Effect of spatial variability of soil properties on slopes stability under rapid water drawdown conditions

Mário V. Ceron[1], Diogo L. Cecílio[1], Renato V. Linn[1], Samir Maghous[1]

[1]*Graduate Program in Civil Engineering PPGEC/UFRGS,*
Av. Osvaldo Aranha, 99, Porto Alegre, 90035-190, RS, Brazil
mario.ceron@ufrgs.br, cecilio.diogo@gmail.com, renatolinn@ufrgs.br, samir.maghous@ufrgs.br

Abstract. Stability analysis of slopes is a fundamental problem of Soil Mechanics. Its main objective consists of evaluating the potential failure of the slope structure under a prescribed loading mode. A major component of the latter refers to the seepage forces induced by pore-pressure gradient, which is known to be responsible for destabilizing effects. An important aspect that is often disregarded is the spatial variability of the material, which directly impacts the stability of slopes, representing a significant source of uncertainty. This study employs a kinematic approach of limit analysis theory to obtain upper-bound solutions to the stability problem of saturated slopes submitted to rapid water level drawdown. In the context of effective stress analysis, the hydraulic problem governing the water filtration velocity is evaluated independently by resorting to a numerical approach. Random fields are considered to take into account the variability related to the spatial distribution of soil cohesion, friction angle, and permeability. The slope failure probability is assessed through Monte Carlo simulations. Results have shown that the overall failure probability of the slope is mainly affected by the variability of the soil cohesion, followed by the friction angle, whereas the variability of permeability has little effect.

Keywords: Slope stability, Seepage forces, Limit analysis, Monte Carlo method, Random fields

1 Introduction

Stability analysis of slopes is a fundamental problem of Soil Mechanics. Its main objective consists of evaluating the potential failure of the structure under prescribed loading conditions from the sole knowledge of the strength properties of its constituent material. Water saturating the soil is known to generally reduce its shear strength. In non-hydrostatic situations, seepage-induced effects can endanger the safety of previously stable structures, especially in rapid drawdown situations, posing a significant risk to riverside, coastal and near dam slopes [1–3]. An important aspect that is often disregarded in most analyses is the spatial variability of the material, which directly impacts the stability of slopes and represents a significant source of uncertainty. Soil variability arises from a combination of geological, environmental, physical, and chemical processes, directly impacting the material properties [4]. Works devoted to this topic typically evaluate the slope failure probability through Monte Carlo Simulations, integrating random fields theory to Limit Equilibrium Methods or numerical approaches to stability analysis (e.g., [5, 6]).

In the present research, the validity of effective stress concept is assumed for the saturated porous medium strength capacities. Following the procedure described by Saada et al. [7], the water flow regime is determined independently through the Finite Element Method. The resulting seepage forces computed from the gradient of pore pressure distribution are regarded as driving body forces in the stability problem. Once the seepage forces are evaluated, the stability analysis is therefore developed within the framework of limit analysis kinematic approach. The latter relies upon the implementation of rotational failure mechanisms involving velocity discontinuity along logarithmic spiral curves. This approach is widely accepted in the scientific community in kinematic approaches of limit analysis [3, 7–9].

The uncertainty concerning the spatial variability of soil properties is modeled by means of random field theory. The soil cohesion, friction angle, and permeability fields are discretized via the Karhunen-Loève expansion with numerically computed eigenfunctions. Monte Carlo simulations are used to evaluate the slope failure probability. The paper ends with the presentation of several numerical simulations aimed at assessing the impact of spatial variability of soil strength and permeability on the stability condition of a slope subjected to seepage forces induced by rapid water level drawdown.

2 Effective stress stability analysis

This section describes a poromechanical framework for the effective stress stability analysis of geotechnical structures in the presence of seepage forces. Along the subsequent analysis, the domain occupied by the soil material shall be referred to as Ω and its external boundary as $\partial\Omega$.

