Dimensionality Analysis of Unsteady-State Core-Flooding Simulations for Homogeneous and Heterogeneous Rock Types

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Abstract. Model quality and computational resources must be carefully balanced in multidimensional numerical simulations due to the high computational cost that three-dimensional (3D) models pose. This is inherently true in the oil and energy industry, where reservoir modelling of complex real experiments are required, in which inaccuracies have a direct impact on financial outcomes. Properly used, one-dimensional (1D) simulations yield significant technological and financial benefits by reducing simulation runtime, allowing more efficient operations, and faster and more accurate decision-making. As a result, wherever possible, reduced models are preferred. This work aims to analyze the feasibility and effectiveness on the dimensional reduction of 3D to 1D models for unsteady-state core-flooding experiments. For that, 1D and 3D multi-phase flow on homogeneous and heterogeneous porous media simulations were performed using the Black Oil IMEX™ software to verify the variability of oil production volume, saturation profiles, and differential pressure numerical responses. According to the results, the oil production volume, water saturation and pressure differential are not affected by the dimensionality of the problem in homogeneous cases. On the other hand, it is found that when dimensionality is reduced, heterogeneities lead to a significantly difference in terms of pressure differential, while the oil production and water saturation were not so impacted

Keywords: Dimensionality Analysis; Multidimensional Modelling; Heterogeneous Porous Media; Special Core Analysis; Advanced Oil Recovery.

1 Introduction

The transition from one-dimensional (1D) to three-dimensional (3D) modeling in numerical simulations of unsteady-state (USS) experiments represents a significant advancement in the accuracy and understanding of two-phase flow processes in porous media. Traditionally, 1D models based on Buckley-Leverett equations have been widely used to determine parameters such as relative permeability. However, these simplified models do not adequately capture the complexity of saturation distributions and spatial variations that occur in real experiments. 3D modeling, on the other hand, allows for a more realistic representation of physical phenomena, incorporating variations in the axial, radial, and transverse directions. This is especially crucial in core plugs, where boundary effects and the geometry of the inlet distributor can significantly influence experimental results Yang et al. [1]. Multidimensional numerical simulations can provide a more detailed and accurate understanding of flow processes, improving the correlation between experimental data and theoretical models, and leading to advancements in reservoir characterization and the efficiency of oil recovery operations.

Despite the significant advantages in accuracy and realistic representation of physical processes, the use of

3D models in numerical simulations faces several limitations. Firstly, 3D models require substantially more computational power compared to 1D or two-dimensional (2D) models. Additionally, the complexity of 3D modeling demands more sophisticated and detailed algorithms, which may cause numerical errors and demand rigorous validation of input data. These limitations can make 3D modeling quite difficult to implement for some studies or applications, particularly in environments where computational and financial resources are limited. Therefore, it is crucial to carefully evaluate the cost-benefit ratio when considering the implementation of 3D simulations in reservoir engineering projects.

In response to the limitations of existing models, the work of Rom and Müller [2] presents a new approach that incorporates a 1D porous medium flow solver incorporated within a two-domain model for transpiration cooling in 2D and 3D. This methodology effectively divides the porous domain into 1D problems, enabling analytical solutions for fluid and solid temperatures with only a single ordinary differential equation required to determine fluid density and velocity. Validated through rigorous numerical testing and experimental data, the model demonstrates a significant reduction in computational time compared to conventional 2D/3D approaches, offering enhanced efficiency and stability for practical applications in high-temperature, gas flow environments.

Chen et al. [3] suggested a dimension-reduced line element method to help with the difficulties of simulating free-surface flow that changes over time in three-dimensional porous media. Their innovative approach conceptualizes permeable pores as an orthogonal network of tubes, effectively transforming the 3D problem into a 1D solution space. This simplification significantly enhances computational efficiency while maintaining accuracy. The method's effectiveness was demonstrated through comparisons with various scenarios, including unconfined aquifers, trapezoidal dams, sand flumes, and wells. The proposed methodology not only offers improved numerical efficiency but also holds promise for modeling other flow problems in porous media, such as two-phase flow and thermal problems, by leveraging the principles of Darcy, Buckingham, and Fourier's law.

