Probabilistic Analysis of Slope Stability Considering Random Fields

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Abstract. The study of slope stability is of paramount importance for society, necessitating a comprehensive assessment not only of the slope's safety factor but also of its probability of failure. Therefore, the use of probabilistic techniques becomes imperative, with the Monte Carlo Method (MCM) emerging as a prevalent choice in geotechnical investigations. Employing MCM for slope stability involves modeling the target profile while varying crucial material parameters—such as friction angle, cohesion, and specific weight—in each iteration. This approach results in a distribution of safety factors, aiding in the assessment of both average safety levels and failure probabilities. This study introduces a case analysis where failure probability is quantified through MCM, integrating random fields in each iteration. For assessing variability, the PLAXIS software was utilized, employing classical limit equilibrium methods alongside the LAS technique (Local Average Subdivision). This facilitates a comparative analysis of probabilities derived from distinct equilibrium methodologies. The case study focuses on a river channel slope located in Itajaí/SC, Brazil, where a comprehensive geotechnical investigation—including field and laboratory tests—has been conducted. The results indicated a predominance of rupture surfaces in the first layer, with varying probability of failure values depending on whether a fixed or floating search mechanism was used.

Keywords: variability analysis, random fields, slope stability.

1 Introduction

The spatial variability of geotechnical properties, influenced by the mineralogical composition of soils, presents significant challenges for engineering practices. This variability complicates the process of obtaining accurate soil data and adds economic considerations to geotechnical projects. Traditionally, engineers address this uncertainty by using characteristic values of soil properties along with a conservative safety factor (Fenton and Griffiths [1]). However, advances in geotechnical analysis have led to the development of methods that better account for spatial variability and uncertainty by utilizing random fields.

Vanmarcke ([2]; [3]) introduced the Random Field Model (RFM) to describe the variability of geotechnical materials. This model rationally considers the correlation of soil properties at different locations, known as autocorrelation. This correlation varies based on a specific distance known as the fluctuation scale. The fluctuation scale indicates how far apart two locations can be while still having similar soil properties. A longer fluctuation scale suggests that soil characteristics remain consistent over greater distances, meaning that changes in those properties occur gradually. Conversely, a shorter fluctuation scale implies that soil properties change more rapidly with distance, leading to greater variability between nearby locations. The RFM framework allows for a more nuanced understanding of how soil properties change both vertically and horizontally within a mass.

To incorporate these variations into engineering designs, various approaches are employed, including the Local Average Subdivision (LAS) method, which maintains internal consistency by ensuring that local averages match the global average (Fenton and Vanmarcke [4]; Fenton and Griffiths [1]). This method facilitates the generation of random fields that reflect spatial variability more accurately. Using these fields, engineers can perform probabilistic analyses such as Monte Carlo simulations to estimate the probability of failure under different scenarios.

Additionally, the use of geotechnical investigation methods like the Piezocone Penetration Test (CPTu) is crucial for characterizing soil properties and assessing their variability (Schnaid and Odebrecht [5]). The CPTu provides detailed information on soil stratigraphy, material properties, and pore pressure, which are essential for evaluating stability and predicting foundation performance.

2 Probabilistic Approach

Probabilistic analysis in geotechnical engineering frequently aims to assess the probability of failure for specific sections by integrating the variability of soil parameters into numerical analyses. This approach is crucial for understanding the risks associated with soil behavior and helps in making more informed engineering decisions (Cao et al. [6]).

A key aspect of accurate probabilistic analysis is the proper characterization of soil spatial variability. Typically, this variability is characterized in the vertical direction, as vertical profiles provide valuable insights into subsurface conditions by revealing different soil layers at various depths. Horizontal variability, on the other hand, presents a greater challenge due to the need for extensive sampling across larger areas (Campello [7]).

To incorporate spatial variability into numerical models, a mathematical approach is required. One effective method is the decomposition technique, where the 'real' value of a geotechnical property, denoted as $\xi(z)$, is decomposed into a smoothly varying trend function $t(z)$ and a fluctuation component $w(z)$, which represents inherent soil variability (Phoon et al. [8], Uzielli et al. [9]). The fluctuation component is characterized by statistical properties such as mean μ, standard deviation σ, and correlation length θ.

For spatial autocorrelation measurements, statistical formulations like autocorrelation and covariance are utilized. These measures depend on the distance between points and are calculated to determine the fluctuation scale or correlation length (Phoon et al. [8]). The autocorrelation function, plotted against the separation distance, helps in determining the fluctuation scale, which indicates the distance over which soil properties show strong correlation. The calculation of this scale can be performed using theoretical autocorrelation models, such as the exponential, quadratic exponential, exponential cosine, and second-order Markov models (Uzielli et al. [9]).