2.1 Limit Analysis of saturated slopes under plane strain condition

The primary objective of limit analysis theory and related kinematic approach is to evaluate the stability conditions of a material system under the applied loading. The plane strain stability analysis under consideration refers to a soil slope with height H and inclination β, as depicted in Figure 1. In the context of effective stress analysis, the strength capacities of the soil will be described by means of an isotropic frictional-cohesive condition governing the Terzaghi effective stress. The strength parameters of the Mohr-Coulomb criterion c and φ denote respectively the soil cohesion (effective cohesion intercept) and friction angle fields.

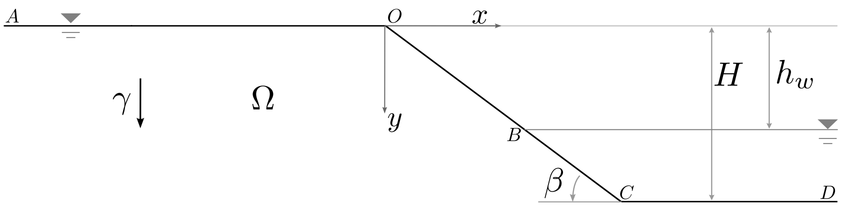

Figure 1. Slope geometry and loading configuration.

As regards the loading conditions, the slope structure is subjected to two forces: the mechanical components, which are defined by the gravity forces (saturated unit weight γ parallel to Oy direction) and boundary conditions; and the hydraulic component, originated from the pore fluid pressure p distribution associated with the flow through the slope that is induced by water drawdown (rapid decrease h_w in water level). The spatial variation of excess pore pressure distribution u, which is related to its gradient field $-\mathrm{grad}\, u$, is expected to be the driving force for the seepage loading component. The excess pore pressure $u = p - p_w$ is defined relative to the hydrostatic pore pressure $p_w = y\gamma_w$, where γ_w denotes the water unit weight.

The kinematic approach is formulated within the framework of effective stress analysis. Denoting by $\underline{U}$ any given kinematically admissible velocity field (referred to as failure mechanism), the upper-bound theorem of limit analysis states the following necessary condition for the stability of the slope structure under applied external loading [10]:

$$\forall \underline{U} \quad P_{ext}(\underline{U}) \leq P_{mr}(\underline{U}) \tag{1}$$

where P_{ext} stands for the rate of external work performed by the external forces and P_{mr} stands for the maximum rate of resisting work developed in the failure mechanism $\underline{U}$.

In this research, the set of possible velocity fields is restricted to the class of rotational failure mechanisms involving velocity discontinuity along logarithmic spiral curves [11]. Due to the heterogeneity of the slope material properties, the velocity discontinuity surface is discretized into logarithmic spiral segments, with their angles set to be greater than or equal to the local soil friction angle. This consideration is necessary in order to comply with the normality condition of the Mohr-Coulomb criterion and obtain a non-trivial inequality (1) (i.e., $P_{mr} < +\infty$).

In this context, P_{ext} and P_{mr} are given by:

$$P_{ext}(\underline{U}) = \int_{\Omega} (\gamma - \gamma_w)\underline{e}_y \cdot \underline{U}\, d\Omega + \int_{\Omega} -\mathrm{grad}\, u \cdot \underline{U}\, d\Omega, \quad P_{mr}(\underline{U}) = \int_{\Sigma} \frac{c}{\tan\varphi} [\underline{U}] \cdot \underline{n}\, dS \tag{2}$$

where $[\underline{U}]$ denotes the velocity jump at a point $\underline{x}$ when crossing a velocity discontinuity surface Σ following its normal unit vector $\underline{n}$. In the spirit of the kinematic approach, the optimal upper-bound estimate to the stability problem, denoted by the stability factor Γ, is achieved by means of the minimization of:

$$\Gamma \leq \min_{\underline{U} \in \zeta} \frac{P_{mr}(\underline{U})}{P_{ext}(\underline{U})}, \quad \text{where } P_{ext}(\underline{U}) > 0 \tag{3}$$

where ζ symbolically represents the class of admissible rotational failure mechanisms considered. Values of Γ lower than unity mean necessary failure of the slope.