The study conducted by Yang et al. [1] highlighted the dimensional differences between 1D and 3D simulations on steady-state (SS) two-phase tests. The 3D simulations showed longer stabilization times and incomplete fluid mixing post-injection compared to the 1D simulations. In 3D, fluid saturations are non-uniform, reflecting the geometry of the distributor, especially in cores with anisotropic permeability. Although gravitational effects are insignificant in typical reservoir cores, the choice of distributor (half-moon versus spiral) significantly impacts the results. The authors also observed that the relative permeability and capillary pressure curves derived from the 3D simulation closely match laboratory data, supporting the use of SSTT to determining these properties.

With the advancements made by studies such as Yang et al. [1], which investigated the 3D effects in steady-state two-phase tests, this work aims to analyze the feasibility and effectiveness of reducing 3D models to 1D models in unsteady-state fluid injection experiments in heterogeneous plugs. This analysis sought to determine how heterogeneity, represented by a probability distribution, influences dimensional reduction and its impact on the accuracy of the results obtained, as well as on the computational efficiency of the models. The primary focus is to evaluate these aspects in the context of two-phase flow experiments in porous media, aiming to optimize both accuracy and computational performance.

2 Methodology

2.1 Numerical Simulation

The flow simulations were performed using CMG™ IMEX, a fully implicit, isothermal black oil simulator. This simulator utilizes the finite difference method to solve the governing equations for multi-phase fluid flow through porous media. The principle of mass conservation is represented for each phase in the following manner,

$$\phi\frac{\partial S_\alpha}{\partial t} + \nabla \cdot u_\alpha = 0 \tag{1}$$

$$u_a = -\frac{k k_{r\alpha}(S_\alpha)}{\mu_\alpha}\nabla p_\alpha \tag{2}$$

where α denotes the fluid phase, ϕ the rock porosity, S_α the phase saturation, u_α the fluid phase velocity, k the rock absolute permeability, $k_{r\alpha}(S_\alpha)$ the fluid phase relative permeability, μ_α the fluid viscosity, and ∇p_α the fluid phase pressure gradient. Once this study investigates a multi-phase flow of water and oil, one may denote α as w for water and o for oil.

The correlations between phase saturation and pressure, as represented by capillary pressure P_c, can be described by the following equations:

$$S_w + S_o = 1 \tag{3a}$$

$$P_c(S_w) = P_o - P_w \tag{3b}$$

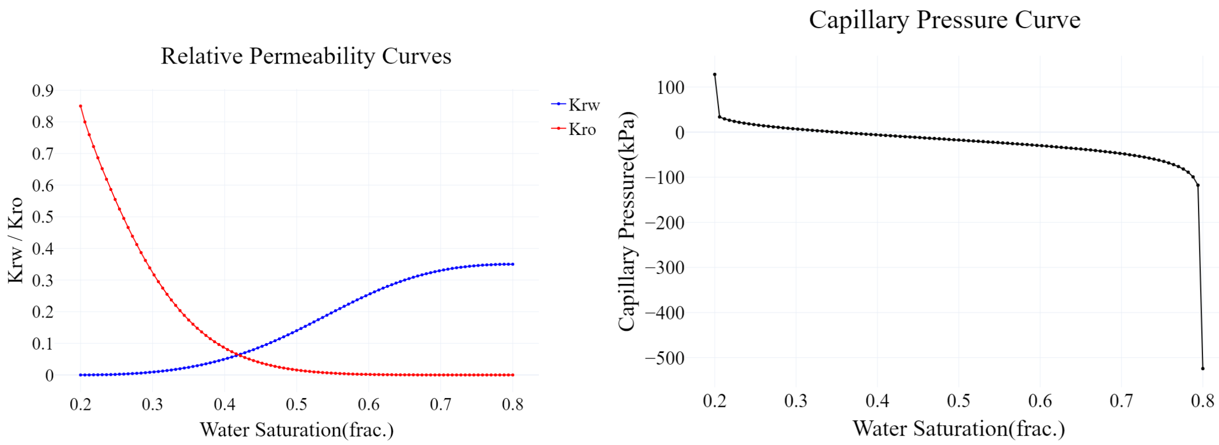
The relative permeability and capillary pressure are both dependent on the water saturation. These parameters are often determined through a history-matching process using data collected from core flood experiments. In this study, one employed the LET correlation [4], a commonly utilized model that offers more flexibility in accurately matching production data points throughout time.

