Streamlined workflow for the simulation of submarine landslides with the material point method

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Abstract. Industry demand for numerical simulations is growing every year due to the advances in computer hardware that make increasingly complex and detailed simulations possible. Numerical tools are particularly useful for situations in which the available analytical and experimental tools are limited or unavailable. For that reason, they are a powerful tool for the analysis of natural phenomena, such as submarine landslides. Submarine landslides are commonly accompanied by large deformations and large displacements, which makes them challenging for traditional numerical tools, such as the finite element method. Thus, a numerical tool which is equipped to handle this behavior is necessary. In this paper, the Generalized Interpolation Material Point (GIMP) is applied. GIMP unites the best traits of mesh-based methods with the best traits of particle-based methods, by combining a fixed background grid of finite elements (i.e., a grid that remains still throughout the simulation) with material points that store the kinematic data. This approach favors the simulation of large displacements and deformations. Using the material point method, a streamlined workflow for the simulation of submarine landslides is provided with a practical example that details the process of modelling and execution.

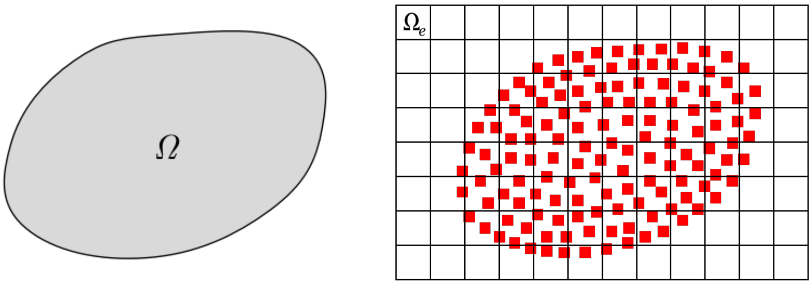
Keywords: Submarine landslides, Material point method, Workflow, Numerical simulation.

1 Introduction

Submarine landslides are of importance to the energy industry, for such events could disturb the supply chain of offshore oil reservoirs, leading to economic and environmental losses, such as oil leakages and pipeline rupture. These natural phenomena are commonly accompanied by large deformations and large displacements, which makes them challenging for traditional numerical tools, such as the finite element method.

In this work, the Generalized Interpolation Material Point (GIMP) method, which was developed as an extension of the original Material Point Method (Sulsky et al. [1]), is adopted. MPM has already been successfully used in the simulation of many challenging problems, such as anchor modelling (Coetzee et al. [2]), runout of landslides (Andersen and Andersen [3]), impact in general (Chen et al. [4]), collapse of granular columns (Mast et al. [5]), avalanches (Mast et al. [6]), hydromechanics (Abe et al. [7]), impact of submarine landslides (Dong et al. [8]), and slope stability (Wang [9]).

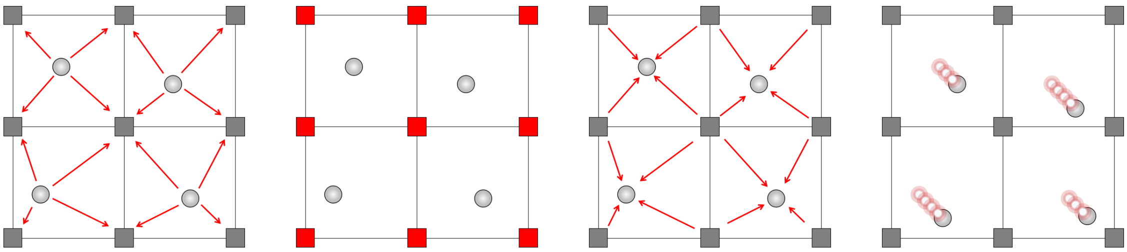
GIMP unites the best traits of mesh-based methods with the best traits of particle-based methods, by adopting a background grid of finite elements that can be reset at the end of each time step, thus, being virtually identical to a fixed background mesh, alongside material points where kinematic data (such as displacement, velocity, and acceleration) is stored (Figure 1).



