Homogenised damage model for concrete

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Abstract. This work deals with numerical simulation of the mechanical behaviour of concrete using a homogenised damage model based on the concept of Representative Volume Element (RVE). The RVE is composed of phases with different mechanical behaviours leading to heterogeneous characteristics on the microstructure level. We adopt a simple damage model capable to simulate the behaviour of the mortar and transition zone while the aggregates are modelled as linear elastic material. To perform the numerical analysis, a based homogenisation technique is used to obtain a damage model that represents the macromechanical behaviour of the material. Results show the capabilities of the model to capture complex phenomena including a comparison with experiments tests performed in concrete specimens.

Keywords: damage mechanics, RVE, concrete.

1 Introduction

This work presents a numerical approach recently developed in Borges and Pituba [1] to model the mechanical behaviour of concrete at mesoscopic level leading to macroscopic homogenised response of the material. For this propose, the main dissipative phenomenon studied here is the damage process that occurs in interface transition zone (ITZ) and mortar. Nowadays, many works deal with the modelling of quasi-brittle materials, particularly the concrete. It can be cited some works which used or developed different numerical approaches based on homogenisation concepts, as example, Guo and Zhao [2], Storm, Qinami and Kaliske [3], Moumen, Kanit and Imad [4] and Borges and Pituba[5]. The incorporation of damage processes in the modelling of the ITZ and mortar lead us toward on more accurately understanding of the mechanical response of quasi-brittle materials. As a result, the approach taking into account the damage process becomes a proper tool for the analysis of concrete structures under diverse application scenarios.

Therefore, this work deals with damage process at mesoscopic level of concrete using a computational homogenisation technique based on Representative Volume Element (RVE) concept. For details, see Borges and Pituba [5]. So that, a constitutive law has been assigned to each material phase within the RVE. Linear elastic behaviour has been adopted to the aggregates. The damage model proposed by Mazars [6] has been used for the mortar and ITZ. As a result, this paper presents the development and application of a based damage homogenisation macroscopic model to consider local damage using the concepts of the multiscale approach [5] and Continuum Damage Mechanics (CDM).

2 Homogenisation Approach based on RVE concept

The homogenisation approach used in this work is presented in detail by several authors, Giusti et al. [7] and

Fernandes, Pituba and Souza Neto [8] and Borges and Pituba [5]. The RVE is assumed as a continuous medium in this approach, which preserves the validity of stress concepts at the microscale. In this approach, the RVE is identified with a sub-index μ, where the volume of the RVE is denoted by V_μ, the domain is given by Ω_μ, and its boundary is given by $\partial\Omega_\mu$. The RVE domain comprises of a solid part denoted by Ω_μ^S, which contains the coarse aggregates and mortar.

Each material point x in the macroscale body with domain Ω is associated with an RVE. The RVE comprises two parts: the mortar part, denoted by Ω_μ^m, and the inclusion (coarse aggregate) part, denoted by Ω_μ^i. The domain of the RVE is denoted by Ω_μ, and its boundary by $\partial\Omega_\mu$. The definitions of the domains are given by:

$$\Omega_\mu = \Omega_\mu^m \cup \Omega_\mu^i \quad (1)$$

$$\partial\Omega_\mu^m = \partial\Omega_\mu \cup \partial\Omega_\mu^i \quad (2)$$

The symbol ∂ denotes the boundary of the domain. In order to simplify the method, inclusions were not allowed on the boundary of the RVE. Figure 1 shows the geometric characteristics of the multiscale modelling, where the macroscale is continuous, and each material point x in the body with domain Ω is associated with an RVE.

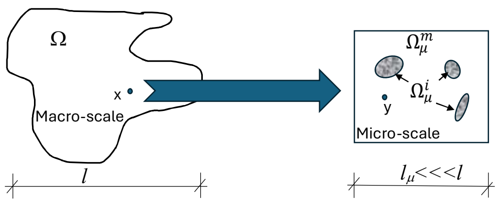

Figure 1. Macro and micro domains

At time instant t, the macroscopic strain tensor $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(x,t)$ and macroscopic stress tensor $\boldsymbol{\sigma} = \boldsymbol{\sigma}(x,t)$ at a specific point x in the macroscale body are calculated as the average volume of their respective microscopic fields $\boldsymbol{\varepsilon}_\mu = \boldsymbol{\varepsilon}_\mu(y,t)$ or $\boldsymbol{\sigma}_\mu = \boldsymbol{\sigma}_\mu(y,t)$ over the RVE associated with the point x at any given time t:

$$\boldsymbol{\varepsilon}(x,t) = \frac{1}{V_\mu}\int_{\Omega_\mu} \boldsymbol{\varepsilon}_\mu(y,t)dV \quad (3)$$

$$\boldsymbol{\sigma}(x,t) = \frac{1}{V_\mu}\int_{\Omega_\mu} \boldsymbol{\sigma}_\mu(y,t)dV \quad (4)$$

where V_μ is the volume of the RVE, Ω_μ is the domain of the RVE, and dV represents the volume element. Equations (3) and (4) describe a homogenisation technique that uses volumetric averaging to obtain macroscopic strain ($\boldsymbol{\varepsilon}$) and stress ($\boldsymbol{\sigma}$) from microscopic counterparts ($\boldsymbol{\varepsilon}_\mu$ and $\boldsymbol{\sigma}_\mu$) over the RVE. The microscopic strain can be expressed in terms of the RVE's microscopic displacement field $\boldsymbol{u}_\mu$ as $\boldsymbol{\varepsilon}_\mu(y,t) = \nabla^S \boldsymbol{u}_\mu(y,t)$, where ∇^S denotes the symmetric gradient. Meanwhile, the microscopic stress can be determined as $\boldsymbol{\sigma}_\mu(y,t) = f_y(\boldsymbol{\varepsilon}_\mu(y,t))$, where f_y is the constitutive functional that can be defined by the Mazars' model [6] when the material starts to present damage process, or by Hooke's law for other cases. The microscopic displacement field is given by:

$$\boldsymbol{u}_\mu(y,t) = \boldsymbol{\varepsilon}(x,t)y + \tilde{\boldsymbol{u}}_\mu(y,t) \quad (5)$$

The component $\boldsymbol{\varepsilon}(x,t)y$ varies linearly with respect to y. The field $\boldsymbol{u}_\mu$ is referred to as the fluctuating displacement field and represents the strain variation of the RVE, which is considered null in this case since uniform microscopic strains $\boldsymbol{\varepsilon}_\mu$ are assumed. Therefore, the microscopic strain field can be decomposed into the following sum:

$$\boldsymbol{\varepsilon}_\mu(y,t) = \boldsymbol{\varepsilon}(x,t) + \tilde{\boldsymbol{\varepsilon}}_\mu(y,t) \quad (6)$$

In this work, the Finite Element Method (FEM) formulation is used to solve the equilibrium problem. The variables and parameters of the RVE are distinct from those of the material in the macrocontinuum, and these characteristics are defined based on a standard RVE that is extrapolated to all RVEs in the macroscopic structure. The solution of an RVE involves calculating the homogenized displacements, internal forces, stresses and constitutive matrix, which are obtained when the convergence of the equilibrium problem is achieved considering an adopted tolerance. However, to solve this equilibrium problem, it is necessary to define the boundary conditions to be imposed on the RVE, and the response obtained may vary depending on the boundary condition adopted.

The choice of the V_μ space must be appropriate according to the kinematic constraints of the RVE. Therefore, the equilibrium of the RVE involves finding a field $\dot{\tilde{\boldsymbol{u}}}_\mu \in V_\mu$ for each instant t given a macroscopic strain tensor ε. With the finite element domain discretization of the RVE, the microscopic equilibrium incremental equation should be valid for a time load increment $\Delta tn = t_{(n+1)} - t_n$. Discretizing h in the domain allows one to find the displacement fluctuation field $\tilde{\boldsymbol{u}}_{\mu(n+1)} = \tilde{\boldsymbol{u}}_{\mu_n} + \Delta\tilde{\boldsymbol{u}}_{\mu_n}$.

$$G_h^{n+1} = \int_{\Omega_\mu^h} B^T f(\boldsymbol{\varepsilon}^{n+1} + \boldsymbol{B}\tilde{\boldsymbol{u}}_\mu^{n+1})dV = 0 \tag{7}$$

where $\boldsymbol{B}$ is the global strain-displacement matrix and Ω_μ^h indicates the discretized domain of the RVE. The described RVE formulation is completed by choosing an appropriate V_μ space, which involves selecting the kinematic constraints to be imposed on the RVE. This leads to different classes of multiscale models and, consequently, to different numerical results. In this work, the Periodic boundary conditions model is adopted.