Capillary pressures are also input parameters, which are functions of water and gas saturation, their modeling was carried out using the Log(beta) function [5].

Table 1 displays the rock and fluid properties used in numerical simulation for unsteady state experiments (USS) involving the homogeneous core. These studies were performed to graphically represent simulated oil production, pressure differential, and water saturation as a function of time, providing a basis for each heterogeneous simulation.

Table 1. Fluid and rock simulation properties.

Property	**Diameter (cm)**	**Length (cm)**	**Water Viscosity (cp)**	**Oil Viscosity (cp)**	**Water Density (g/cm3)**	**Oil Density (g/cc)**
Value	3.8	5	0.5	2	1	0.8



(a) (b)

Figure 1. Relative permeability (a) and capillary pressure (b) curves used in the simulations.

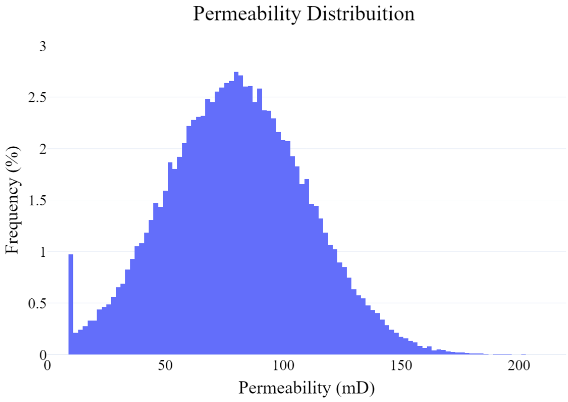
2.2 Mesh Convergence

A mesh convergence study was conducted on a homogeneous base case, consisting of a cylindrical plug with a permeability (k) of 256 mD and a porosity (ϕ) of 0.24. An analysis was performed to examine the relative error by increasing the number of blocks in all three dimensions. The cases are represented as M1 (7x7x252), followed by M2 (9x9x252), M3 (13x13x302), M4 (15x15x352), and M5 (17x17x352). The relative error was evaluated for pressure differential and oil production at six distinct timesteps: the initial five points, identified by the increase of flow rate in the multistep experiment, and the last one corresponding to the conclusion of the experiment. The M5 mesh, consisting of 79,200 blocks, was chosen for its low error, as well as its reasonable simulation runtime.

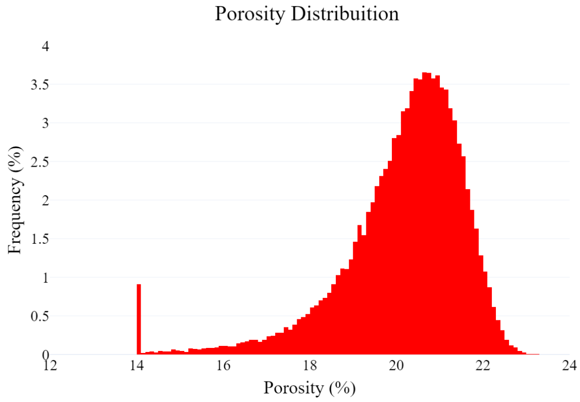
2.3 Model Construction — Heterogeneity

To construct a synthetic model incorporating heterogeneity as proposed by AlMansour et al. [6], the distribution of porosity and permeability within the cells must be carefully designed. Initially, the permeability map is defined by a gaussian distribution with a mean (μ) of 80 mD and a standard deviation (σ) of 30 mD. This distribution is truncated at 10 mD to prevent negative permeability values, ensuring that all cells in the model have physically plausible values. Next, the porosity map is determined using the correlation specified in Eq. 4, extracted from Peters [7]. Following this correlation, the porosity follows a log-normal distribution, ensuring that the natural variations observed in heterogeneous porous media are adequately represented. This procedure generates a model with realistic heterogeneity characteristics, which is essential for the analysis of flows in complex porous media.