(a) Continuum domain (b) Discrete domain

Figure 1. Discretization of the physical domain in GIMP. In this method, the continuum domain (a) is represented by a set of material points (or particles), represented in the color red (b). Alongside the particles, a fixed background grid is created. As the background grid remains still during the simulation, it must cover the entire region that the particles may occupy.

In a typical GIMP time step (Figure 2), data is transferred between the particles and the background grid, i.e. particle data is mapped to the background grid, where the equations of motion are solved. The results are then mapped back from the grid to the particles. The update of particle kinematics is performed at the end of the time step.



(a) P2G (b) Equation solving (c) G2P (d) Kinematics update

Figure 2. The four basic steps in a typical GIMP time step. a) Particle data is mapped to the background grid (P2G). b) The equations of motion are solved in the background grid. c) Grid results are mapped back to the particles. d) Update of particle kinematics with the mapped results.

2 Formulation

A brief discussion of the mathematical aspects of the numerical method and the material equation is carried on in this section.

2.1 Generalized Interpolation Material Point Method (GIMP)

GIMP is built upon the variational form for conservation of momentum:

$$\int_\Omega \rho \boldsymbol{a} \cdot \delta \boldsymbol{v} \; d\boldsymbol{x} + \int_\Omega \boldsymbol{\sigma} : \nabla \delta \boldsymbol{v} \; d\boldsymbol{x} = \int_\Omega \rho \boldsymbol{b} \cdot \delta \boldsymbol{v} \; d\boldsymbol{x}, \tag{1}$$

in which:
Ω: current volume,
ρ: mass density,
$\boldsymbol{a}$: acceleration,
$\delta \boldsymbol{v}$: admissible velocities,
$\boldsymbol{x}$: current position,
$\boldsymbol{\sigma}$: Cauchy stress,

$\boldsymbol{b}$: specific body force.

The particle characteristic function $\chi_p^i(\boldsymbol{x})$, which is a partition of unity, is introduced to the variational form of conservation of momentum. Then, the initial particle volume is defined as $V_p^i = \int_{\Omega^i} \chi_p^i(\boldsymbol{x}) d\boldsymbol{x}$, in which Ω^i is the initial volume of the continuum body.

This work adopts the Contiguous Particle GIMP Method (cpGIMP). In this variant of GIMP, the particle domain is updated as it deforms, i.e., its physical dimensions changes when forces are applied (Bardenhagen et al. [10]). In this setup, particles are represented as rectangles with a side length of $2l_p$. A set of weighting functions $\overline{N}_{np}$ and gradient weighting functions $\overline{\nabla N}_{np}$ cover the interface between the particles and the nodes is defined:

$$\overline{N}_{np} = \frac{1}{2l_p} \int_{x_p - l_p}^{x_p + l_p} N_n(x) dx, \tag{2}$$

$$\overline{\nabla N}_{np} = \frac{1}{2l_p} \int_{x_p - l_p}^{x_p + l_p} \nabla N_n(x) dx, \tag{3}$$

in which x_p is the particle position, $N_n(x)$ is the nodal shape function and $\nabla N_n(x)$ the gradient nodal shape function.

After careful numerical manipulation of the terms in the resulting equations, the equation of motion can be summed as:

$$\dot{\boldsymbol{p}}_n = \boldsymbol{f}_n^{ext} - \boldsymbol{f}_n^{int}, \tag{4}$$

in which:

$\dot{\boldsymbol{p}}_n = \sum_p \dot{\boldsymbol{p}}_p \overline{N}_{np}$: nodal momentum (in terms of the particle momentum $\dot{\boldsymbol{p}}_p$ and the weighting function $\overline{N}_{np}$),

$\boldsymbol{f}_n^{ext} = \sum_p m_p \boldsymbol{g} \overline{N}_{np}$: external nodal force or self-weight (in terms of the particle mass m_p and the gravitational acceleration $\boldsymbol{g}$),

$\boldsymbol{f}_n^{int} = \sum_p V_p \boldsymbol{\sigma}_p \cdot \overline{\nabla N}_{np}$: internal nodal force (in terms of the particle volume V_p, the particle stress $\boldsymbol{\sigma}_p$ and the gradient weighting function $\overline{\nabla N}_{np}$).