2.2 The hydraulic problem

Traditionally, the seepage influence on the stability of saturated slopes is accounted for by employing empirical methods based on pore pressure coefficients. The major drawback of such an approach is that the value of the coefficients to be used for a given stability configuration is a priori unknown. In the present study, the alternative approach described by Saada et al. [7] is considered.

As stated in the effective stress formulation of the slope stability problem presented in the previous section, the effects of seepage forces may be accounted for through the distribution of $-\text{grad}\ u$ (Eq. 2), which act as external body forces that should therefore be previously evaluated. In that respect, the gradient of excess pore pressure is the driving force of the hydraulic flow network through the slope and is classically related to the filtration velocity field $\underline{v}$ by Darcy's law:

$$\underline{v} = -\underline{\underline{K}} \cdot \text{grad}\ u \tag{4}$$

where $\underline{\underline{K}}$ denotes the fluid permeability tensor expressed in $\text{m}^2 \cdot \text{Pa}^{-1} \cdot \text{s}^{-1}$ unit. Under the assumptions of steady flow, incompressibility of the solid as well as fluid particles, and neglecting the skeleton volume strain (one-way coupling framework), the field equation controlling the uncoupled hydraulic problem is given by:

$$\text{div}\ \underline{v} = 0 \tag{5}$$

The formulation for the uncoupled hydraulic boundary value problem in terms of excess pore pressure u relies upon field equation (5) and Darcy's law (4) together with the hydraulic boundary conditions (see Figure 1):

$$\begin{cases} \text{div}\ (-\underline{\underline{K}} \cdot \text{grad}\ u) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \overline{AO} \\ u = -y\gamma_w & \text{on } \overline{OB} \\ u = -h_w\gamma_w & \text{on } \overline{BC} \cup \overline{CD} \end{cases} \tag{6}$$

Equation (6) is a classical elliptic differential problem, which will be addressed numerically using the Finite Element Method. Nine-noded quadrilateral elements are considered. Once the distribution of $-\text{grad}\ u$ is obtained, it is utilized to evaluate the rate of external work (Eq. 2) over the failure mechanisms in the minimization problem. Throughout this study, the isotropic permeability ($\underline{\underline{K}} = k\underline{\underline{1}}$) is considered.

3 Spatial variability of soil properties: a stochastic slope stability approach

Most of stability analyses in soil mechanics applications generally rely upon deterministic approaches in which the poromechanical soil properties are described by average values. It is however well-established that both mechanical and fluid transport properties of soil masses are rarely homogeneous. This section aims to extend the preceding reasoning formulated for the slope stability under seepage forces to reliability-based analysis.

3.1 Probabilistic formulation of the stability problem

In the framework of spatially variable soil properties, the concept of stability factor should be viewed in a probabilistic sense (as described by Cho [5]) as a function of random variables (or vectors of correlated random variables), denoted by $\underline{\text{X}}$, describing the uncertainty related to soil poromechanical properties. The stability factor $\Gamma(\underline{\text{X}})$ represents a probability distribution. Based on this definition, the slope probability of failure can be evaluated from the following integral [12]:

$$P_f = P(\Gamma(\underline{\text{X}}) < 1) = \int_{\Gamma(\underline{\text{X}})<1} f_{\underline{\text{X}}}(\underline{\text{X}})\ d\underline{\text{X}} \tag{7}$$

where $f_{\underline{\text{X}}}(\underline{\text{X}})$ stands for the joint probability density function.

In the present analysis, Equation (7) is approximated by resorting to Monte Carlo Simulations, which revealed efficient to accurately solving the slope stability that involves large variability and strong non-linearity of the soil properties. The accuracy of the obtained failure probability approximation will be evaluated by the corresponding Coefficient of Variation $CoV(P_f)$ [13]. Values below $CoV(P_f) < 5\%$ are usually taken as standard in reliability analysis and, therefore, will be considered as the criterion for convergence in the present study [12].

