will propagate, taking into account the stress intensity factors in Modes I and II, is then given by:

$$\theta = 2tan^{-1} \left(\frac{1}{4} \frac{K_I}{K_{II}} \pm \frac{1}{4} \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8} \right) \tag{3}$$

Where: Propagation angle θ, Stress Intensity Factor Mode I K_I, Stress Intensity Factor Mode II K_{II}.

2.5 Boundary Element Method (BEM) and Dual Contour Element Method (DBEM)

The Boundary Element Method (BEM) is a numerical technique for analyzing problems by discretizing a boundary, Γ. Unlike finite element methods (MEF), BEM stands out for discretizing only the boundary or external surface, resulting in computationally more efficient models.

The Dual Contour Element Method (DBEM) approach, proposed by [4], emerged as a solution to the challenges associated with cracking problems that could not be resolved via conventional BEM. The method has had significant contributions, in relation to the accuracy of the analysis, carried out by researchers such as S. Parvanova and G. Gospodinov [20] and Cordeiro and Leonel [21]. The dual equations on which DBEM is based are the displacement and tension boundary integral equations. By using them on each face of the crack, it was possible to eliminate the singularity that previously existed. On the other hand, when using FEM, it is necessary to work with the enrichment of functions to address the problem of cracks, due to the continuous refinement (remeshing) required in the mesh. The dual boundary integral equations on which the DBEM is based are those of displacement (u_i) and traction (σ_{ij}), in the absence of body forces they can be expressed as:

$$u_j(X') + \int_\Gamma T_{ij}(X',x)u_j(x)d\Gamma(x) = \int_\Gamma U_{ij}(X',x)t_j(x)d\Gamma(x) \quad (4)$$

$$\sigma_{ij}(X')\int_\Gamma S_{ijk}(X',x)u_k(x)d\Gamma(x) = \int_\Gamma D_{ijk}(X',x)t_k(x)d\Gamma(x) \quad (5)$$

where i and j are the Cartesian components; T_{ij} and U_{ij} are the fundamental Kelvin solutions for traction and displacement, respectively, at a point x belonging to the contour; $\int_\Gamma$ line integral over the contour; the distance between X' and x is defined as r. The terms S_{ijk} and D_{ijk} are linear combinations of T_{ij} and U_{ij}, respectively; Γ is the crack contour, Portela et al., [4].

3 Methodology

To investigate the effects of crack propagation under elastoplastic conditions, this study employs the Dual Boundary Element Method (DBEM) in a two-dimensional model. DBEM is chosen for its capability to resolve the singularity problem at crack surfaces. Non-homogeneous terms in the Boundary Integral Equations (BIE) are handled using the Dual Reciprocity Method (DRM), allowing for elastoplastic effects to be included in the analysis. Evaluation of elastoplastic behavior around the crack tip is performed using the von Mises yield criterion to analyze induced plastic stresses. Key parameters in Fracture Mechanics, such as crack propagation path and Stress Intensity Factor (SIF), are determined using the BEMCRACKER2D [22]. software. Numerical results from this study are presented to validate the effectiveness of the proposed model. Continuous and discontinuous quadratic elements are employed along the remaining boundaries of the problem domain and the crack boundaries, respectively. To demonstrate the efficiency and robustness of the method, a C++ program is used for handling two-dimensional crack models, featuring a MATLAB interface for previewing the propagation path. Additionally, two case studies are utilized to showcase the method's capabilities.

3.1 Numerical Modeling

The simulation is conducted using both BEMLAB2D [23] and [22]. [23] is a graphical user interface used for preprocessing and postprocessing in boundary element method (BEM) simulations. It assists in modeling geometry, defining boundary conditions, and generating meshes. The preprocessing steps include defining geometric points, constructing contours, specifying material zones, and applying boundary conditions. After setting up the model, [23] integrates with [24] for detailed analysis in the postprocessing phase, it visualizes various results, including deformation plots, stress distributions, stress intensity factors, fatigue life, crack propagation, and plastic zones. [23] is employed to set up the problem geometry, create the boundary element mesh, and input material parameters such as elasticity modulus and Poisson's ratio. On the other hand, [22] handles the actual processing: it reads pre-processing data, employs programmed methods to compute Stress Intensity Factors (SIFs) via the J-integral, determines crack propagation direction using the MCS, and performs other calculations. Finally, [23] is revisited for post-processing, generating deformation plots, crack propagation analyses, and other outputs based on data exported from [22].

