Comprehensive reliability-based well integrity analysis: application to worst case discharge (WCD) scenario

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Abstract. The global increase in energy demand, combined with the availability of hydrocarbons, positions oil and gas among the main non-renewable sources in the world's energy matrix. In the scope of well design, ensuring their structural integrity under increasingly severe conditions, especially in deep and ultra-deep water, has become a critical factor for the oil and gas industry. Consequently, there has been an investment in modeling extreme environmental scenarios found in challenging wells to ensure their integrity throughout their lifecycle. This work proposes a new methodology for probabilistic analysis of structural integrity during the construction and operation phases of wells, taking into account the uncertainties associated with variables related to the behavior of tubulars, cement sheath, formation, and applied loads. It aims to analyze well integrity through safety barrier failure events in a scenario of severe blowout known as Worst Case Discharge (WCD), using structural reliability theory. The calculation of failure probability will be carried out using the First Order Reliability Method (FORM). It is understood that the proposed methodology can be applied in the design and monitoring stages of wells, contributing to decision-making process.

Keywords: Cement sheath, Thermo-poroelastic model, Probabilistic model, Wellbore integrity

1 Introduction

Within the context of oil and gas industry it is paramount to ensure productivity without sacrificing operational safety across the different assets involved. Regarding production and injection Wells, their integrity refers to the capability of preventing undesirable fluid flow from and to the environment. It relies on the adequate performance of different physical barriers, such as casing and production tubulars, cement sheath, formation, safety valves, fluids, and other equipment.

Offshore oil and gas well structure is exposed to significant challenges due to the diverse environmental and operational conditions. According to Araújo et al. [1], this is primarily because offshore fields often contain reservoirs at considerable depths, which are subject to severe geological loads, typically found in high-pressure and high-temperature (HPHT) regions. The Worst Case Discharge Load (WCD) scenario is a major concern in the oil and gas industry and refers to an extreme uncontrolled flow of fluids – oil or natural gas – from the production zone into the wellbore, resulting in severe oil spill.

In addition to the severity of these load scenarios, the uncertainties inherent to the problem stand out. Variability associated to the element dimensions, material properties and intensity of loads significantly impact the well structure performance. These uncertainties can be accounted for through the statistical modeling of design variables within the framework of structural reliability theory. This approach enables the estimation of failure probability of the structure under specified limit states.

Since the 1990's, in works as Payne and Swanson [2] and Adams et al. [3], the limitations of deterministic well structure design have been highlighted. The use of Safety Factors (SF) as the sole safety measure can lead to either overly conservative or overly bold designs, compromising the risk-cost balance. The reliability-based assessment of well tubulars has been extensively studied, e.g. in Gouveia et al. [4], Yang et al. [5] and Várady Filho et al. [6]. Regarding the probabilistic response of the cement sheath in offshore wells, Silva et al. [7], Estrela et al. [8] and Zhang et al. [9] can be cited.

This paper addresses the probabilistic assessment of well integrity in terms of two well barriers, tubulars and cement sheath, by applying the First-Order Reliability Method (FORM). The methodology is applied to the analysis of oil production well subject to a WCD scenario.

2 Well integrity and Worst Case Discharge

Well integrity refers to the capability of a well to consistently contain and control fluids throughout its entire lifecycle, spanning from drilling and production to abandonment. This encompasses the preservation of the structural integrity of key elements such as the well casing, cement, and other components to prevent any potential leakage of fluids into the surrounding formations or environment. Adhering to principles of well integrity significantly diminishes the risk of accidents that may result in environmental, economic, and human life damages. While significant strides have been made in safety and risk assessment in offshore wells over recent decades, serious accidents still occur and are likely to persist, albeit with decreasing frequency (Corneliussen et al. [10]). The Deepwater Horizon accident in the Macondo prospect, Gulf of Mexico, in April 2010, stands as a landmark failure event in the oil and gas industry.

In this context, guidelines for ensuring the integrity of wells are outlined in regulatory standards and internal documents defined within the well operators. These documents establish the minimum number of well barrier elements required for each operation.