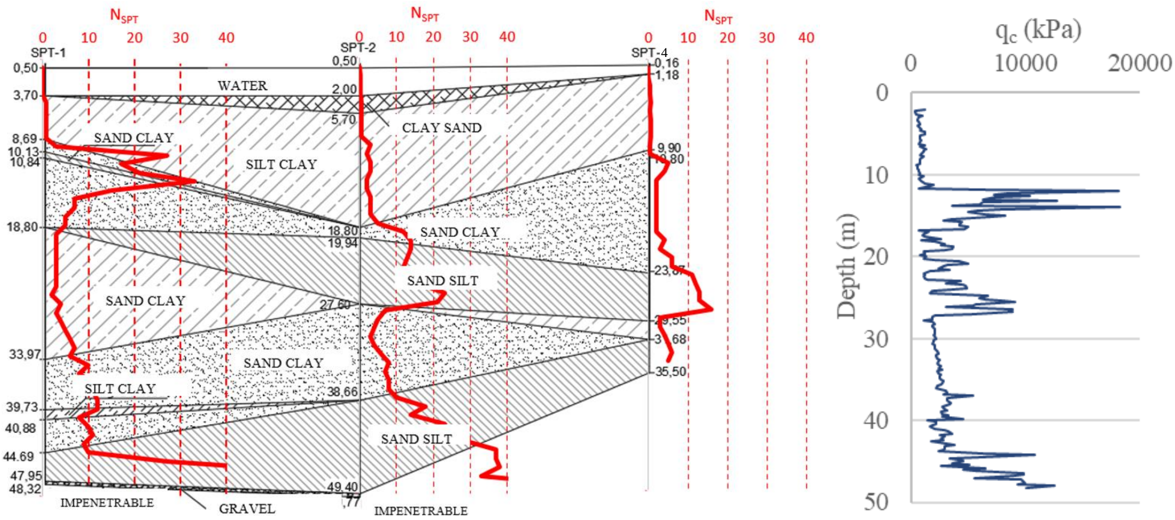
3 Materials and Methods

Focusing on a numerical application, a river channel slope located in Itajaí, SC, Brazil, was defined as a case study. The region has a consistent history of field and laboratory tests. In this work, emphasis is placed on field characterization through piezocone tests, which provide a substantial vertical dataset. This extensive dataset enables the establishment of reliable relationships and minimizes potential biases in statistical and probabilistic applications (Salgado et al., 2015).

The aim of this research in the Itajaí port area is to evaluate the dredging slopes that would be feasible given an increase in the depth of the river channel. Figure 1 shows the study region and the locations of the standard penetration tests (SPT) and cone penetration tests (CPT). In total, six CPT tests and three SPT boreholes were performed. Figure 2a presents a summary of the SPT results, providing an overview of the soil stratigraphy, while Figure 2b displays the peak resistance profile of CPT 03, located closer to the riverbank, for comparison.

For the stability analyses, however, the soil profile was segmented into four distinct layers to simplify data interpretation, due to the alternating sand and clay layers. Layer 1 extends to a depth of -12 meters, Layer 2 ranges from -12 to -17 meters, Layer 3 spans from -17 to -37 meters, and Layer 4 covers from -37 to -50 meters. The profile exhibits significant stratification, with layers of clayey soil interspersed with sandy layers of varying thickness. The upper section is dominated by a soft clay layer, followed by sandier intervals where penetration resistance increases (as indicated by CPT and SPT results), and then decreases upon reaching the clay substrate. Close to the impenetrable zone, the soil becomes sandy once more, culminating in an impenetrable layer at approximately 49 meters. The depth of the river channel was considered to be 13.50 m, to be obtained after dredging, and the slope (V:H) of the embankments was evaluated from 1:3 to 1:1.

Figure 1. Sounding locations



(a) (b)

Figure 2. Geotechnical profile (a) and CPT 03 profile (b)

For the variability analysis, only the cone resistance readings (q_c) from CPT 03 were considered, with the statistical parameters presented in Table 1 for each previously defined soil layer. The fluctuation scale was estimated using the AMA method, following the methodology described in Ziesmann [10]. The horizontal fluctuation scale used to generate the random field was 10.0 meters, a value commonly adopted in the literature.