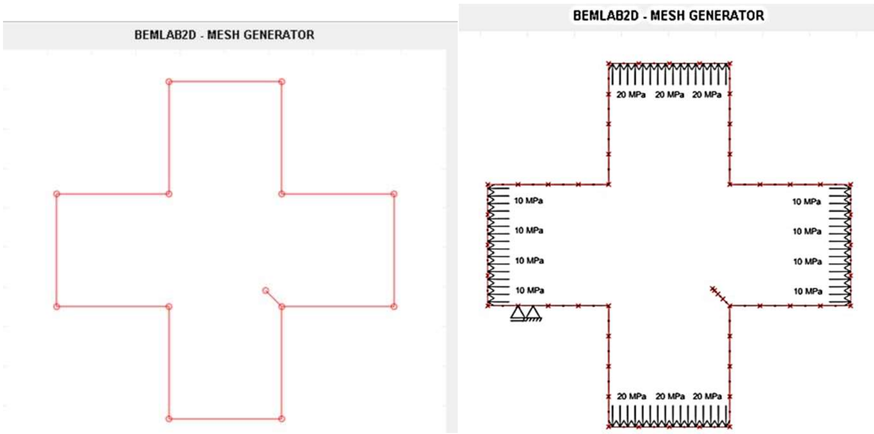
This section presents two case studies to illustrate the process of modeling and visualizing incremental crack propagation analysis.

3.2 Case Study

3.2.1 Cruciform Plate with Inclined Crack

This application involves the analysis of a cruciform plate with an inclined crack, where the initial crack length-to-width ratio is $a/L = 0.2$, $L = 0.2$, and the angle between the crack and the vertical axis is 45°. The applied tensile stresses are $T = 20\ MPa$ and $T = 10\ MPa$, while the boundary conditions are traction (y, x) and displacement, as illustrated in Fig. 4 (a). The material properties used were an elastic modulus of $E = 218400\ Pa$ and a Poisson's ratio of $\nu = 0.3$. Fig. 4 (a) shows the model generated by the [24] interface using the user-defined actions specified in Module I.

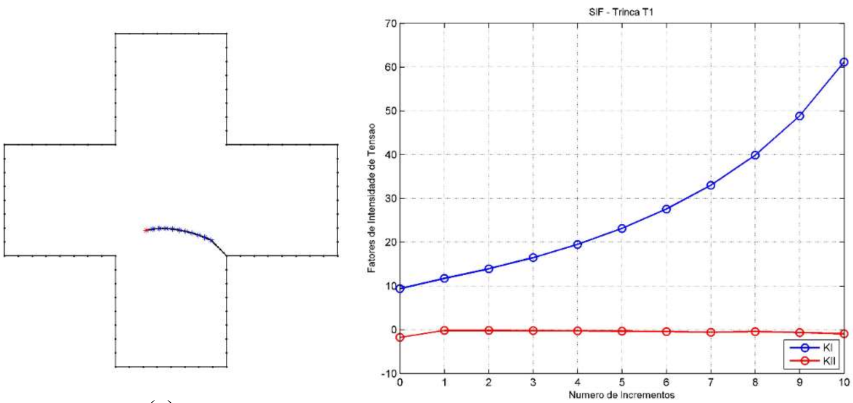
Figure 4 (b) shows the boundary element mesh, generated by Module II after applying boundary conditions with Module III. In this figure, the use of continuous and discontinuous quadratic boundary elements is observed on the domain boundary and the crack, respectively.



(a) (b)

Figure 4. Cruciform Plate with Crack: (a) Model Generated by BEMLAB2D; (b) Mesh Generator by [24]

The analysis begins with Module IV, generating the crack propagation path mesh, created using Module V, as illustrated in Fig. 6 (a). For this analysis, a total of 10 crack increments, each with a length of 0.3a m, were considered (see the enlarged detail of the path). Figure 6 (b) displays the graph of Stress Intensity Factors versus crack advancement, which is generated using Module V (SIFs button) and includes a configuration options dialog for display. It is evident that Mode II is nearly zero after the first increment.