$$k = 0.1038\mathrm{e}^{0.3255\phi} \tag{4}$$



(a)



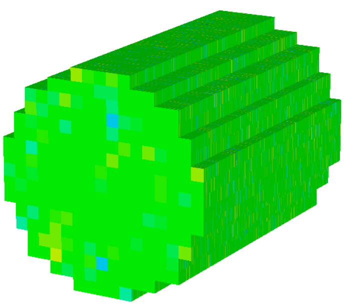
(b)

Figure 2. Relative permeability (a) and capillary pressure (b) curves used in the simulations.

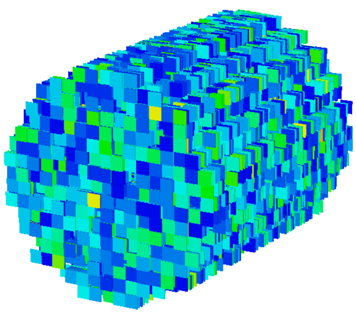
From this synthetic model, multiple scenarios were created to evaluate the impact of heterogeneity on relative permeability. The different scenarios were constructed by altering the permeability and porosity properties of a certain volume of cells, as described in Tab. 2 and visualized in Fig. 3. In case AH, the scenario is homogeneous. In case A1, 12% of the cell volume in the plug was altered based on the homogeneous model, resulting in a total of 9,425 heterogeneous grid cells. In case A2, 25% of the rock volume becomes heterogeneous, resulting in 19,781 heterogeneous grid cells. In case A3, 45% of the cells were altered, resulting in 35,606 heterogeneous grid cells. In case A4, 50% of the cells were altered, with 39,562 heterogeneous grid cells. In case A5, 75% of the cell volume was altered, resulting in 59,343 heterogeneous grid cells. Finally, case A6 shows complete heterogeneity, with 100% of the rock volume being heterogeneous, resulting in 79,125 heterogeneous grid cells.

Table 2. Parameter values for random scenearios

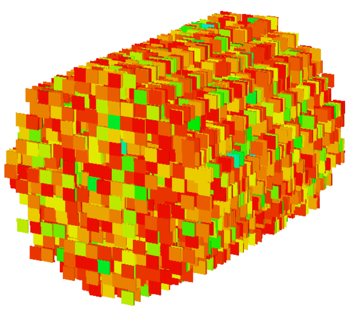
Scenarios	**AH**	**A1**	**A2**	**A3**	**A4**	**A5**
%	0	12	25	50	75	100
Altered blocks	0	9450	19688	39375	59063	78750



(a)



(b)



(c)

Figure 3. Representation of random heterogeneity applied on different rock volumes. (A) Low heterogeneous plug (12%); (B) Medium heterogeneous plug (50%); (C) High heterogeneous plug (75%).

2.4 1D Simplification

Developing a one-dimensional simplification for additive petrophysical properties (such as area, porosity, volume, among others) is a relatively simple task. According to da Silva Guedes [8], the arithmetic mean often provides satisfactory results for these properties. Therefore, Eq. 5 presents the mathematical expression for calculating porosity (ϕ) as a function of the sum of the product of porosity (ϕ_i) with the volume (Vol_i) of each of the n cells that make up the subdomain, divided by the total volume (Vol) of the mesh layer.

$$\phi = \left(\frac{1}{Vol}\right)\sum_{n=1}^{n}\phi_i Vol_i \tag{5}$$

This process was repeated for each mesh layer and the 1D porosity created was aplied in Eq. 4 to generate the 1d permeability map.

3 Results and Discussion

In this study, the numerical simulations with IMEX™ Black Oil software considered three different results: water saturation, oil production volume (NP) and pressure difference (ΔP). Figure 4 presents the ΔP curves for the different scenarios analyzed.