Table 1. Statistical analysis for q_c from CPT 03

Depth (m)	Mean (kPa)	Standard deviation (kPa)	CV	Scale of fluctuation (m)
0 to -12	865.99	315.94	0.36	0.56
-12 to -17	7055.60	4171.71	0.59	0.50
-17 to -37	2898.16	1610.59	0.55	2.35
-37 to -50	7137.39	3001.91	0.42	7.89

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The strength parameters were obtained through empirical correlations with the cone penetration test (CPT) data. For the sandy layers, the proposal by Kulhawy and Mayne [11] was applied, as presented in Equation 1. Thus, Table 2 presents the average strength parameter along with their respective standard deviation. The specific weight of each layer was defined based on the classification derived from the surveys of each material, while the value of the cohesive intercept was set to 1 kPa for numerical stability. As a premise for the analyses, the coefficient of variation (CV) and fluctuation scale values were considered the same as those obtained for q_c, and the value of q_c equals that of q_t for sandy soils (Robertson and Cabal [12]).

$$\varphi' = \tan^{-1}\left[0.1 + 0.38\log\left(\frac{q_t}{\sigma'_{v0}}\right)\right] \tag{1}$$

where φ' is the friction angle, q_t is the total cone resistance, σ'_{v0} are the effective vertical stresses.

Table 2. Geotechnical parameters and φ' variation

Depth (m)	γ (kN/m³)	c' (kPa)	Mean (°)	Standard deviation (°)
0 to -12	14	1	34.67	12.48
-12 to -17	16	1	41.31	24.37
-17 to -37	16	1	31.07	17.09
-37 to -50	18	1	33.15	13.92

In the PLAXIS software, modeling was performed using a random field mesh for the first and second layers, which are considered more representative of the expected failure mode. The failure surface was obtained using the auto-refinement method and tested using both fixed and floating approaches. In the fixed approach, an initial stability evaluation was conducted using deterministic values, after which the defined failure surface remained constant for subsequent analyses that considered random fields. In contrast, the floating approach involved searching for a new failure surface in each random field. To calculate the limit equilibrium, simulations were conducted using the Morgenstern-Price (M-P) method. A total of 1,000 Monte Carlo simulations were performed for each slope inclination.

4 Results

After the simulations, the results presented in Table 3 were obtained, which include the values of the average safety factor (*FS*) and failure probability (P_f), both for the fixed and floating rupture surface, for each slope inclination. The only situation that presented a failure probability was the 1:1 slope, with 16.6% for the fixed case and 25.4% for the floating case, despite its average safety factor being above one.

Table 3. Values of safety factor and failure probability

Slope	Average FS	P_f (fixed)	P_f (floating)
1:3	2.566	0%	0%
1:2.5	2.182	0%	0%
1:2	1.835	0%	0%
1:1.5	1.453	0%	0%
1:1	1.076	16.6%	25.4%

Figure 3 presents the geometries of the rupture surfaces that generated the average safety factors for each of the slope inclinations. It is noted that for steeper inclinations, the ruptures are shallower, becoming more circular as the slope becomes more moderate. Additionally, it can be observed that the rupture is almost entirely confined to the first soil layer, due to the lower resistance parameters.

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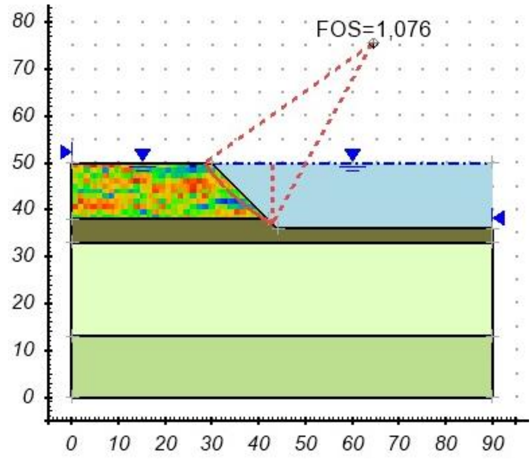
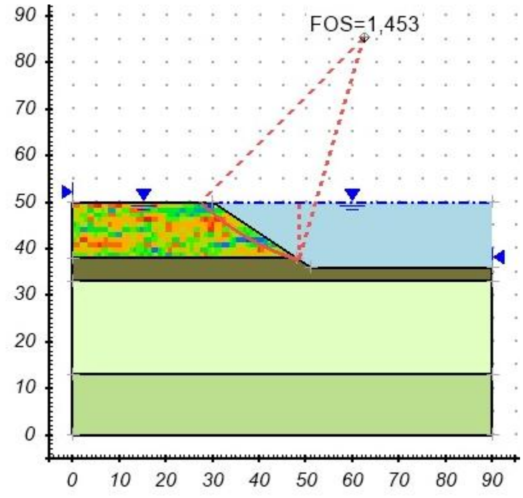