3.2 Random fields

Following the description provided by Vanmarcke [14], a random field can be viewed formally as an indexed set of random variables with or without correlation structure. The Karhunen-Loève (KL) method [15], used in this work, is a series expansion method that decomposes a stochastic field into an infinite linear combination of orthogonal functions and uncorrelated stochastic random variables. In such a procedure, a random field H (denoting c, φ or k) with log-normal distribution in a geometrical domain Ω is approximated from a set of deterministic orthonormal functions $\{f_n(\underline{x})\}$ as:

$$H(\underline{x},\,\theta) \approx \exp\left[\mu'\left(1 + CoV'\sum_{n=1}^{M}\sqrt{\lambda_n}\xi_n(\theta)f_n(\underline{x})\right)\right],\ \mu' = \ln\left(\frac{\mu}{\sqrt{1+CoV^2}}\right),\ CoV' = \frac{\sqrt{\ln\left(1+CoV^2\right)}}{\mu'} \tag{8}$$

where $\underline{x} \in \Omega$ represents the local position vector, $\theta \in \Theta$ represents an element of the space of random events, μ denotes the mean and CoV the relative standard deviation of H, M is the truncation number of the series expansion, and $\{\xi_n(\theta)\}$ is a set of orthogonal random variables with standard normal distribution $\mathcal{N}(\mu = 0,\ \sigma = 1)$. The log-normal distribution is chosen to enforce the non-negativity of the considered soil properties.

In the above karhunen-loève Expansion, $\{\lambda_n\}$ and $\{f_n(\underline{x})\}$ represent respectively the eigenvalues and associated eigenfunctions of the covariance function $C(\underline{x}_1,\ \underline{x}_2)$. In this study, a two-dimensional exponential covariance function will be considered. The set of eigenvalues $\{\lambda_n\}$ and eigenfunctions $\{f_n(\underline{x})\}$ are then obtained as the solution for the Fredholm Integral Equation of the Second Kind:

$$\int_{\Omega} C(\underline{x}_1,\ \underline{x}_2)f_n(\underline{x}_1)\,d\underline{x}_1 = \lambda_n f_n(\underline{x}_2); \quad C(\underline{x}_1,\ \underline{x}_2) = \exp\left(-\frac{|x_1 - x_2|}{L_x} - \frac{|y_1 - y_2|}{L_y}\right) \tag{9}$$

where $(x_i,\ y_i)$ are the coordinates of point $\underline{x}$, L_x and L_y denote the horizontal and vertical autocorrelation distances, and $d\underline{x}_1$ the product of the coordinates $(dx_i)_i$. Solutions to the integral equation (9) will be obtained numerically from a finite element analysis. It is noted that the present study considers that all the random fields c, φ and, k exhibit the identical autocorrelation function C defined in domain Ω. Additionally, the cross-correlation prevailing between each pair of these random fields, such as the cohesion and friction angle, is disregarded throughout the subsequent analysis.

3.3 Probabilistic analysis procedure

Once the statistical parameters that model the random cohesion, friction angle and permeability fields are defined and the KL Expansion is discretized in the given slope domain, a typical Monte Carlo simulation cycle may be summarized into three steps:

1. Generate a single realization of the cohesion, friction angle and permeability random fields.
2. Evaluate the seepage forces from the solution to the hydraulic problem considering the permeability random input field generated in the previous step.
3. Compute the slope stability factor from deterministic effective stress analysis considering the soil properties (cohesion and friction angle) defined by the realization of the random fields.

