On the probabilistic assessment of casing applied to top hole design by FORM

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Abstract. This study employed reliability-based models to optimize the design of top-hole casing sections considering the uncertainties associated with soil behavior and casing manufacturing. The oil and gas well integrity significantly depends on the casing system throughout its life cycle, ensuring tightness, stability, and load support. Various load scenarios were analyzed to estimate the probability of the occurrence of different soil-casing system failure modes. Analyses of various types of top-hole designs are included in this work. Reliability-based techniques have emerged as interesting tools for structural analyses and design. This research leverages soil characterization data from piezocone tests (CPTu) to statistically define the mechanical parameters crucial for conductor and surface casing design. Additionally, random variables linked to the geometric properties of tubular are incorporated, drawing from the casing manufacturing data outlined in API/TR 5C3 (2018). Probabilistic models are developed using the first-order reliability method (FORM), an efficient optimization-based procedure, and applied across multiple load scenarios to gauge the failure probability in top-hole casing design. The analysis focused primarily on the variability in undrained soil strength derived from the CPTu data, which was deemed the most influential random variable due to its spatial heterogeneity. These results underscore the viability and importance of estimating the probability of relevant failure modes in accordance with internal regulations concerning the conductor casing load capacity, surface casing triaxial stress, and wellhead displacement. The work in progress considers random variables obtained from correlated soil test data and related to casing manufacturing (outer diameter and wall thickness) in a combined probability density function applied to failure functions. Moreover, the analysis indicated that the outer diameter did not significantly influence the probabilistic response owing to its low dispersion. This novel approach combines soil statistics information and casing manufacturing data within a reliability-based framework, achieving a balance between cost and safety while aiding decision-making in top-hole design.

Keywords: Reliability Analysis, Casing design, Soil load capacity

1 Introduction

The design of top-hole casing in oil and gas wells is a critical aspect of well integrity management. Traditional deterministic methods often fall short in addressing the The design of top-hole casings in oil and gas wells is a critical aspect of well integrity management. Traditional deterministic methods often fail to address the complexities and uncertainties inherent in geological and operational environments. The well structure must prevent unintentional fluid flow into the external environment and between well intervals. The initial steps include drilling and installing the conductor and surface casing strings. These top-hole sections are crucial for providing structural stability to the wellhead system and ensuring strength against various loads encountered during construction and operational phases.

This paper applies reliability-based models to the design of top-hole casing sections, considering uncertainties in soil behavior and casing tubular manufacturing. It addresses typical load scenarios to estimate the probability of different failure modes in the soil-casing system. Uncertainties in the mechanical properties of offshore soils stem from multiple factors, such as natural variability within a soil volume, potential errors in model interpretation and inaccuracies in geotechnical parameter measurements. These factors include the heterogeneity of soil characteristics, limitations in sampling methods, variability in laboratory test outcomes, and the complexities of applying theoretical models to real-world scenarios.

Some studies investigate the quantification of uncertainty and the estimation of characteristic values of mechanical soil parameters, such as undrained shear strength and submerged unit weight. Notable examples include Ching et al. [1], Hu and Wang [2], and Phoon et al. [3]. Varady et al. [4] proposed Bayesian-based data-driven site characterization methods for estimating soil parameters for top-hole casing design. The results of the statistical modeling of soil parameters obtained support the reliability-based analysis in this work.

The structural performance of casing pipes depends on the main geometric parameters, such as the outer diameter (OD) and wall thickness (W_t), as well as material properties such as Yield strength. Manufacturing inconsistencies introduce uncertainties in these parameters, which affect the tube performance. This topic has been examined since the 1990s, with notable contributions like Adams et al. [5], Tallin et al. [6] and recent studies include Liao et al. [7], Long et al. [8], Gouveia et al. [9], Tessari [10], Beck et al. [11], Yang et al. [12] and Gouveia et al. [13].

In this context, the API/TR5C3 [14] standard promotes probabilistic approaches in casing design, providing statistical descriptions of design parameters based on manufacturing data. These descriptions were used to characterize parameters in this study, as done by Várady Filho et al. [15]. The probabilistic approach evaluates uncertainties in structural analysis to estimate the probability of failure of a structural element or system, as discussed by Gouveia et al. [9] and Yang et al. [12]. The reliability theory enables estimation of specified limit states using statistical descriptions of design variables. Melchers and Beck [16] address the fundamental concepts of structural reliability analysis and prediction, including methods for modeling and statistical analysis.

By integrating probabilistic models, we can better predict potential failure modes and optimize casing design to mitigate risks. This approach improves structural integrity and contributes to more efficient and cost-effective operations. This new paradigm is being slowly integrated into design practices through research and consulting efforts.

