On the accuracy of prediction models for the collapse strength of worn casing

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Abstract. Oil and gas wells are typically constructed in highly complex and harsh environments. Casing is a crucial component of the well structure, enduring a wide range of loads throughout the well life cycle, including internal and external pressures, and axial forces. After installation, these tubulars may develop wear grooves on their inner walls due to contact with the tool joints of the drill string. This reduction in wall thickness, combined with initial geometric imperfections such as ovality and eccentricity, as well as residual stresses, can significantly reduce tubular resistance, especially under external pressure (collapse). Thus, models capable of accurately estimating collapse pressure, validated with realistic data, are extremely relevant. In this context, some collapse prediction models of worn casing have been proposed in the literature, based on experimental, analytical, or numerical approaches. However, in several cases, the data used to derive these models are limited in quantity and variety. The present study aims to investigate and propose improvements in the equations for the collapse pressure of worn casing by utilizing a large database of numerical simulations. Several Finite Element (FE) simulations are performed to generate a substantial database of collapsed pipes. The accuracy of collapse prediction models from the literature is then evaluated, and new model parameters are calibrated to enhance precision. The study adopts material and geometric configurations commonly observed in worn casing tubulars of oil and gas wells. Two subsets of the large database are generated: one for fitting the models and the other for testing them. The exploratory analysis of the FE database provides insights into the collapse strength deration concerning relevant parameters such as damage depth, tool joint radius, and tube slenderness. The results compare the accuracy of the models, and a parallel discussion about the influence of various features is conducted.

Keywords: Well integrity; collapse pressure; worn casing; casing tubulars

1 Introduction

Casing tubulars are a crucial part of wells, which are built in highly complex and harsh environments and are subjected to a large variation of loading conditions during their lifecycle. During this period, the tubulars experience a reduction in wall thickness, especially during the construction phase, due to mechanical contact with the tool joints. This reduction in wall thickness significantly decreases the collapse pressure, which, when combined with specific loading conditions, may result in well failure.

The analysis of casing tubulars has been conducted through experimental campaigns [1-3] or by Finite Element Analysis (FEA), using bidimensional models [4-6] or three-dimensional models [3, 7]. In the first case, specific laboratory setup is required, and high costs are always involved, leading to a lack of experimental data in the literature [5]. In the second case, although there are no high monetary costs, modeling must be performed by specialists, and high computational costs can be associated. Therefore, the use of methods that combine efficiency and accuracy is crucial for worn casing analysis.

Analytical models to predict the collapse pressure have also been proposed in recent years. The main advantage of these models is their efficiency in analysis, due to fast calculations. Each model admits different considerations, such as modeling the wear groove as an uneven eccentricity [1], being applicable for a wide range of outer diameter to thickness (*OD/wt*) ratios [5], or presenting better results for thin tubulars [3].

Therefore, the aim of this paper is to investigate and propose improvements to the prediction models for collapse pressure of casing tubulars proposed by Moreira Junior et al. [3] and Teigland et al. [5]. The models were recalibrated using a wider dataset obtained by FEA, with a larger variation of parameters and consideration of nonlinear effects. The updated models were compared to the original ones using a second FEA database, which allowed for the definition of each model's accuracy by the mean and standard deviation of the normalized collapse pressures.

2 Worn casing

Determining the residual collapse pressure of casing tubulars is a challenging issue that has been addressed over the past decades. The primary reason is the number of variables involved in the process, such as geometry, initial imperfections (ovality, eccentricity), and wear variables (number of grooves, wear depth, wear position). Moreover, the use of finite elements is often required, which can entail high computational costs. Therefore, it is useful to define analytical models able to yield accurate results. The models proposed by Moreira Junior et al. [3] and Teigland et al. [5] are briefly presented in this section.

2.1 Moreira Junior et al. (2015)

The authors [3] proposed an analytical model to describe the collapse pressure of casing tubulars (P_c), based solely on the intact collapse pressure (P), which may be determined by the model proposed by Klever and Tamano [8] (Equation (7)), and in the wear to thickness percentage (ω), as presented in Equation (1):

$$P_c = P\kappa e^{-a\omega} \tag{1}$$

where κ and a are parameters of the proposed model.

According to Moreira Junior et al. [3], the model was calibrated using a dataset with the following variations: 5 *OD/wt* ratios, in the range of 11.19 to 20.39, 3 different steel grades, and 5 wear to thickness percentages, equally spaced from 0% to 40%. Note that there was not a direct combination of these parameters, totaling 45 numerical results. The simulations were carried out considering a three-dimensional tubular model in Abaqus [9] and the collapse pressure was determined using the Riks arc-length method [10], to solve the nonlinear problem. The authors achieved parameters κ and a equal to 0.9782 and 1.58, respectively.