3 Damage Homogenisation Macroscopic Model

A based damage homogenisation model was proposed in [1]. The proposed model starts from the relationship between stress and strain tensors on the macroscale:

$$\boldsymbol{\sigma}(x) = \boldsymbol{C_h}(x)\boldsymbol{\varepsilon}(x) \tag{8}$$

where $\boldsymbol{\sigma}(x)$ is the stress tensor on the macroscale, the $\boldsymbol{C_h}$ (x) is the homogenised constitutive tensor of the material and $\boldsymbol{\varepsilon}(x)$ is the strain tensor on the macroscale. The homogenised constitutive tensor can present characteristics of an anisotropic or isotropic tensor depending on the imperfections, phase debonding, microcracking and dissipative process that occurred in the microstructure and captured by constitutive models applied in each phase. In your turn, $\boldsymbol{C_h}$ (x) depends on the damage constitutive model used in the mortar and ITZ.

Following Pituba and Fernandes [9], the way that the damage tensor acts in the constitutive tensor can change the characteristics of the $\boldsymbol{C_h}$ for anisotropic ones induced by damage, if the damage tensor is anisotropic and the $\boldsymbol{C_h}$ is initially isotropic. On the other hand, if the damage tensor is isotropic, then when applied in the $\boldsymbol{C_h}$, this last one remains isotropic or anisotropic depends on its initial characteristic. Considering the Continuum Damage Mechanics (MDC) and Pituba and Fernandes [9], the following relation on macroscale is valid:

$$\boldsymbol{C_h}(x) = (\boldsymbol{I} - \boldsymbol{D}(x))\boldsymbol{C}(x) \tag{9}$$

where the ***C(x)*** is the homogenized elastic constitutive tensor of the material, e. g., this tensor represents the elastic properties of the homogenized material when dissipative process is not activated yet. In Equation (9), $\boldsymbol{I}$ is the identity tensor. Considering the concepts of volumetric average and the Equation (4), the relation (9) follows:

$$(\boldsymbol{I} - \boldsymbol{D}(x))\boldsymbol{C}(x) = \frac{\frac{1}{V_\mu}\int_{\Omega_\mu} \boldsymbol{\sigma}_\mu(y)dV}{\boldsymbol{\varepsilon}(x)} \tag{10}$$

Considering the stress-strain relationship for damaged microstructure as follows:

$$\boldsymbol{\sigma}_\mu(y) = (\boldsymbol{I} - \boldsymbol{d}(y))\boldsymbol{C}_y(y)\boldsymbol{\varepsilon}_\mu(y) \tag{11}$$

where $\boldsymbol{C}_y$(y) is the elastic constitutive tensor at point y at the microscale and $\boldsymbol{d}(y)$ is a damage tensor related to the point y at the microscale. Using the Equation (11) in Equation (10) gives:

$$(\boldsymbol{I} - \boldsymbol{D}(x))\boldsymbol{C}(x) = \frac{\frac{1}{V_\mu}\int_{\Omega_\mu} (\boldsymbol{I} - \boldsymbol{d}(y))\boldsymbol{C}_y(y)\boldsymbol{\varepsilon}_\mu(y)dV}{\boldsymbol{\varepsilon}(x)} \tag{12}$$

After mathematical manipulations, it is found:

$$\boldsymbol{D}(x) = \boldsymbol{I} - \frac{\frac{1}{V_\mu}\int_{\Omega_\mu} (\boldsymbol{I} - \boldsymbol{d}(y))\boldsymbol{C}_y(y)\boldsymbol{\varepsilon}_\mu(y)dV}{\boldsymbol{C}(x)\boldsymbol{\varepsilon}(x)} \tag{13}$$

The Equation (13) represents the damage state of the point x of the macrocontinuum arising from the homogenized microstructure through a multiscale approach. It shows that the macroscopic strain state cannot be obtained directly as a volumetric average of the microscopic strain when there are microcracks, as noted by Ren et al. [10].