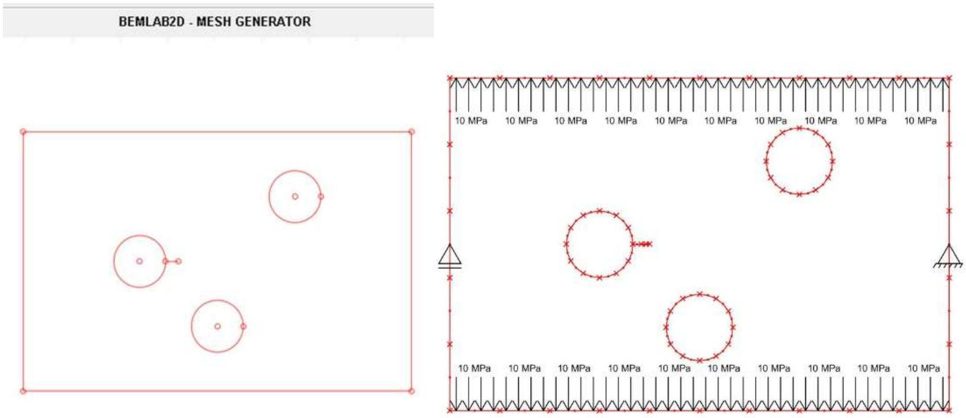
(a) (b)

Figure 6. (a) Propagation mesh generated by [24] and enlarged detail of the path; (b) SIFs vs. Crack Increment Advancement graph.

3.2.2 Rectangular Plate with Holes and Crack

This example examines a rectangular plate (3m x 2m) with three holes (each with a radius of 0.2m) and an initial crack ($a = 0.1m$) emanating from one of these holes. The characteristic dimensions are $h = 1m$, $b = 0.6m$, and $d = 0.5m$, as depicted schematically in Fig. 7 (a). A uniform tensile stress of $t = 10\ MN/m^2$ is applied along the plate's edges, perpendicular to the x-axis of the initial crack. The plate's material properties include an elastic modulus of $E = 200{,}000\ MN/m^2$ and a Poisson's ratio of $\nu = 0.25$. Figure 7(b) illustrates the model created by [23].

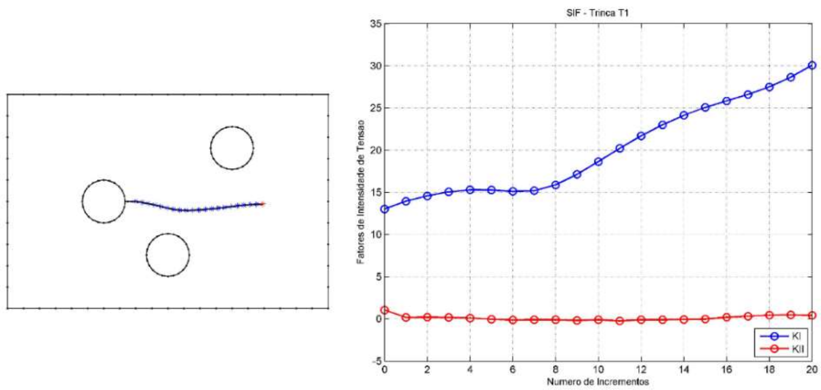
Figure 7 (b) shows the boundary element mesh created by Module II following the application of boundary conditions by Module III. The figure displays the use of continuous and discontinuous quadratic boundary elements along the domain's boundary and the crack's edges (see enlarged detail).



(a) (b)

Figure 7. Rectangular Plate with Three Holes: (a) Problem Model; (b) Mesh Generator by [24]

Following the analysis with Module IV, the crack propagation path mesh is generated using Module V, as shown in Fig. 8(a). For this analysis, a total of 20 crack increments, each with an advance of 0.6 mm, were considered (see the enlarged detail of the path). Figure 8(b) presents the graph of Stress Intensity Factors versus the number of increments.



(a) (b)

Figure 8. (a) Propagation mesh generated by [23] and enlarged detail of the path; (b) SIFs vs. Crack Increment Advancement graph.

4 Conclusions

In this work, two boundary integral equations—displacement and traction—were applied independently in the process of modeling two-dimensional cracks. To predict the crack propagation path, an incremental analysis using the Boundary Element Method (BEM), incorporating both equations, was also conducted. The automation of the modeling and incremental analysis process was achieved through the interaction between the [23] and [22] programs. [23] handles the modeling of the crack surface using both boundary integral equations and discontinuous quadratic elements for the crack boundary, continuous quadratic elements for the domain boundary, mesh generation, and visualization of the crack propagation path. [22], invoked through the interface, performs a stress analysis of the structure using BEM. After each increment, the stress intensity factors are computed, and the crack propagation direction is calculated and adjusted using the maximum stress criterion. Finally, the two examples presented demonstrated the functionality of the graphical interface, and the results of the crack propagation path obtained with the [22] program showed that using both boundary integral equations significantly simplifies the modeling process and confirms the robustness of the technique.

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