2.2 Teigland et al. (2022)

Teigland et al. [5] proposed an analytical model based on the Buckingham Pi theorem [11] and the Stress Concentration Factor. According to these authors, the residual strength of a casing tubular with one crescent wear groove may be defined by: the outer and inner (*ID*) diameters of the tubular, the wear to thickness percentage, and the outer diameter of the tool joint (D_{tj}). Applying the Buckingham Pi theorem, there are four variables and only one fundamental dimension, so only three dimensionless parameters are needed. The reduction parameters proposed by the authors are presented in Equation (2), with A_0, A_1, a, b and c being the parameters of the model,

$$K_t = A_0 + A_1 \left(\frac{wt}{OD}\right)^a \left(\frac{D_{tj}}{ID}\right)^b \omega^c. \tag{2}$$

Teigland et al. [5] proposed that different parameters should be calibrated to reduce the yield and elastic collapse pressures in the model proposed by Klever and Tamano [8], defined by Equations (3) and (5), respectively. These calibrated values are then applied to the combined collapse pressure, as given in Equation (7):

$$\Delta p^{yc} = min\left[\frac{1}{2}\left(\Delta p^{yM} + 2\zeta\bar{\sigma}_y\right), \Delta p^{yM}\right] \tag{3}$$

where Δp^{yM}, ζ and $\bar{\sigma}_y$ are the von Mises through-wall yield, characteristic tubular geometry parameter, and the factored yield strength, respectively, as defined in Equation (4):

$$\begin{gathered}\Delta p^{yM} = \zeta\bar{\sigma}_y \frac{4(1+2\zeta)}{3+(1+2\zeta)^2}\left[S_i \pm \sqrt{1 + 3\frac{1-S_i^2}{(1+2\zeta)^2}}\right] \\ S_i = \frac{\sigma_a + p_i}{\bar{\sigma}_y} \\ \zeta = \frac{1}{\frac{OD}{wt} - 1} \\ \bar{\sigma}_y = k_y\left(1 - H_y\right)\sigma_y.\end{gathered} \tag{4}$$

σ_a is the axial stress and p_i is the internal pressure in the tubular, σ_y is the yield strength and k_y and H_y are the bias factor and decrement function for yield collapse, respectively. The elastic collapse pressure is given by:

$$\Delta p^{ec} = \frac{2\bar{E}}{1-\nu^2}\zeta^3(1 + tc\,\zeta) \tag{5}$$

$$\bar{E} = k_e(1 - H_e)E \tag{6}$$

where E is the Young's modulus, $\bar{E}$ the factored Young's modulus, ν the Poisson's ratio, tc is a thickness parameter, k_e and H_e are the bias factor and decrement function for elastic collapse, respectively. Klever and Tamano [8] proposed that the collapse pressure for combined loads may be defined by Equation (7), with H_t the decrement function, that considers initial manufacture imperfections, given by Equation (8),

$$\Delta p^c = \frac{2\Delta p^{y^c}\Delta p^{e^c}}{\Delta p^{y^c} + \Delta p^{e^c} + \sqrt{(\Delta p^{y^c} + \Delta p^{e^c})^2 + 4H_t\Delta p^{y}c\Delta p^{e^c}}} \tag{7}$$

$$H_t = 0.127ov + 0.0039ec - 0.44\frac{rs}{\sigma_y} + h_n \tag{8}$$

in which ov, ec and rs stand for ovality and eccentricity, respectively, besides h_n that refers to a constitutive curve shape factor. Teigland et al. [5] calibrated the model using datasets created by FEA, considering a bidimensional model. Yield and elastic collapse parameters were modeled separately, considering different failure criteria. For yield collapse, failure criterion of onset through-wall yield was used, whereas the elastic collapse was determined by the critical buckling pressure, obtained through an eigenvalue buckling analysis. The reduction parameters for yield and elastic pressures proposed by Teigland are given by Equations (9) and (10), respectively.

$$K_t^{yc} = 1.0598 + 2.7902\left(\frac{wt}{OD}\right)^{-0.0748}\left(\frac{D_{tj}}{ID}\right)^{0.0120}\omega^{1.0598} \tag{9}$$

$$K_t^{ec} = 0.9907 + 4.4198\left(\frac{wt}{OD}\right)^{0.2952}\left(\frac{D_{tj}}{ID}\right)^{0.4178}\omega^{1.6598} \tag{10}$$

3 Methodology

3.1 Finite Element Analysis

The analysis presented in this paper was conducted using FEA software Abaqus [9]. The entire process was automated by a Python script, enabling batch simulation with variations in geometry, physical parameters, and worn conditions, from modeling through to post-processing. The methodology is adapted from Silva [4], employing a bidimensional approach that can yield consistent results, especially given that the wear groove length exceeds ten times the outer diameter (OD), as proposed by Sakakibara et al. [2].