In this work, it is adopted an isotropic damage behaviour for the damaged phases at microstructure level, therefore $\boldsymbol{d}(y)$ is a scalar damage related to the point y at the microscale ($\boldsymbol{d}(y) = d(y)$) following the Mazars′ damage model described in [6] and it can be assumed that $\boldsymbol{D}(x) = D_H$ and it is called homogenized damage.

On the other hand, in the case of homogeneous material in the RVE, it has only one material in the microstructure that it is the same for the macrostructure (C(x)=Ch(x)), and it has that $\boldsymbol{C_y}$ is constant ($\boldsymbol{C_h}(x) = \boldsymbol{C_y}$), as the damage process has the same state at any point in the RVE and, therefore, the Equation (13) is expressed as follows:

$$\boldsymbol{D}(x) = \boldsymbol{I} - \frac{\frac{1}{V_\mu}\int_{\Omega_\mu} (\boldsymbol{I} - d(y))\boldsymbol{C_y}\boldsymbol{\varepsilon}_\mu(y)dV}{\boldsymbol{C_h}(x)\boldsymbol{\varepsilon}(x)} \tag{14}$$

Besides, the damage process is the same over the RVE leading to a constant value d(y) in any y of the RVE and therefore:

$$\boldsymbol{D}(x) = \boldsymbol{I} - \frac{\frac{1}{V_\mu}\int_{\Omega_\mu} \boldsymbol{\varepsilon}_\mu(y)dV}{\boldsymbol{\varepsilon}(x)} + \frac{\frac{d(y)}{V_\mu}\int_{\Omega_\mu} \boldsymbol{\varepsilon}_\mu(y)dV}{\boldsymbol{\varepsilon}(x)} \tag{15}$$

Considering Equation (3), Equation (15) gives:

$$\boldsymbol{D}_H = d_y \tag{16}$$

Therefore, it is concluded that in the case of homogeneous material, a constitutive model can be used directly in the macrocontinuum.

On the other hand, each phase of the constituent material contributes to the effective properties of the composite. These contributions can be dependent only on the volumetric fraction of each constituent materials. Also, the models for obtaining homogenized elastic properties only allow approximate estimative. There are several models to predict the elastic properties of composite materials that can be classified into analytical and numerical methods. In this work, the multiscale approach is used as a numerical method to obtain the elastic properties. For this, an elastic analysis of the RVE is performed and with the stiffness matrix (Equation 7), it is compared with the expression of the relationship between stress and strain in the plane stress state in the cases discussed in this work in order to obtain the elastic parameters of the material.

4 Numerical Application

In this section, experimental test in concrete specimens performed by Delalibera [11] was used to show the potentialities and limitations of the based damage homogenisation model. It deals with the comparison between stress x strain curves obtained by an experimental uniaxial compression test performed by Delalibera [11] and the one by the based damage homogenisation model. The RVE is assumed to be composed of two kinds of materials: coarse aggregates dispersed in a mortar. Besides, for better discretization of the concrete geometry, an ITZ region has been adopted around the aggregates with adopted stiffness characteristics equal to 50% of the mortar. In this case, the proposed RVE has 2678 finite elements, with 1142 finite elements representing the mortar following the Mazars' damage model [6], 872 finite elements representing coarse aggregates with linear elastic behaviour characteristics and 664 finite elements representing ITZ using Mazars' model. The coarse aggregates volumetric fraction is equal to 39.19% (see [11]) and ITZ volumetric fraction is equal to 9.47%. The geometric arrangement of the aggregates was distributed randomly following [12] and considering the requirements for the concrete given by [11]. Figure 2 shows the RVE adopted.

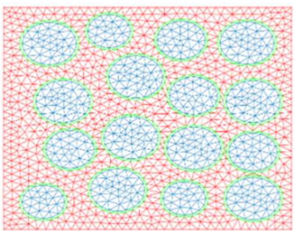

Figure 2. RVE adopted for the analysis

The parameters used for the RVE are the same ones used in the Silva et al. [12]. So, for the Young's Modulus and Poisson's Coefficients for coarse aggregates (E=35 GPa and ν=0.3). Besides, it was adopted the Mazars´ model for the mortar (E=24 GPa, ν=0.2, Reference Deformation (ε_{d0}) = 0.0002, AC=0.3, AT=0.7, BC=200 and BT=2000). For the ITZ, it has assumed a reduction factor of 50% of the Young's Modulus related to the mortar and ν=0.2 following the works [13], [14] and [15].