The equation of motion must be solved at all time steps. Thus, a time integration scheme is needed. In this work, the Euler method (an explicit time integration scheme) is adopted. The critical time step ($\Delta t_{critical}$), i.e. the maximum value of the time step is defined by:

$$\Delta t_{critical} = \frac{\Delta x}{c}, \tag{5}$$

in which,

Δx: element size on the fixed background grid,

c: sound wave propagation speed in the material.

It is of note that the moment on which the strese update is performed, i.e. before or after solving the equation of motion in a given time step, leads to different results. Two commonly adopted algorithms to solve this problem are: *Update Stresses Last* (USL) and *Update Stresses First* (USF). In USF, the stress update step is performed after the G2P step. Conversely, in USL the stress update step is performed before the G2P step. USF is a conservative algorithm, but can lead to an overall increase of the energy level in the system (Bardenhagen [11]). Therefore, this work adopts USL.

2.2 Material equation

In this work, the landslide material is simulated as a Bingham fluid, as this model shows good results for submarine landslide models and allows for the consideration of water effects on the soil without the need of simulating a fluid phase. In the Bingham model, the undrained shear strength is updated according to:

$$s_u = s_{u0} + K\dot{\gamma} \tag{6}$$

in which:

s_u: Undrained shear strength,
s_{u0}: Initial undrained shear strength,
K: Consistency index,
$\dot{\gamma}$: Shear strain rate.

2.3 Contact algorithm

A primary-secondary contact algorithm (Figure 3) is implemented into GIMP, so that tangential and normal force components are computed every time step. In this contact algorithm, a primary grid containing all bodies in the simulation is created alongside secondary grids, which only contain one body each. By comparing nodal grid masses among grids, the contact condition is detected and nodal forces are corrected to account for the collisions. A Coulomb friction coefficient may be adopted to enforce frictional forces among different bodies.

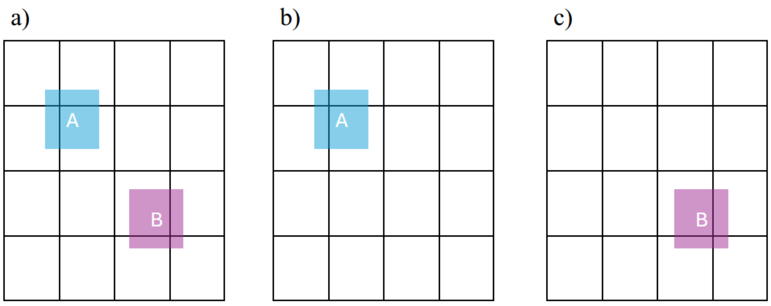


Figure 3. Primary-secondary contact algorithm. a) Primary grid. b) Second A grid. c) Secondary B grid.

3 Modelling workflow

To be able to build the submarine landslide model, some steps (Figure 4) are necessary. First, a field survey of the region of interest must be conducted, so bathymetric and stratigraphic data is collected. That data is used to build an elevation map of the terrain. With the elevation map, a subset of the point cloud must be selected as a subregion of interest, which will be effectively part of the computational model. That subregion is employed to generate a 3D geometric model. In this model, a critical slip surface (in general, an ellipsoid) is defined as the region that might be displaced during the landslide event. The material properties are added to the model, so that MPM may be used to discretize the domain into a set of particles and a background grid.

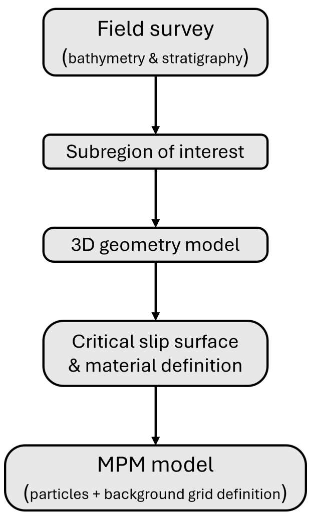


Figure 4. Submarine landslide modelling workflow

4 Submarine landslide simulation

In this work, a simplified submarine landslide model (Figure 5) is simulated. The model is represented by a trapezoidal prism, intersected by a sphere with a radius of 4 m and centered at the point $(12, 0, 5)$ m, which delimits the critical slip surface.

