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A SIMPLIFIED NUMERICAL METHOD OF PREDICTING SETTLEMENTS ON AXIALLY STRESSED PILES

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Abstract. The aim of this article is to simulate soil-structure interaction on multi-story buildings with deep foundations (building + pile cap) using a simple numerical model via the Winkler model, based on Mindlin's equations, considering an empirical load transfer model proposed by Aoki-Velloso to calculate bearing capacity in order to obtain settlements and coefficients of subgrade reaction between soil and pile. The advantage of applying this methodology is supported by its simplicity and easy computational implementation as well as avoiding a significant computational cost and memory. The use of this simple proposed procedure is due to the future application in the analysis of more complex group, considering the geometric non-linearity of the building and the coupling with a dynamic model associated with fluid mechanics in the numerical wind tunnel solution. The formulation proved to be consistent when comparing the responses obtained with analytical model - Poulos and Davis - and by numerical procedure via the Boundary Element Method.

Keywords: Soil-structure interaction, Piles, Settlements, Winkler Model, Boundary Element Method

1 Introduction

Soil-structure interaction is an area of engineering that still has much to be studied and optimized due to the intrinsic complexities of the set, the difficulty in simulating this system is mainly due to the fact of correctly quantifying the soil properties and predict its behavior when request by different actions. The most common practice adopted to evaluate the behavior of the superstructure under a plenty of actions is to consider that it is under a fixed and rigid base, considerably simplifying the calculations but not in accordance with the actual physical behavior because the soil tends to deform when receiving the loads from it. In last decades, it has been shown that the soil-structure interaction not only change the response in the settlements developed by the soil but also generates a redistribution of efforts in the structural elements (beams and columns), Chameki [1], Poulos [2]. Thus, several schemes has been studied to simulate the soil-structure so that the analysis can satisfactorily represents allowable tension, bearing capacity of the foundation and the developed settlements in order to replace a conventional analysis.

In high rise buildings, bridge or offshore platforms is common for the foundations solution to be based on piles, given the magnitude of the loads and problems related to soft soil in surface layers. Therefore, the most popular methods to describe this type of soil-structure interaction are: analytical models, the Finite Element Method (FEM), Boundary Element Method (BEM) or the equivalent spring model.

The analytical method has currently little use, since the formulations and applications are frequently limited to certain foundation element geometry, type of applied load and soil type (clay or sand). The FEM and BEM methods are used for numerical simulation and covers several applications in computational mechanics. The disadvantage in both method application is the high computational cost and processing time when the domain of the problem requires a three-dimensional treatment as is the case of soil. However, the BEM is more efficient in modelling the soil-structure interaction, because in addition to reducing a three-dimensional problem to a two-dimensional case, it has weighting functions attended to undisturbed distance, Almeida [3]. In Equivalent Spring method, the soil mass is replaced by a set of springs, known as the Winkler model. The advantage in applying this method is its simplicity and easy computational implementation. It is also convenient for integrate the soil-structure interaction into a building based on FEM, since the characteristics such as sparsity and symmetry of the structure stiffness matrix are not lost in the process, resulting in less data storage and faster processing speed, even if the analysis is performed by fewer powerful personal computers. Because of its simplicity, the Equivalent Spring is the method applied on this work to simulate soil-structure interaction of deep foundation.

The present article proposes an alternative numerical method to discretize the soil-pile system of piles submitted to axial loadings, so that it is possible to predict approximately settlements in foundation element by pile by incorporated the Mindlin's equations together with a load transfer model and the Aoki and Velloso [4] bearing capacity method associated with Standard Penetration Test (SPT) for characterizing the soil properties. The quotient between force and settlement results in spring, which is incorporated in the node of each pile finite element that is coupled with the building. The use of this simple proposed procedure to represent the soil-pile group is due to the future application in the analysis of more complex group, considering the geometric non-linearity of the building and the coupling with a dynamic model associated with fluid mechanics in the numerical wind tunnel solution. The results obtained demonstrate good agreement between the analytical model of Poulos and Davis [5] and the numerical model of the BEM and FEM verifying that the equivalent spring model for soil-structure simulation is a feasible path for coupling more complex systems.

2 Numerical model

The total settlement of the foundation must be considered in two important parts: deformation related to the material that constitutes the pile and deformation of the soil. For the present paper, the pile has rigid body behavior without deformation due to the applied load, therefore, only the soil deformation is considered for calculation purposes. Furthermore, a group of pile in the same pile cap has influence in the final settlement of the foundation, one way of taking into account this phenomenon is by the superposition of effects, where the final settlement for a group may be obtained by summing the individual settlement of each pile. The proposed method can be easily adapted to consider this effect, however only the settlement mobilized by a single pile is considered in the present work.