In the aftermath of the Macondo incident, Bureau of Ocean Energy Management (BOEM) defined guidelines for calculating Worst Case Discharge (WCD) to enhance wellbore safety. According to BOEM, the worst case discharge is described as the daily rate of an uncontrolled flow from all producing reservoirs into the open wellbore (Bowman [11], Moyer et al. [12]). According to well drilling planning, the WCD encompasses all hydrocarbon-bearing zones within each open-hole well section, accounting for potential uncontrolled flow scenarios.

This scenario assesses the collapse risk during uncontrolled flow to the seafloor. The temperature and pressure assumptions for the calculation of this load are:

- Internal pressure – seawater hydrostatic at the mudline casing hanger, hydrocarbon gradient below to the depth of interest;
- External pressure – fracture gradient at the previous casing shoe depth (1) plus hydrostatic of the heavier mud to below previous shoe and (2) minus lighter mud hydrostatic above previous casing shoe depth;
- Temperature: influx temperature and depth.

3 Mechanical Modeling

After the cement hardening process, the casing-cement-formation system form a thick hollow cylinder subject to the internal pressure of the fluid inside the casing and the external pressure applied by the rock formation . As a result of the interaction between the materials, contact pressures arise at the casing-cement and cement-formation interfaces (Figure 1).

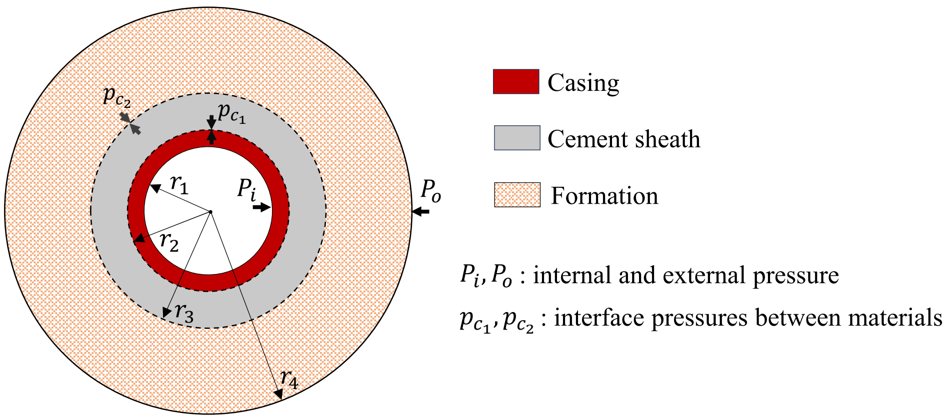

Figure 1. Conceptual model for interaction between casing-cement-formation system

The proposed thermo-poroelastic model is built on the following assumptions:

- Casing, cement sheath and formation are all linearly described in plane strain state;
- Interfaces between the casing, the cement and the formation present perfect adherence;

- Temperature variation in the radial direction is constant;
- Casing is a thermo-elastic material while the cement and formation are thermo-poroelastic materials.

Detournay and Cheng [13] present Hooke's law for a thermo-poroelastic solid in cylindrical coordinates with the following expressions:

$$\begin{cases} \varepsilon_r = \dfrac{1}{E}\left[\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right) + \alpha\left(1-2\nu\right)p\right] + \beta\Delta T \\ \varepsilon_\theta = \dfrac{1}{E}\left[\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right) + \alpha\left(1-2\nu\right)p\right] + \beta\Delta T \\ \varepsilon_z = \dfrac{1}{E}\left[\sigma_z - \nu\left(\sigma_r + \sigma_\theta\right) + \alpha\left(1-2\nu\right)p\right] + \beta\Delta T, \end{cases} \tag{1}$$

where E defines the Young's modulus, ν is the Poisson's ratio, p is the pore pressure, β refers to thermal expansion coefficient, ΔT stands for temperature variation and α is the Biot coefficient. Applying the plane-strain conditions (i.e. $\varepsilon_z \approx 0$), the radial and tangential strains in a thermo-poroelastic material can be expressed as:

$$\begin{cases} \varepsilon_r = \dfrac{1}{E^*}\left(\sigma_r + \nu^*\sigma_\theta - \alpha^* p\right) + \beta\Delta T \\ \varepsilon_\theta = \dfrac{1}{E^*}\left(\sigma_\theta - \nu^*\sigma_r + \alpha^* p\right) + \beta\Delta T, \end{cases} \tag{2}$$

using the following plane-strain coefficients:

$$E^* = \frac{E}{1-\nu^2},\ \nu^* = \frac{\nu}{1-\nu},\ \alpha^* = \frac{1-2\nu}{1-\nu}\alpha,\ \beta^* = \frac{(1+\nu)}{\beta}. \tag{3}$$