Regarding Abaqus meshing, the 8-node hybrid biquadratic finite element with reduced integration (CPE8R) was adopted. A mesh convergence analysis was carried out, determining that 12 radial elements provided more accurate estimations than 20 radial elements. Additionally, the casing tubulars were modeled with a ring-shaped geometry, adopting the plane strain hypothesis and using the Riks method [10]. The steel's nonlinear physical behavior was defined based on the elastoplastic model with nonlinear hardening proposed by ASME BPVC [12].

The boundary conditions were defined using a coupling mechanism linked to a reference point located at the geometric center of the cross-section. This condition was applied to restrict the rigid body movement of the tubular. The load was applied as a hydrostatic pressure over the external surface. Given the solver method (arc-length) and the unitary load, the maximum value of the Load Proportionality Factor (LPF) corresponds to the collapse load of the tubular.

The failure criterion was defined as the maximum capacity of the casing tubular, irrespective of the failure mechanism (yield or elastic). It is important to note that this consideration, along with the inclusion of nonlinear effects (both physical and geometrical), leads to accurate collapse pressure results.

3.2 Datasets

The quality of the prediction model depends not only on the model itself but also on the dataset used for calibration. In this paper, the datasets were defined based on typical values for the oil and gas industry, as obtained from the DEA-130 technical report [13].

Two different datasets were created: one for recalibrating the prediction models presented in sections 2.1 and 2.2, and the other for testing the models. Each dataset considered different parameters, as detailed in Table 1. The datasets were created with a direct combination of the selected parameters. Consequently, the calibration dataset contains a total of 540 numerical results, while the testing dataset contains 120 numerical results.

Table 1. Parameters of the datasets.

Parameter	Value	
	Calibration	Testing
OD (inches)	11.75	9.0, 15.0
OD/wt	12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34	12, 16, 20, 24, 28, 32
ω (%)	0, 10, 20, 30, 40	0, 10, 20, 30, 40
σ_y	N-80, P-110, Q-125	M-65, T-95
D_{tj} (inches)	3.0, 4.0, 5.0	6.0

4 Results

4.1 Updated regression Models

The procedure proposed in this paper aims to recalibrate the models proposed by Moreira Junior et al. [3] and Teigland et al. [5], using a larger dataset with nonlinear FEA results and a broad range of *OD*/*wt* ratios, addressing both yielding and elastic failure mechanisms.

The minimization of the usual relative error function may not be appropriate for this application due to the large variation in collapse pressure results. For instance, the absolute error for a tubular that fails by yielding results in a relative error much lower than the same absolute error for a tubular that fails due to elastic collapse. Therefore, the objective function in the minimization problem was set as Equation (11), based on normalizing the collapse pressure of the reference value (P_{FEA}) to that obtained by the prediction models ($P_{proposed}$).

$$f_{objective} = \sum \left(\frac{P_{FEA}}{P_{proposed}} - 1 \right)^2 \tag{11}$$

After calibrating the model with the new dataset and objective function, the parameters κ and a of the model proposed by Moreira Junior et al. [3], referred to here as "*Moreira Junior et al. Updated*" were equal to 1.0263 and 1.2964, respectively. For the prediction model proposed by Teigland et al. [5], referred to as "*Teigland et al. Updated*", the yield and elastic reduction parameters are presented in Equations (12) and (13), respectively.

$$K_t^{yc} = 1.1223 + 1411.5900 \left(\frac{wt}{OD} \right)^{1.8570} \left(\frac{D_{tj}}{ID} \right)^{0.1251} \omega^{1.8359} \tag{12}$$

$$K_t^{ec} = 0.9800 + 0.0132 \left(\frac{wt}{OD}\right)^{-1.9491} \left(\frac{D_{tj}}{ID}\right)^{0.1444} \omega^{1.1763} \tag{13}$$

4.2 FEA validation

The prediction models must be validated using a different dataset from the one used for calibration, as presented in Table 1. Figure 1 shows the results normalized to the FEA values for different OD/wt ratios, for the steel grade T-95 and OD equal to 9.0 inches. It can be noticed that the updated prediction models produce results close to the reference values, with the normalized results approaching 1.0 across the entire range of OD/wt ratios.