Step 3 is illustrated in Figure 2 for a slope of height $H = 5$ m and inclination $\beta = 45°$, submitted to total water level drawdown $h_w = H$. The autocorrelation distances $L_x = 20$ m and $L_y = 2$ m were considered in the random fields generation. The excess pore pressure distribution was obtained by solving the hydraulic problem over a random permeability field with $\mu(k) = 10^{-6}\ \mathrm{m^2Pa^{-1}s^{-1}}$ and $CoV(k) = 100\%$.

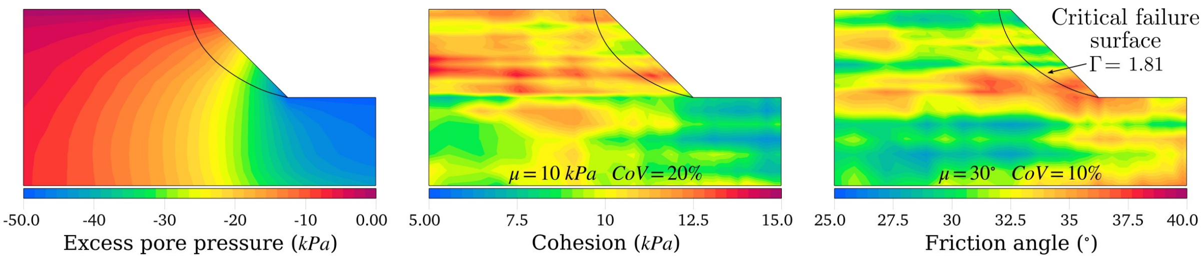

Figure 2. Critical failure surface associated with a typical Monte Carlo simulation cycle.

For the subsequent analysis, the procedure described in this section is repeated in blocks of 10000 simulation cycles until the convergence criterion $CoV(P_f) < 5\%$ is met.

4 Influence of the spatial variability of material parameters

The parametric study presented considers a fixed slope geometry, defined by height $H = 5$ m and inclination $\beta = 45°$ (Figure 3a). Based on typical soil properties and related statistical parameters (e.g., [5, 16, 17]), the statistical reference model data is summarized in Table 1. As regards the seepage forces, the slope is subjected in reference configuration to a total water level drawdown $h_w = H$.

Table 1. Statistical reference soil model data.

	Parameters	μ	CoV	L_x [m]	L_y [m]
c	Effective cohesion	10 kPa	30%	20	2
φ	Effective friction angle	30°	10%	20	2
k	permeability	10^{-6} m^{2}Pa^{-1}s^{-1}	60%	20	2
γ	Soil unit weight	20 kN/m^3	-	-	-

In the present case associated with the reference configuration, the analysis yields an overall failure probability of $P_f = 11.50\%$ (Figure 3b). The stability factor distribution shows a mean value of 1.353, a median value of 1.313, and a coefficient of variation of 23.54%. The deterministic analysis solution presented in Figure 3a was performed considering the mean value of the cohesion, friction angle and permeability fields.

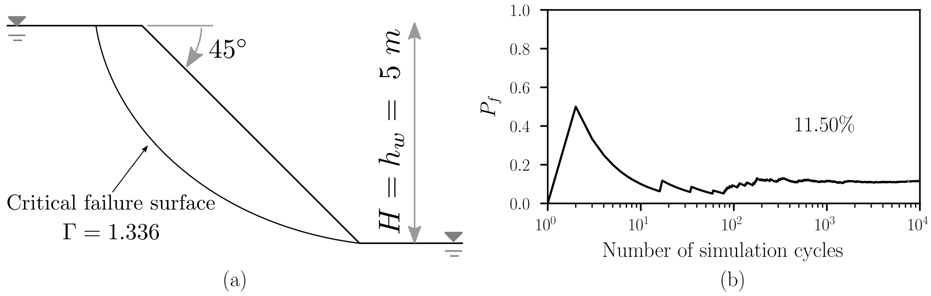

Figure 3. (a) Slope geometry and critical failure surface obtained from deterministic analysis; (b) Convergence of the failure probability.