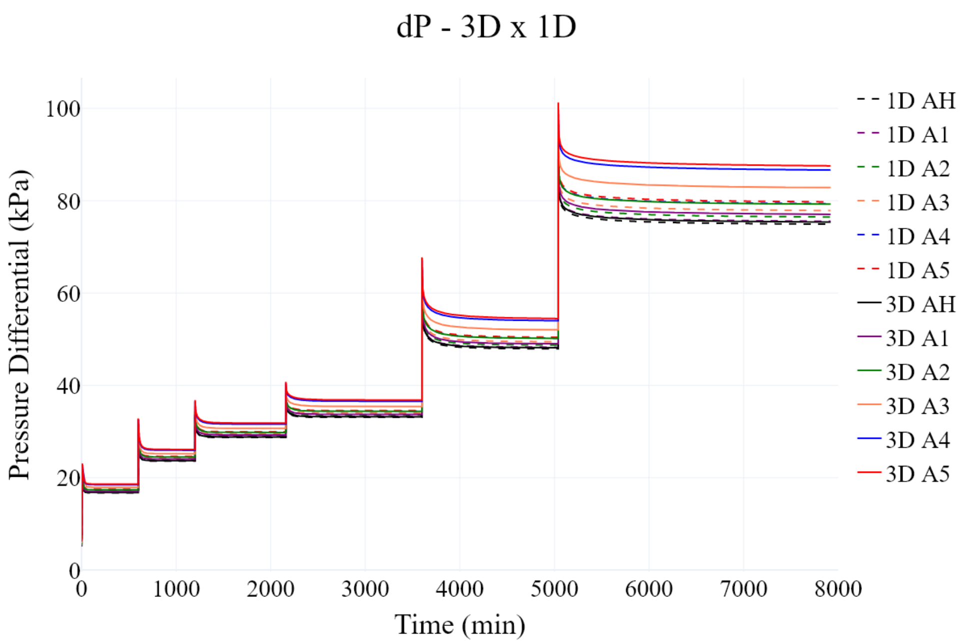


Figure 4. Differential pressure (ΔP) curves for the different 3D (solid line) and 1D (dashed line) scenarios.

It was initially noticed that increasing the flow rate causes greater variation on differential pressure in the the 3D models compared to the 1D models. For homogeneous models, whether 1D or 3D, the ΔP curves almost overlap, regardless of the flow rate used, indicating good agreement between dimensions.

As heterogeneity increases, whether 1D or 3D representations, especially from the scenario with 50% of modified cells (scenario A3). This larger deviation indicates that heterogeneity begins to significantly impact the results, making accurate representation by 1D models more challenging.

Furthermore, from the point where 75% of the cell volume is altered (scenario A4), the effect of heterogeneity on the ΔP curves varies little. This suggests that after a certain threshold of heterogeneity, the differential pressure curves reach an almost asymptotic behavior, where further increases in heterogeneity do not result in significant variations in the results.

Figure 5 presents the NP curves given 1D and 3D simulations. The numerical curves presents distinct behaviors compared to ΔP curves. Contrary to what was observed for ΔP, the difference of the 3D NP curves from their respective 1D curves is not influenced by the increase in test flow rate. Regardless of the applied flow rate, the NP curves of the 3D models maintain a consistent correlation with the corresponding 1D curves.

Furthermore, the NP curves show minimal sensitivity to variations in Gaussian heterogeneity. Even with the application of different heterogeneity levels in the models, the NP curves did not show significant changes. This