The collapse pressures predicted by the models are compared directly with the FEA results, as shown in Figure 2(a). It is evident that all models exhibit high coefficients of determination R^2, with a minimum value of 94.13% for the model by Moreira Junior et al. [3]. However, it is important to examine the dispersion of the results, which can be observed in the histogram of normalized collapse pressures (Figure 2(b)). While the original models show high R^2 values, they have mean values that are 6.3% lower than the reference for the models (Moreira Junior et al. [3]) or with the highest dispersion of 13.30% (Teigland et al. [5]). In contrast, the updated models present mean values closer to 1.0 and with low dispersion results, equal to 8.49% and 8.03%, for the updated models of Moreira Junior et al. [3] and Teigland et al. [5], respectively, as shown in Table 2. Note that the "Teigland et al. Updated" model presented a mean value lower than the one obtained by the "Moreira Junior et al. Updated" model. This may be explained due to the steel grade values, which M-65 is out of the range used for calibration.

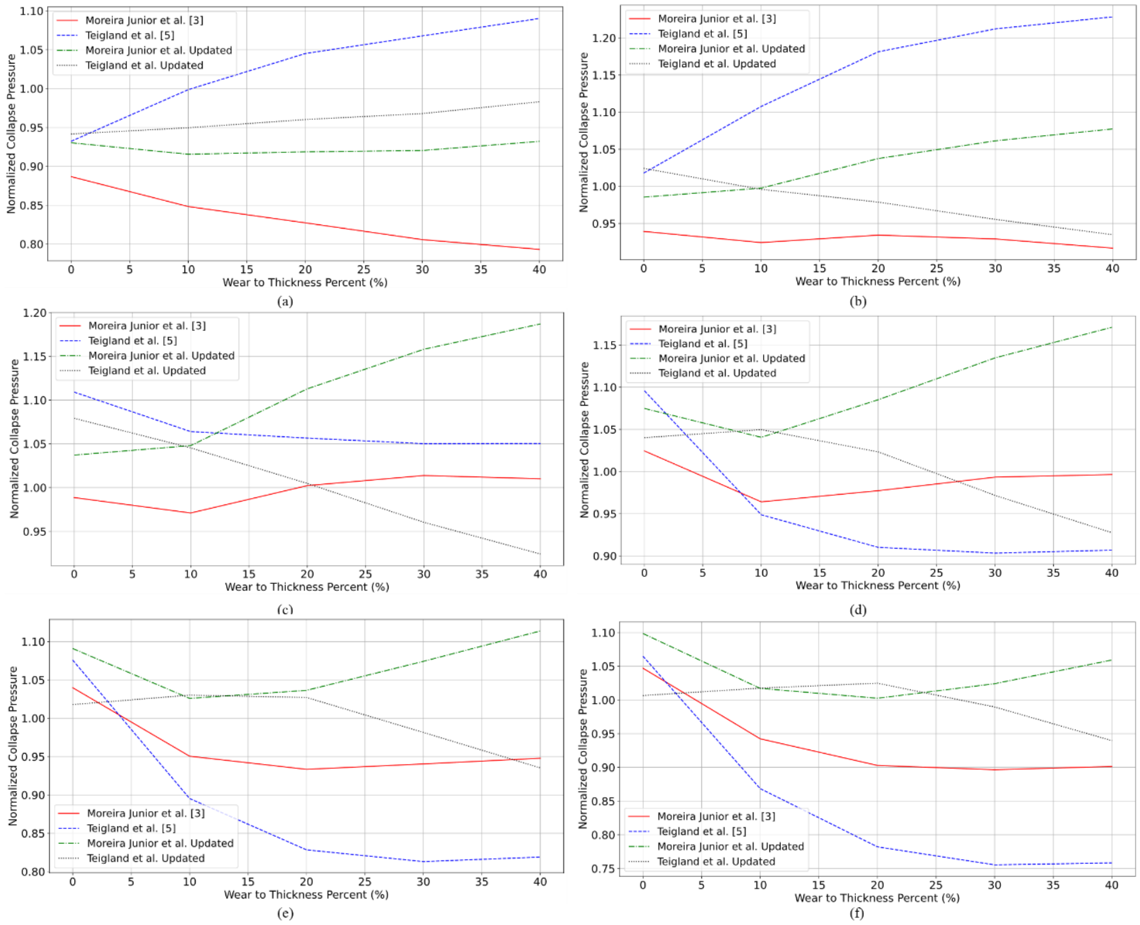

Figure 1. Normalized collapse pressure for (a) OD/wt equal to 12.0, (b) OD/wt equal to 16.0, (c) OD/wt equal to 20.0, (d) OD/wt equal to 24.0, (e) OD/wt equal to 28.0, and (f) OD/wt equal to 32.0