2 Concepts of Structural Reliability Analysis

Structural reliability theory has the aim of assessing the safety of structures by quantifying the probability of failure while considering the uncertainties inherent in the problem. Limit state equations, or failure functions, are used to describe the potential failure modes of a structure under typical operational situations or under extreme survival conditions, like kicks and blowout scenarios. The ability of the structural assembly to continue in service depends on the relationship between resistance and the stresses imposed on the system.

Consider a limit state function $G(x)$, where X is an n-dimensional vector containing the design variables treated as random variables (r.v.). This function establishes a boundary between the failure domain (Ω_f) and the safe domain (Ω_s) of the structure. Positive values of $G(x)$ represent a safe event, whereas the condition $G(X) \leq 0$ indicates failure events.

The probability of failure (P_f) is calculated by integrating the joint probability density function (PDF) of the r.v. ($f(x)$) over the failure domain (Ω_f), as illustrated in eq. (1):

$$P_f = \int_{\Omega_s} f(x)dx. \tag{1}$$

The solution to Eq. (1) can be complex depending on the complexity of the probability distribution function (PDF), which requires the use of structural reliability analysis methods.

The Monte Carlo (M-C) method is the most widely used approach, and it has been applied across various well engineering domains, such as predicting well construction cost and time (Kitchel et al. [17], Quang-Hung et al. [18]) and casing design (Zhang and Feng [19], Muoghalu et al. [20]).

Also, the method involves simulation based on generating random numbers to produce N random events to be evaluated using the function $G(x)$. The probability of failure was estimated as the ratio of the number of failure events to the total number of events. The accuracy of the results depends on the number of scenarios tested. For problems with very low failure probabilities, numerous simulations are required to obtain an adequate response, leadingto high computational costs.

In this study, the First Order Reliability Method (FORM) was considered as an alternative to other reliability methods with higher computational costs.

The FORM is a semi-analytical approach based on transforming the original random variables (X) into standardized Gaussian random variables (Y) and linearizing the failure function of the problem. The problem is framed as a nonlinear constrained optimization problem to identify the design point, which contains the values of the random variables most likely to cause structural failure.

In this context, the concept of the reliability index (β) is introduced, which represents the shortest distance between the origin of the transformed space and the failure surface. The optimization problem is traditionally solved using the HLRF method, which was named after Hasofer [21] and Rackwitz and Flessler [22]. Further details about the method used are provided in Melchers and Beck [16].

In this study, the limit-state equation $G(X)$ contains expressions for the resistance and load models, which are implicitly defined by a numerical model. A Finite Element (FE) solution was used to evaluate the soil-casing response.When coupling a mechanical model to a FORM routine, it is necessary to calculate the value of the failure function and its gradients at each iteration, which are used in the estimation. Given the implicit nature of $G(X)$, its derivatives are numerically computed using finite differences for each call of the mechanical FE model throughout the optimization process.

The present work highlights that FORM can be combined with any external mechanical model by simply requesting a problem response several times throughout the iterative process. Therefore, despite being an unusual technique for most well design teams, it can be integrated into currently used casing design software because the probabilistic design paradigm is gradually being implemented in the oil industry.

3 Mechanical Analysis and Design of Top-Hole Casing Tubulars

In casing design, various load cases must be considered, corresponding to service and survival loads throughout the well's lifecycle, from drilling to abandonment. Conductor and surface casing sections are designed to handle various loads from: equipment installation and removal, casing and production/injection column installation, fluid changes during drilling and completion, and thermal loads.

Mechanical analysis was conducted using an in-house Finite Element Method software to evaluate the system stresses and displacements, ensuring compliance with internal design criteria defined internally by the oil company. This process determines the material selection for the conductor and surface strings, the length and cement extension of each column, and other parameters.

The top-hole casing design criteria must satisfy the following:

$$LC_{cond} > CL \tag{2}$$

$$SF_{triaxial} \geq 1.25 \tag{3}$$

$$D_{wellhead} < 50cm, \tag{4}$$

where LC_{cond} is the load capacity of the conductor casing, CL is the critical demand within the lifecycle of the well, SF is the triaxial Safety Factor of the non-cemented part of the surface casing, and $D_{wellhead}$ is the maximum displacement of the wellhead.

These criteria ensure the project's safety and serve as reference points for analyses. Using soil and casing design input data, FEM analysis was used to verify whether the design meets the criteria. During each iteration of the FORM method, this process is repeated, ultimately providing a probability of failure value for the top hole system.