Once these strain fields are defined, Lamé equations for thick-walled cylinders are applied, allowing the definition of radial (σ_r), tangential (σ_θ) and vertical (σ_z) stresses in casing, cement sheath and formation:

$$\begin{cases} \sigma_r = \dfrac{r_i^2}{r_o^2 - r_i^2}\left(1 - \dfrac{r_o^2}{r^2}\right)P_i - \dfrac{r_o^2}{r_o^2 - r_i^2}\left(1 - \dfrac{r_i^2}{r^2}\right)P_o \\ \sigma_\theta = \dfrac{r_i^2}{r_o^2 - r_i^2}\left(1 + \dfrac{r_o^2}{r^2}\right)P_i - \dfrac{r_o^2}{r_o^2 - r_i^2}\left(1 + \dfrac{r_i^2}{r^2}\right)P_o \\ \sigma_z = \dfrac{2\nu}{r_o^2 - r_i^2}\left(r_i^2 P_i - r_o^2 P_o\right) - \alpha\left(1-2\nu\right)p - \beta E\Delta T \end{cases} \tag{4}$$

where P_i, P_o are internal pressure (pressure at the inner wall of a hollow cylinder) and external pressure (pressure at the outer wall of a hollow cylinder), respectively, and r_i, r_o are inner and outer radius, respectively. The effective radial $\sigma_r^{'}$, tangential ($\sigma_\theta^{'}$) and vertical stresses ($\sigma_z^{'}$) are calculated by subtracting the portion corresponding to pore pressure, which corresponds to αp. Radial displacements are calculated as follows:

$$\begin{aligned} u(r) = \frac{r}{E^*\Big(r_o^2 - r_i^2\Big)}\Big[\Big(\Big(1-\nu^*\Big)r_i^2 + \Big(1+\nu^*\Big)\frac{r_i^2 r_o^2}{r^2}\Big)P_i - \\ \Big(\Big(1-\nu^*\Big)r_o^2 + \Big(1+\nu^*\Big)\frac{r_i^2 r_o^2}{r^2}P_o\Big)\Big] + \beta^*\Delta T r + \frac{1}{E^*}\alpha^* p r \end{aligned} \tag{5}$$

Imposing displacement compatibility conditions at both interfaces (casing-cement and cement-formation), the contact pressures (Figure 1) pc_1 and pc_2 can be obtained. The tubular collapse resistance was estimated using the Klever and Tamano [14] equation. Further details on the formulation are provided in their work.

4 Probabilistic approach proposed

The probabilistic model for cement sheath failure is developed based on the Mohr–Coulomb criterion, widely adopted for brittle materials: $\tau = c + \sigma_n \tan\phi$, where τ is the shear stress, σ_n the normal stress, c is the cohesion of material and ϕ refers to its internal friction angle. Al-Ajmi and Zimmerman [15] present the Mohr–Coulomb criterion in the form $\sigma_1 = \sigma_c + q\sigma_3$, in which σ_1 and σ_3 are the maximum and minimum principal stresses respectively, σ_c is the compressive strength of the material and q is a parameter related to internal friction angle. These parameters can be calculated by the following equations:

$$\begin{cases} q = \tan^2\left(45 + \dfrac{\phi}{2}\right) = \dfrac{1+\sin\phi}{1-\sin\phi} \\ \sigma_1 = \max\left(\sigma_r^{'}, \sigma_\theta^{'}, \sigma_z^{'}\right) \\ \sigma_3 = \min\left(\sigma_r^{'}, \sigma_\theta^{'}, \sigma_z^{'}\right). \end{cases} \tag{6}$$

De Andrade and Sangesland [16] conducted a series of experiments to estimate the correlation between compressive strength (MPa) and Young's modulus (GPa). The authors then developed a fitted model to describe the relationship between these two parameters (eq. (7)).