(a)



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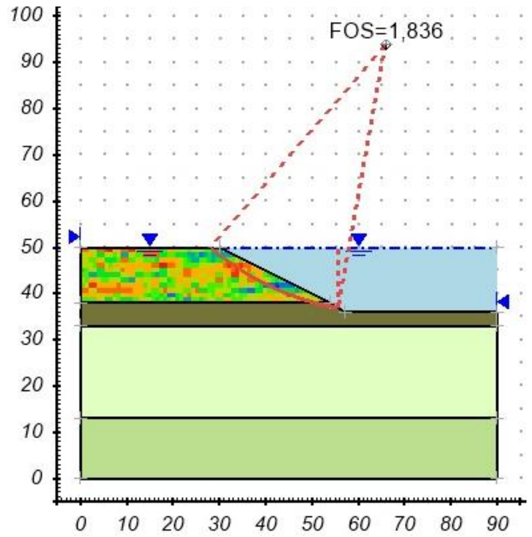
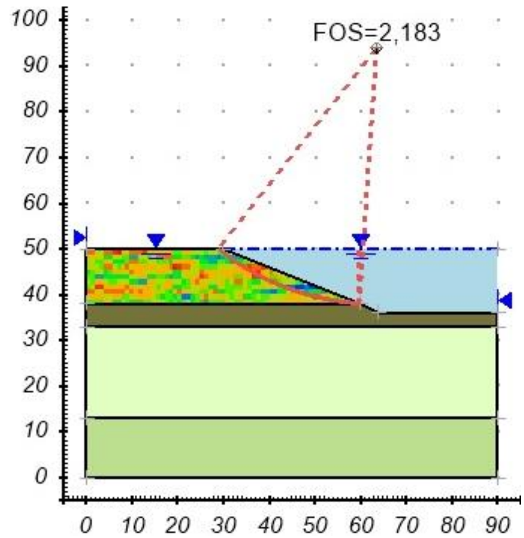
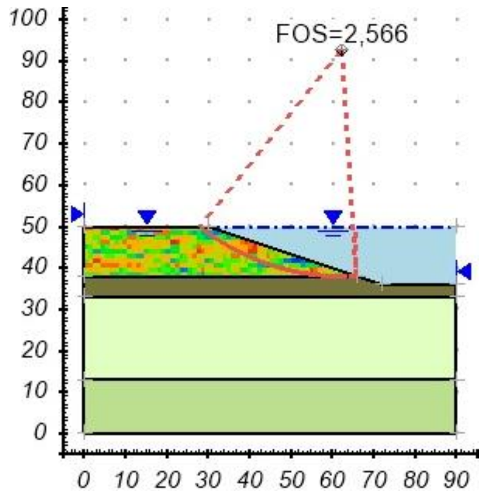


(c)



(d)



(e)

Materials

	Name	Spatial Variation	Strength Type	Unit Weight (kN/m^3)	Cohesion (kPa)	Phi (deg)
	Camada 1	Phi	Mohr Coulomb	14,000	1	34,67
	Camada 2	Phi	Mohr Coulomb	16,000	1	41,31
	Camada 3	-	Mohr Coulomb	16,000	1	31,07
	Camada 4	-	Mohr Coulomb	18,000	1	33,15

Figure 3. Failures surfaces: slope 1:1 (a), 1:1.5 (b), 1:2 (c), 1:2.5 (d) and 1:3 (e).

5 Conclusions

In the present research, a variability analysis was conducted for the dredging slope of a river canal in Itajaí/SC. The material studied was classified as sandy silt, and its geotechnical parameters were obtained through the analysis of SPT and CPT tests. The variability analysis was performed using PLAXIS software for five different slope inclinations, using the Monte Carlo method with 1000 simulations for each scenario, considering spatial variability through the LAS method. Among the obtained safety factor values, only the 1:1 inclination produced results with a probability of failure, being 16.6% for the fixed case and 25.4% for the floating rupture surface. In evaluating the geometry of the rupture surface, an increase in depth and greater circularity was observed for slopes with lower inclinations. Additionally, almost all of the ruptures were located within the first soil layer.

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