Figure 3 shows the homogenised stress in x direction versus imposed macroscopic strain in the same direction curves obtained by the multiscale approach using Mazars´ damage model and the based damage homogenisation model compared to the experimental test [11]. In this example, there is a similarity between the responses of the experimental model and the numerical models up to 90% of the peak stress region. The evidenced loss of rigidity is caused by the damage processes in the ITZ and mortar.

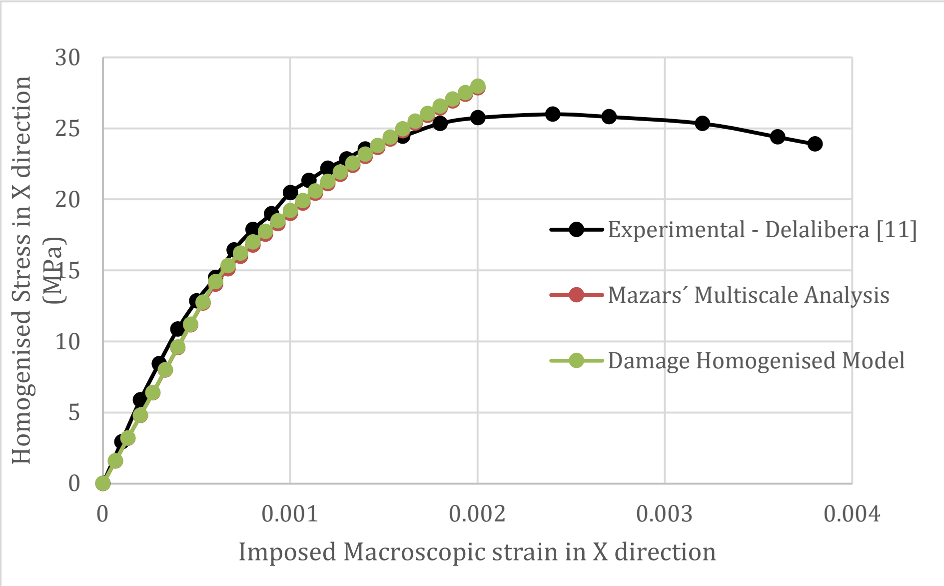

Figure 3. Numerical and experimental responses

In this analysis, the stress values are constrained due to the presence of damaged elements within the RVE approaching the limit (D =1). Near this threshold, the obtained results become inconsistent presenting convergence issue since that the model is not capable to capture the damage localization process and consequent crack nucleation leading to softening regime. The corresponding damage values are depicted in Figure 4.

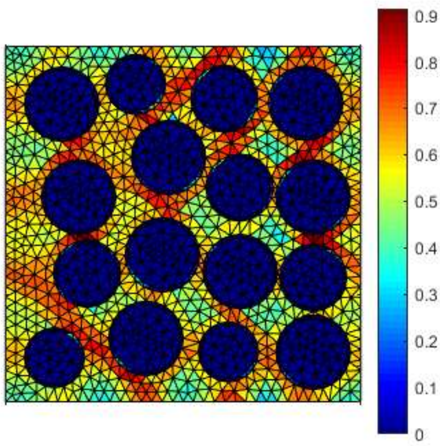

Figure 4. Damage distribution on the RVE and damage scale from 0 to 0.9

5 Conclusions

This paper presents a recently proposed based damage homogenisation model using the concept of RVE. In order to evidence the potentialities and limitations of the homogenised damage model, a numerical analysis has been presented comparing the numerical and experimental response in concrete specimen. The result shows the good agreement between the experimental and the based damage homogenisation model responses. However, it has been evidenced that when damage localization occurs, the homogenised damage model is not capable to reproduces the response presenting softening behaviour.

In future works, the authors intend to expand the homogenised damage model to take into account the crack nucleation using the Strong Descontinuity Approach (SDA) to be able to perform numerical analysis considering softening regime. On the other hand, the homogenised damage macroscopic model will be used in concurrent multiscale analysis with commercial software to model the mechanical behaviour of solids with complex geometries subject to complex loading conditions.

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