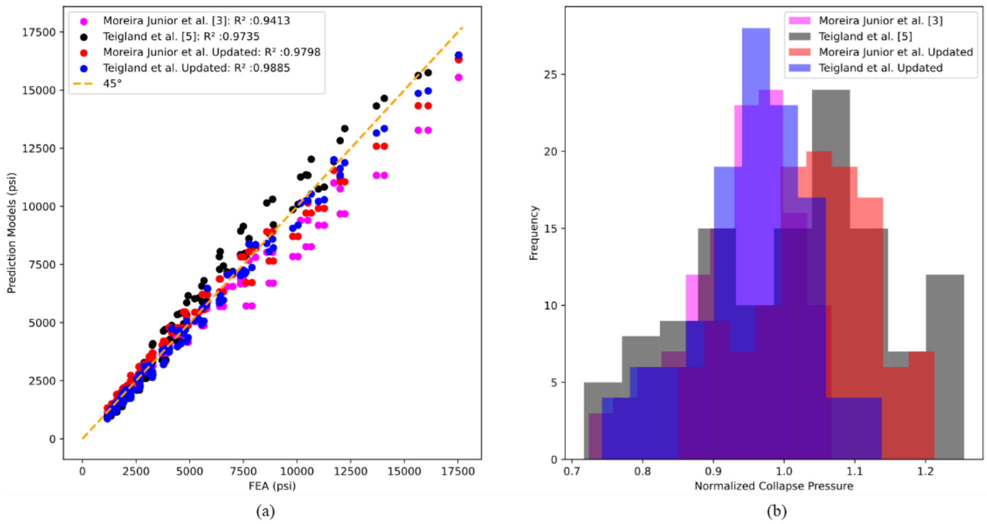

Figure 2. Comparison to FEA results: (a) FEA vs. Prediction Models collapse pressures and (b) Histogram of normalized collapse pressures.

Table 2. Statistical results.

	Mean	St. Deviation	COV (%)
Moreira Junior et al. [3]	0.937	0.0769	8.21%
Teigland et al. [5]	1.012	0.1330	13.15%
Moreira Junior et al. Updated	1.040	0.0849	8.16%
Teigland et al. Updated	0.956	0.0803	8.39%

Regarding the coefficient of variation (COV), it is defined as the ratio of the standard deviation to the mean, which is useful for comparing the dispersion of datasets with different mean values. Based on the results obtained from the FEA testing dataset (Table 2), it is possible to note that the updated models and the one presented by Moreira Junior et al. [3] have COV values that are quite close, with the updated model by Moreira Junior et al. having the lowest COV.

Based on the present analyses, the updated models provided the most accurate results, with a mean value from 0.956 to 1.040 (closest to 1.0) and low COVs. Therefore, it can be confirmed that improving the dataset for calibration leads to models that can more accurately describe the collapse pressure of casing tubulars. The "Teigland et al. Updated" model still may be improved in future research, by incorporating additional dimensionless parameters in the model.

5 Conclusions

The accurate description of casing tubulars is crucial due to their significant role in the oil and gas industry. Prediction models offer a viable alternative to Finite Element Analysis (FEA) models because of their lower computational cost. This paper analyzed the prediction models proposed by Moreira Junior et al. [3] and Teigland et al. [5] and introduced improvements by recalibrating these models. This recalibration used a dataset created through FEA, incorporating all nonlinear effects and a different objective function for the nonlinear regression. The datasets were developed using typical geometries found in the oil and gas industry, as detailed in the DEA-130 report [13].

From the outcomes of the study, it was possible to confirm that the updated models produced more accurate results in terms of collapse pressure compared to the original ones. This analysis, conducted using normalized collapse pressure, showed that the updated models had mean values ranging from 0.956 to 1.040. Moreover, the "Moreira Junior et al. Updated" model yielded the best results, followed by "Teigland et al. Updated", with the lowest dispersion. Therefore, it is possible to confirm that calibrations with more accurate databases can lead to best results.

Future research should explore additional parameters in prediction models, such as material yield strength, residual stress, and wear groove position, by adding new dimensionless parameters in the model proposed by Teigland et al. [5]. Moreover, comparing these models with experimental data would determine their accuracy in predicting actual residual collapse pressures.

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