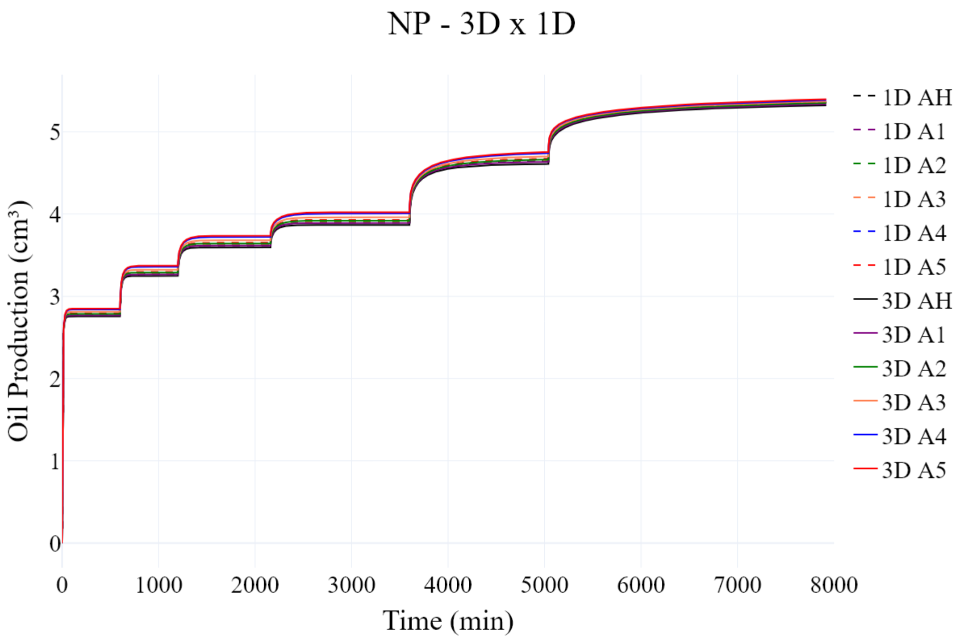


Figure 5. Net production (nP) curves for the different 3D (solid line) and 1D (dashed line) scenarios.

behavior suggests that oil production, as measured by the NP curves, is less impacted by the complexity and distribution of heterogeneities in the porous medium, in contrast to the ΔP curves, which displayed greater variability due to heterogeneity. This suggest that, for NP, 1D models can be sufficiently representative, even in scenarios with varying levels of heterogeneity. The insensitivity of NP curves to random heterogeneity and increased flow rate reinforces the viability of 1D models for predicting NP. This approach enables a more simplified and efficient analysis without compromising the accuracy of the results.

Figure 6 presents the water saturation curves (S_w) for 1D and 3D models of the different heterogeneity scenarios at five distinct points along the sample (10%, 25%, 50%, 75%, and 90% of the core length). The distancing of the 3D S_w curves from their respective 1D curves is primarily influenced by the position where S_w is measured.

These water saturation curves (S_w) exhibit a similar behavior to NP. This is because the total fluid saturation must always equal 100% of the porous volume of the sample. However, some differences can be observed between the S_w curves at different collection points. For instance, at higher flow rates, the saturation profiles reach a *plateau* at smaller times at the beginning of the sample (10% and 25%). The location where S_w is collected also directly influences the time it takes for the saturation to reach its *plateau* for the injection flow rate. This difference in time arises because the water breakthrough front takes longer to form at higher flow rates.

Given the correlation between S_w and NP, the similar behavior was expected, and the conclusions are analogous: 1D models can effectively represent scenarios with varying levels of heterogeneity. This approach enables a more simplified and efficient analysis without compromising the accuracy of the results. However, caution is advised for samples exceeding 5 cm in length, as there is a tendency for the distance between 3D and 1D cases to increase with sample length.

4 Conclusions

This study investigated the feasibility and limitations of dimensionality reduction from 3D to 1D models for non-steady-state fluid injection experiments in heterogeneous plugs. The findings suggest that in homogeneous scenarios, oil production, water saturation, and pressure difference are not significantly affected by the problem's dimensionality. However, the presence of heterogeneities in the porous medium exhibited a substantial influence on the simulation results, particularly on the behavior of the pressure difference. Accumulated production and water saturation were found to be relatively insensitive to heterogeneity variations, even though saturation exhibited sensitivity to sample size

It was observed that for cases where the where the level where porosity and permeability of the sample deviated from the mean corresponded to more than 50%, all analyzed data experienced a significant increase in the relative distance between their 1D simplifications. In the case of the pressure difference, this difference ranged