4 Statistical Description of Random Variables

This section aims to statistically describe the random variables involved in the analyzes. The probability distribution and their respective characteristic parameters of the random variables associated with the casing tubes were adopted as indicated in API/TR5C3 [14]. The API/TR5C3 [14] report consolidates extensive production data for pipes manufactured between 1977 and 2004, encompassing a wide range of manufacturing technologies and quality standards. The key variables considered include Yield strength (Y_s), outer diameter (OD), and wall thickness (t), along with imperfections such as cross-sectional ovality and eccentricity.

This report highlights that variability in the manufacturing process can introduce significant uncertainties in these geometric and mechanical parameters, which in turn can impact the structural performance of the pipes. To address this issue, API/TR5C3 [14] advocates for the standardization of probabilistic procedures in casing design. This approach aims to enhance the assessment of structural strength by incorporating statistical descriptions of design parameters based on empirical production data.

The statistical parameters utilized in the analyses are evaluated using coefficients specified in Table 1,the mean factor is the actual mean value divided by the nominal value, while COV is the standard deviation divided by the actual mean value. These values were taken from Annex F of API/TR5C3 [14].

Table 1. Statistical description of the r.v. related to cross-section geometry.

Random Variable	Mean Factor	Coefficient of Variation - COV (%)	Distribution
Wall thickness (t)	1.0069	2.590	Gaussian
Outer diameter (D)	1.0059	0.181	Gaussian

The statistical characterization of the undrained shear strength (S_u) was conducted using the random field estimation methodology outlined by Phoon et al. (2004) and subsequently applied by Várady Filho et al. (2024). This study used CPTu data collected by an oil company in the Campos Basin in eastern Brazil. The data obtained from various piezocone tests included measurements of tip resistance, sleeve friction, and pore pressure. From this dataset, the coefficients of variation and probability distribution can be calculated based on statistical tests for undrained shear strength.

5 Analysis and Results

5.1 Data

This study considered scenarios based on real offshore oil wells off the Brazilian coast and provided by the partner company, as shown in Table 2. These data were required to calculate the probabilistic assessment of each scenario.

Table 2. Description of anlysis cases.

Top Hole Method	**A - Drilling and cementing (5 phases)**	**B - Jetting (4 phases)**
Water depth	600 m	1822 m
Final drill depth	5170 m	3638 m
Conductor casing	72m: 36 x 33 in (X-60)	54 m: 36 x 33 in (X-60)
Surface casing	775 m (20 x 18 in, X-70)	977 m (20 x 18 in, X-70)
Cement data	Second cementation 220 m of cement extension	No second cementation 910 m of cement extension
Production casing	Without casing	With casing
Production loads	Acidizing	Injection of water at 6 °C
Setup time	-	4 days
Undrained shear strength (Su)	100kPa - 100m	145 kPa - 100m

5.2 First Analysis

Case A has 5 phases and among the loads expected in the project, operations such as the installation and removal of casing equipment/columns, fluid changes, and thermal loads (for example, from acidizing processes) can lead to displacement of the wellhead. The random variable with the highest importance factor in the FORM method is S_u, as indicated in the Table 3.

Note that when analyzing scenario A from a deterministic point of view, the maximum displacement of the wellhead was 49.9 cm, reaching the acceptable limit when evaluated by the third criterion. This justified the higher probability of failure.

Table 3. Results of the parametric analysis of the drilling and cementing case.

Random variable (r.v.)	**Soil**	**Conductor Casing**		**Surface casing**	
	S_u	W_t	OD	W_t	OD
Importance factor	9.9997e-01	3.7241e-06	1.1854e-05	6.4669e-06	8.3117e-06

Table 4 shows the results of the failure probability and respective reliability index calculated for case A with the installation of the conductor casing by the drilled and cemented method. The results are presented for the three evaluated failure modes based on previously mentioned top-hole design criteria mentioned previously (eq. (2), eq. (3) e eq. (4)). It is important to note that in this analysis, the largest random variable is the undrained shear strength (S_u) of the soil and follows a Gaussian distribution with mean 100 kPa and 10% COV.

The results show that the probability of failure found for the failure modes that evaluate the load capacity of the first casing and the safety factor associated with the second casing is low. However, the failure mode that evaluates the displacement of the wellhead system is the most critical failure mode among the considered situations.

Table 4. Results of the parametric analysis of the drilling and cementing case.

Failure mode	**Failure Probability (P_f)**	**Reliability index (β)**
Conductor load capacity	3.69e-15	7.77
Surface casing triaxial stress	1.16e-13	7.33
Maximum wellhead displacement	1.04e-2	2.31

5.3 Second Analysis

The second case study, Case B, was performed to investigate the influence of the failure mode of the conductor load capacity, considering that the casing design for jetted cases mostly depends on the increase in the load bearing capacity of the soil-structure system, i.e., the first design criterion (eq. (2)).