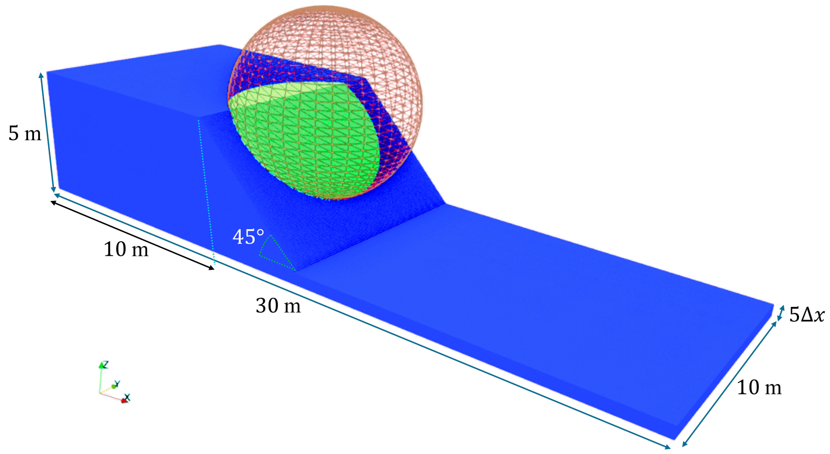


Figure 5. Schematics of the submarine landslide model with display of the critical slip surface geometry and model dimensions

In this model, the critical slip surface region, i.e. the mobilized soil, is modelled as a Bingham fluid (Table 1). The intact part of the of the model is modelled as a rigid body. The numerical parameters of the model domain are presented in Table 2.

Table 1. Mobilized soil material parameters from the Bingham fluid model

Parameter	Value
Mass density (ρ)	1,800 kg/m^3
Young's modulus (E)	0.5 MPa
Poisson's ratio (ν)	0.49
Initial undrained shear strength (s_{u0})	5 kPa
Consistency index (K)	80 Pa·s

Table 2. Numerical domain parameters

Parameter	Value
Background grid element size	0.15 m
Number of mobilized particles	465,856
Number of intact particles	5,724,144
Gravitational acceleration	9.81 m/s^2
Percentage of the critical time step	10%
Coulomb friction coefficient	1
Simulation time	5 s

The simulation timeline (Figure 6) shows, as expected, a landslide that reaches the peak velocity at the beginning of the simulation, followed by a gradual reduction of velocity, as the soil stabilizes, which happens at, approximately, time $t = 5$ s.