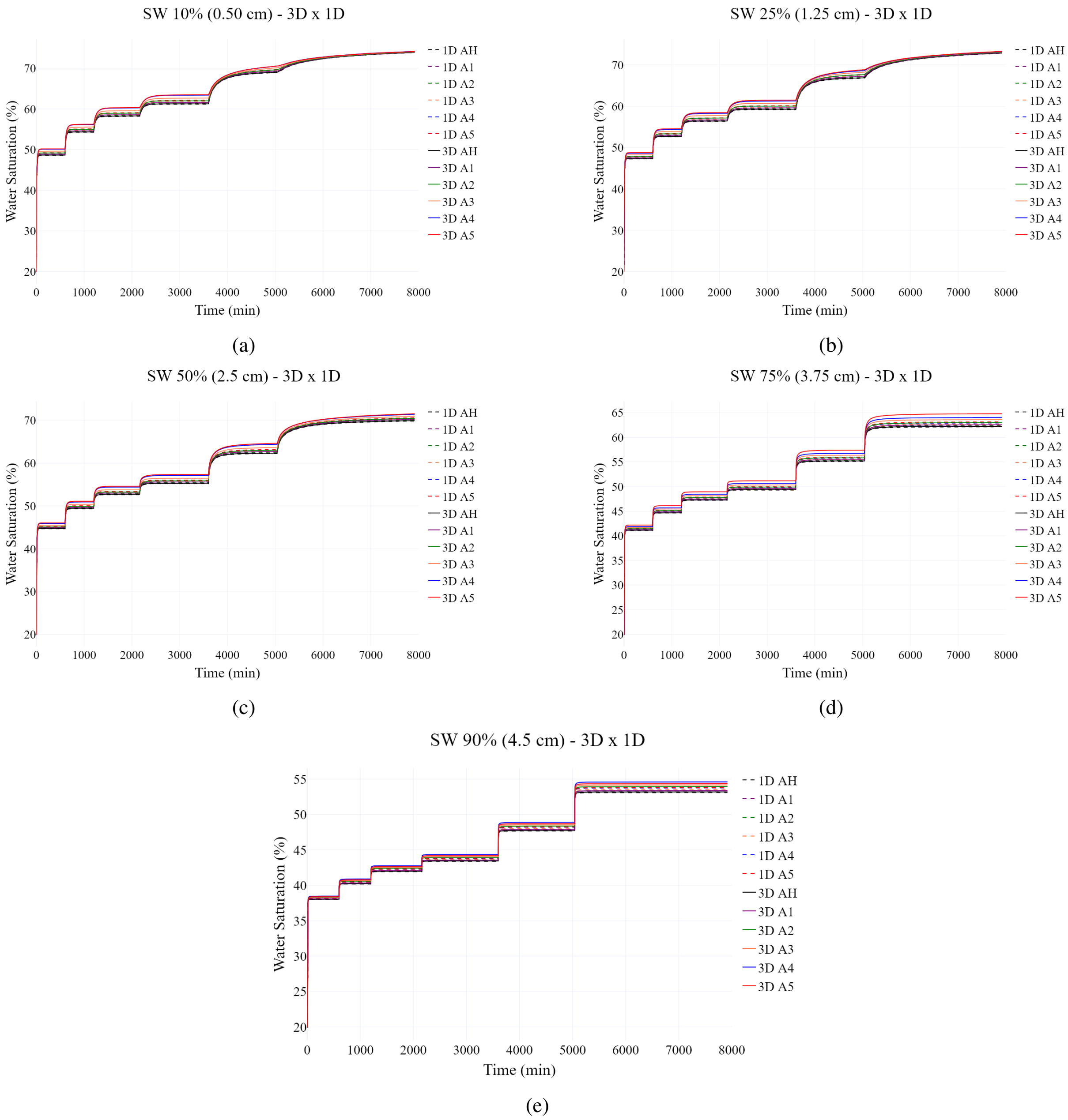


Figure 6. S_w curves for the different 3D (solid line) and 1D (dashed line) scenarios.

from 6% to 10% in the most impactful case ($A5$).

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