In the first analysis, the three failure modes were explored for the same scenario. For Case B, we varied the coefficient of variation of r.v. S_u and evaluated its effect on the probabilistic assessment of the jetted case. Table 5 presents the results obtained.

Table 5. Results of the parametric analysis of the jetted case.

COV S_u **Setup: 4 days**	**Failure Probability (P_f)**	**Reliability index (β)**
10 %	4.29e-11	6.49
15 %	2.72e-1	0.61

Analyzing the results in Table 5, it can be observed that the reduction in P_f represents COV reduction, which is expected considering the reduction in the coefficient of variation of S_u. In this failure mode, the undrained soil strength directly corresponds to the resistance component of the failure function. Small changes in this parameter reflect large variations in the P_f value.

In this context, another analysis was performed on the influence of the setup time on the probability of failure. Setup is the effect of increasing the load capacity of the soil-conductor system over time. This effect was considered in conductors installed by jetting and driving. The graph in Figure 1a shows load capacity behavior in relation to time.

Figure 1b presents the failure probability results obtained for Case B with S_u COV 10%. The result corresponds to what was expected because increasing the setup time reduced the failure probability.

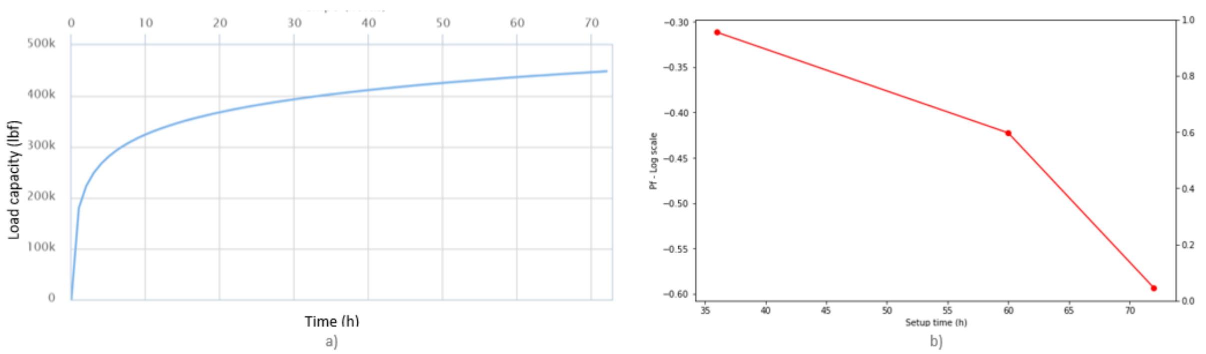

Figure 1. a) Increase load capacity with time; b) Probabilistic curve decline with Increase of setup.

It is necessary to understand that these results serve as indicative measures of the probabilistic behavior of specific case studies, and caution should be exercised when interpreting the P_f values because they are derived from the dataset obtained for API/TR5C3 [14]. A more robust application of probabilistic methodology involves the statistical characterization of variables provided by the manufacturer for a specific product.

6 Conclusions

Probabilistic evaluation of the top-hole design has proved to be a promising tool. Incorporating a probabilistic assessment into casing design practices can be achieved through methodologies like the one presented in this study. The use of a numerical model based on finite elements allowed the application of different loading scenarios, the verification of multiple design criteria, and an integrated response of soil and casing strings. Results obtained demonstrate the relevance of the probability-based analysis for estimating the occurrence of relevant failure modes defined under the oil company's internal regulations (conductor casing load capacity, surface casing triaxial stress in the non-cemented region and maximum wellhead displacement). The proposed approach considers random variables obtained from the correlated soil test data and casing manufacturing to the conductor and surface casing, which are considered in a FORM-based reliability model.

It is necessary to understand that these results serve as indicative measures of the probabilistic behavior of specific case studies, and caution should be exercised when interpreting the P_f values because they are derived from the dataset obtained from API/TR5C3 [14]. There is no agreement on the acceptable P_f values because this depends on a case-by-case analysis considering the critical failure mode observed and the risk tolerance defined by the operator. NORSOK D-010 (2004) referenced an admissible safety level in terms of an allowable P_f of $10^{-3.5}$. However, a revision (NORSOK D-010, 2023) reaffirms the necessity of specific studies to validate target P_f values.

Future works can address the incorporation of r.v. regarding the material properties of the casing, which has a certain difficulty due to the API/TR5C3 [14] dataset, besides the reliability-based optimization of the top-hole design. Evaluation of scenarios with the conductor casing installed by driving and a complete analysis of the setup effect could be part of the next steps in this study.

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