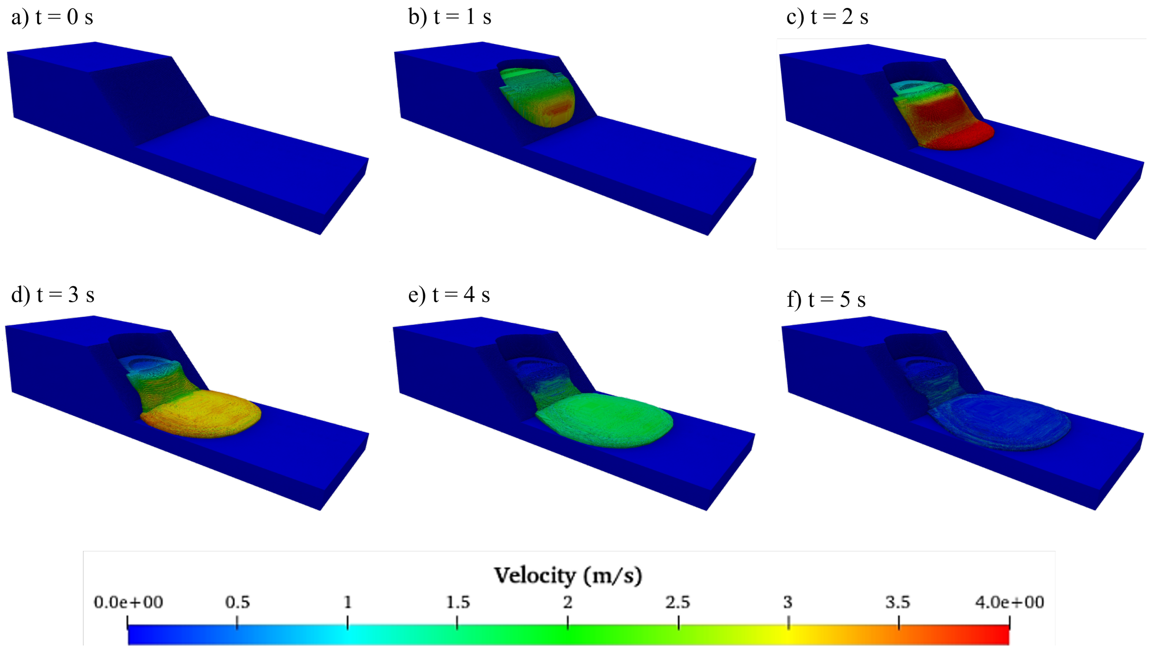


Figure 6. Landslide flow timeline with emphasis on the magnitude of the velocity field

5 Concluding remarks

In this work, it was shown that, following a simple workflow with the delineation of a critical slip surface and material properties, the GIMP is suitable for simulating submarine landslides. Naturally, aspects such as CPU times may vary greatly depending on the available machines and the requirements for refinement level of the background grid and particles, as well as the time necessary for the complete simulation.

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References

[1] D. Sulsky, Z. Chen, and H. L. Schreyer. A particle method for history-dependent materials. *Computer Methods in Applied Mechanics and Engineering*, vol. 118, pp. 179–196, 1994.

[2] C. J. Coetzee, P. A. Vermeer, and A. H. Basson. The modelling of anchors using the material point method. *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 29, n. 9, pp. 879–895, 2005.

[3] S. M. Andersen and L. V. Andersen. Modelling of landslides with the material-point method. *Computational Geosciences*, vol. 14, n. 1, pp. 137–147, 2010.

[4] Z. Chen, Y. Han, S. Jiang, Y. Gan, and T. D. Sewell. A multiscale material point method for impact simulation. *Theoretical and Applied Mechanics Letters*, vol. 2, n. 5, pp. 051003, 2012.

[5] C. M. Mast, P. Arduino, P. Mackenzie-Helnwein, and G. R. Miller. Simulating granular column collapse using the Material Point Method. *Acta Geotechnica*, vol. 10, n. 1, pp. 101–116, 2014a.

[6] C. M. Mast, P. Arduino, G. R. Miller, and P. Mackenzie-Helnwein. Avalanche and landslide simulation using the material point method: flow dynamics and force interaction with structures. *Computational Geosciences*, vol. 18, n. 5, pp. 817–830, 2014b.

[7] K. Abe, K. Soga, and S. Bandara. Material Point Method for Coupled Hydromechanical Problems. *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 140, n. 3, pp. 04013033, 2014.

[8] Y. Dong, D. Wang, and M. F. Randolph. Investigation of impact forces on pipeline by submarine landslide using material point method. *Ocean Engineering*, vol. 146, pp. 21–28, 2017.

[9] B. Wang. *Slope failure analysis using the material point method*. PhD thesis, TU Delft University of Technology, 2017.

[10] S. G. Bardenhagen, E. M. Kober, and others. The generalized interpolation material point method. *Computer Modeling in Engineering and Sciences*, vol. 5, n. 6, pp. 477–496, 2004.

[11] S. G. Bardenhagen. Energy conservation error in the material point method for solid mechanics. *Journal of Computational Physics*, vol. 180, n. 1, pp. 383–403, 2002.