$$\sigma_c = 0.0354E_c^2 + 3.1509E_c + 4.0642 \tag{7}$$

The limit state equations (tubular and cement sheath) are defined in the case study presented in the next section. The random variables adopted refers to geometric and mechanical properties of the casing and cement sheath. The First Order Reliability Method (FORM) is employed to the estimation of reliability index and, consequently, the probability of failure. According to Silva et al. [17], the method is based on the transformation of the original random variables (r.v.) into equivalent normal ones. It also involves the linearization of the limit state equation. The main advantage of the method lies in its ability to use all the statistical information of the r.v., dealing with any statistical distributions, including correlation between pairs of variables. The reliability problem is formulated as a constrained nonlinear optimization problem, which is iteratively solved by the HLRF algorithm. Further details about the method are presented in Melchers and Beck [18].

5 Case study

An oil production well under a water depth of 2156 meters is adopted (Figure 2). The casing program is presented in Table 1, regarding borehole and tubular parameters, as well as the weight of the drilling fluid used in each drilling phase. It is important to emphasize that, although the described well is hypothetical, its parameters have been selected to resemble those of real oil wells. Figure 3 shows the pressure profiles (internal and external) considered in the WCD analysis. The 10.75-inch production casing exposed to WCD loading is analyzed, with the loading calculation parameters provided in Table 2. Tables 3 and 4 detail the geometric and mechanical properties of the cement sheath and casing, with Table 3 treating these properties as deterministic and Table 4 treating them as random variables (r.v.).

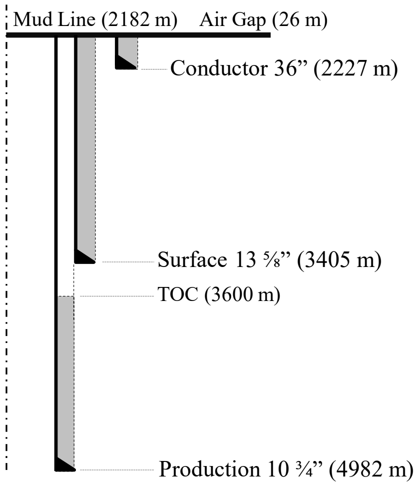

Figure 2. Wellbore schematic diagram.

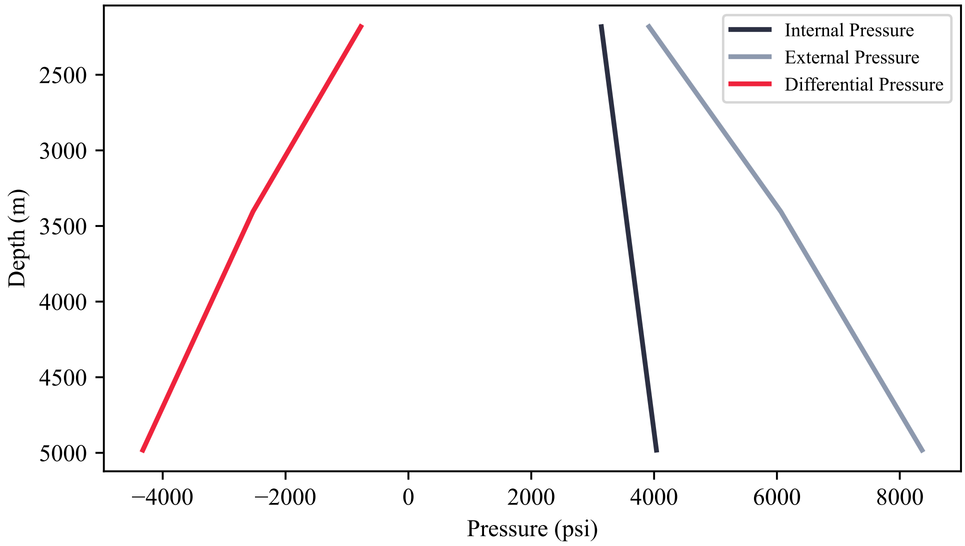

Figure 3. Pressure profile considered for the WCD scenario.

Table 1. Casing program.