4.1 Effects of the coefficient of variation of soil strength and permeability properties

The numerical simulations in this section investigate how the coefficient of variation of soil properties affects probabilistic slope stability assessment. Following Gu et al. [18], a range of typical values is individually considered for each soil property. The remaining parameters are kept fixed to the reference data. Figure 4 and 5 display the variations of the statistical parameters that characterize the slope stability conditions. As expected, higher values of the coefficient of variation result in flatter PDF distributions, thus reflecting higher uncertainty regarding the stability factor distribution. This feature is more pronounced in the particular case of cohesion property, being associated with a shift in peak value towards lower stability factors.

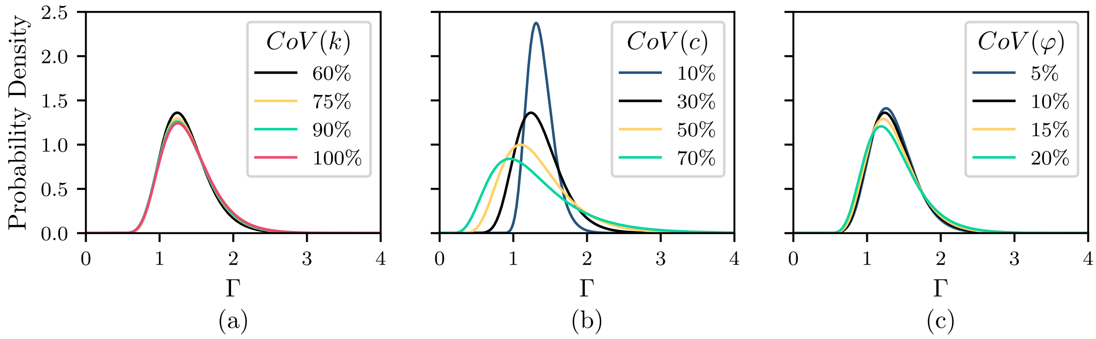

Figure 4. Effects of the coefficient of variation of soil properties on probability density function of the stability factor: (a) soil permeability; (b) Cohesion; (c) Friction angle.

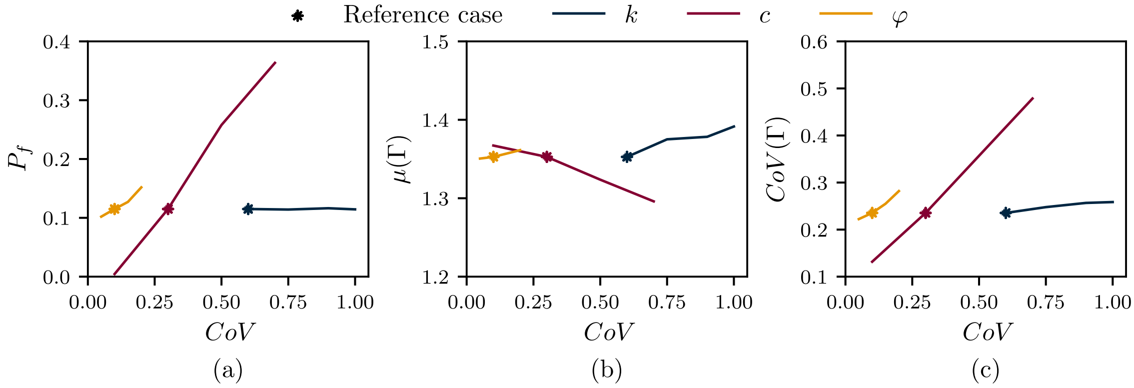

Figure 5. Failure probability (a), stability factor mean value (b), and stability factor coefficient of variation (c) as a function of the coefficient of variation of soil Permeability, soil cohesion and friction angle.

The simulations indicate that increasing the coefficient of variation of the soil strength properties leads to higher failure probabilities, particularly for the soil cohesion. In contrast, the results derived from the simulation suggest the coefficient of variation of the permeability very slightly affects the overall probability of failure. The significant impact of the cohesion variability on slope stability may be explained by observing that the maximum rate of resisting work P_{mr}, defined by Equation (2) in the case of Mohr-Coulomb strength criterion, is proportional to the local value of soil cohesion. Results of Figure 5b suggest that the coefficient of variation for the soil properties slightly affects the mean of stability factor distribution, at least within considered ranges. As regards the variations of $CoV(\Gamma)$, a quasi-linear dependence of this statistical parameter is observed in Figure 5c with respect to the coefficient of variation of soil strength properties.

4.2 Influence of the autocorrelation distances

This section investigates the impact of autocorrelation distances on the failure probability and statistical distribution of the slope stability factor. The study is restricted to couples $(L_x,\ L_y)$ that are proportional to their value in the reference configuration, that is $(L_x,\ L_y) = s \times (20\ \text{m},\ 2\ \text{m})$, where $s \geq 1$ can be viewed as a scale factor. The soil mass will exhibit lower spatial variability in strength and permeability properties for increasing scale factors. Figure 6 displays the variations of three statistical parameters that characterize the slope stability conditions.

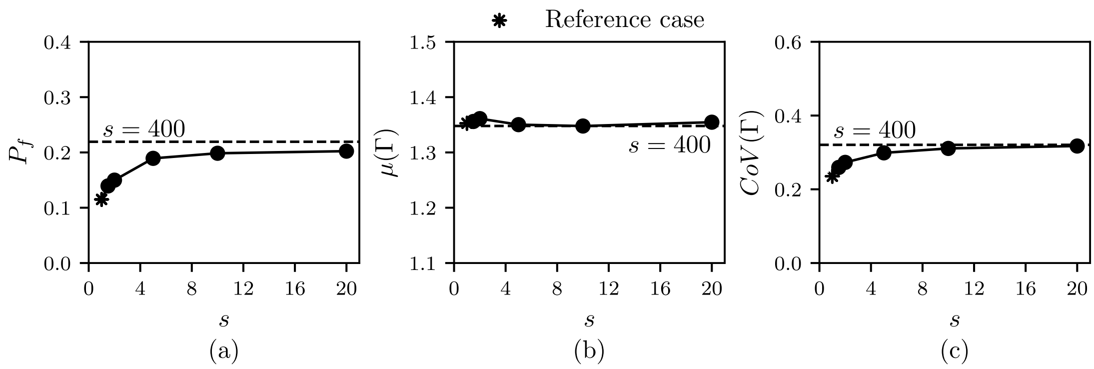

Figure 6. Effects of autocorrelation distance on the overall failure probability (a), coefficient of variation of stability factor (b) and mean value of stability factor (c).

Increasing the scale factor results in a monotonic increase in failure probability or, equivalently, to a decrease in the slope stability. The results also corroborate that higher values of the scale factor asymptotically characterize the configuration of a homogeneous soil mass.

5 Conclusions

In this research, the limit analysis kinematic approach formulated within the effective stress context has been applied to address the stability problem of slopes subjected to rapid water level drawdown. Taking into account the uncertainties and spatial variability of soil strength and permeability properties, a stochastic approach to the slope stability problem in the presence of seepage flow has been developed, the primary objective being to take into account the inherent uncertainties and spatial variability of soil strength and permeability parameters. In that respect, soil cohesion, friction angle, and permeability are modeled as random fields that are numerically generated. The Monte Carlo simulation method has been employed to evaluate the probability density function of the slope stability factor and the associated overall failure probability.

A parametric study relying upon a series of numerical simulations has been performed to investigate the impact of some statistical parameters defining the distributions of strength and permeability on the slope stability conditions. As could be expected from such a study, the results corroborated that the overall failure probability of the slope is mainly affected by the variability of the soil cohesion and, to a slightly lesser extent, by that of the friction angle, whereas the variability of permeability has little effect on this stability parameter.

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