Phase	Name	Type	OD (in.)	Weight (ppf)	Grade	Hanger (m)	Base (m)	TOC (m)	Hole size (in.)	Fluid density (ppg)
1	Conductor	Casing	36	554	X60	2182	2227	2182	42	8.55
2	Surface	Casing	13.625	88.2	L80	2182	3405	2182	16	11.5
3	Production	Casing	10.75	65.7	L80	2182	4982	3600	12.25	8.55
4	-	Open hole	-	-	-	-	5139	-	8.5	9.1

Table 2. Parameters used to calculate pressure profiles in the WCD scenario.

Internal pressure attributes		External pressure attributes	
fluid gradient (psi/m)	0.3202	mud weight above previous shoe (m)	10.3
influx depth (m)	5139	mud weight below previous shoe (m)	8.55
influx temperature (°C)	69.3	previous shoe depth (m)	3405
seawater density (ppg)	8.55	previous shoe gradient (m)	10.45

Table 3. Deterministic parameters used in the case study.

Parameter	Casing	Cement	Formation
Inner Radius (mm)	121.41	136.53.90	155.58
Outer Radius (mm)	136.53	155.58	1555.75
Young's Modulus, E (GPa)	200	r.v	8.19
Poisson Ratio, ν (-)	0.27	r.v	0.2
Biot Coefficient, α (-)	-	0.75	0.8
Thermal Expansion Coefficient, β (°C^{-1})	1.30E-05	9.00E-06	1.00E-05
Friction Angle, ϕ (°)	-	35	30

Table 4. Statistical parameters adopted for casing and cement random variables.

Random Variable	Symbol	Distribution	Mean coeff.	COV
Outside diameter	D	Gaussian	1.0059	0.00181
Wall thickness	t	Gaussian	1.0069	0.0259
Yield strength	f_y (L80)	Gaussian	1.10	0.036
Ovality	ov^*	Weibull	0.217	0.541
Eccentricity	ec^*	Weibull	3.924	0.661
Residual stress	rs^*	Gaussian	-0.138	0.507
Model uncertainty (KT)	mu^*_{KT}	Gaussian	0.9991	0.067
Cement Young's modulus	E_c	Lognormal	1.00	0.01
Cement Poisson's ratio	ν_c	Lognormal	1.00	0.25

The limit state equation presented in Eq. 8, based on the Mohr–Coulomb failure criterion, assesses two r.v. and is used to calculate the failure probability of the cement sheath. Null or negative values of G indicate failure.

$$G\left(E_c, \nu_c\right) = \sigma_c - \left(\sigma_1 - q\sigma_3\right) \tag{8}$$

Regarding the tubulars, the Klever and Tamano [14] equation is used, being the failure function for the external pressure resistance model defined in terms of seven r.v., as follows:

$$G(D, t, f_y, ov^*, ec^*, rs^*, mu^*_{KT}) = R_{prob} - \Delta P \tag{9}$$

where ΔP represents the pressure differential in the scenario and R_{prob} is the distribution of the tubular resistant pressure, calculated by accounting for the uncertainties in the variables.

The reliability levels of the 10.75-inch production casing and the cement sheath are evaluated at a depth of 4038 m, where the highest applied pressure value is observed, as shown in Figure 3. The results for the safety factor, probability of failure, and reliability index are presented in Table 5. It is noted that the casing has a very low probability of collapse failure, with a corresponding safety factor of 1.55. However, in this section, the cement sheath presents a safety factor close to 1.0, which suggests it may be near to failure. The associated probability of failure (P_f) is around 0.0026%, which indicates that, according to the probabilistic approach, the cement sheath seems to perform in an acceptable regime.

Table 5. Results of safety factor, probability of failure and reliability index.

Element	Safety Factor	Failure Probability	Reliability Index
Casing	1.55	$10^{-10.8223}$	6.6460
Cement	1.01	$10^{-4.5788}$	4.0431

6 Conclusions

In this study, the authors presented an thermo-poroelastic analytical approach to the casing-cement-formation interaction in a probabilistic framework. The results highlight the importance of evaluating the casing-cement-formation system as an integrated whole, assessing the safety levels for each component of the system. Works in this direction will contribute to disseminating the philosophy of probabilistic design applied to the design practice of oil wells.

Acknowledgements. The authors acknowledge the financial and technical support provided by Petróleo Brasileiro S.A. - PETROBRAS.

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