The soil mass is considered as homogeneous, isotropic and semi-infinite space, which is a widely and proper assumption for this type of problem.

2.1 Axial load transfer model

Considering a single pile of length L submitted an axial compressive load P and a linear diagram of load distribution proposed initially by [4], the stresses are divided in two parts: along a unit length (Δ_l) at shaft, when slip occurs between the pile and the soil causing shear (Q_l) and uniform base stresse (r_p) takes to base as shown in Fig. 1.a), in order to obtained a static equivalent system of point loads reaction (P_l) along the shaft surface and base force (P_p). Figure 1.b) shows the scheme for deduction of the bearing capacity (P_r), simplified by the balance of forces represented in Eq. (1).

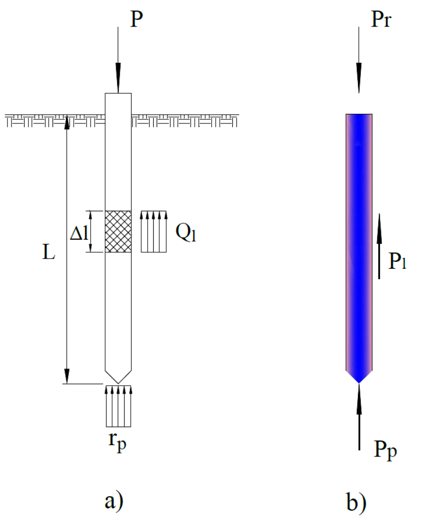

Figure 1. a) stress distribution along the shaft and base; b) bearing capacity of the pile

$$P_r = P_l + P_p. \tag{1}$$

The lateral forces developed along the pile shaft and the base force can be expressed by Eq. (2) and Eq. (3), respectively.

$$P_l = U \sum (Q_l \Delta_l). \tag{2}$$

$$P_p = r_p A_p. \tag{3}$$

Where U is the perimeter of pile shaft, Δ_l is the subdivision of unit segments of length L, A_p is the the cross section area.

The geotechnical variables of the problem (Q_l) and (r_p) are defined from semi-empirical correlations with in situ test results and calibrated with load tests. Therefore, such variables were initially correlated to the Cone Penetration Test (CPT) of static penetration, represented by Eq. (4) and Eq. (5), take to account the lateral friction on the sleeve (f_s) and the base cone resistance (q_c). These variables were obtained according to the semi-empirical load capacity method of [4].

$$Q_l = \frac{f_s}{F_2}. \tag{4}$$

$$r_p = \frac{q_c}{F_1}. \tag{5}$$

The parameters F_1 and F_2 represent the different between pile and cone behavior, which are indicated in Cintra and Aoki [6]. However, with the replacement of CPT by the SPT, it was necessary to procedure a modification in geotechnical variables to correlated them with the blow count, or N-value $(N_{SPT}(z))$, which varies the depth of the soil layer.

$$q_c = KN_{SPT}(z). \tag{6}$$

Such correlation also allowed to express the lateral friction as a function of N_{SPT}.

$$\alpha = \frac{f_s}{q_c}. \tag{7}$$

and

$$f_s = \alpha q_c = \alpha K N_{SPT}(z). \tag{8}$$

The coefficients related to the kind of soil α and K are indicated in [4]. Rewriting Eq. (4) and Eq. (5).

$$Q_l = \frac{\alpha K N_{SPT}(z)}{F_2}. \tag{9}$$

$$r_p = K\frac{N_{SPT}}{F_1}. \tag{10}$$

In Eq. (10) N_{SPT} corresponds to SPT value of the pile support. It is concluded that known the type of soil, by the SPT report, it is possible to obtain the values of lateral forces along the shaft and base force, Fig. 2.

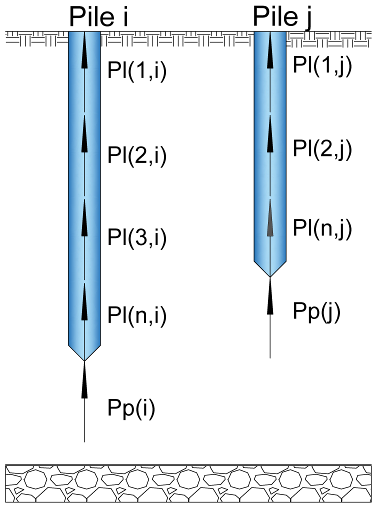

Figure 2. Forces of shaft and base

Experimental observations of [4] showed that: initially, the lateral forces are fully mobilized, occurred for various types and dimensions of piles; the base reaction P_p is mobilized for large displacements depending on the dimension of the pile and occurs just after the lateral resistance has been exhausted. Figure 3 shows the axial transfer mechanism, where the applied load mobilizes all the shaft resistance and part of the base reaction, being lower than the total bearing capacity of the pile.

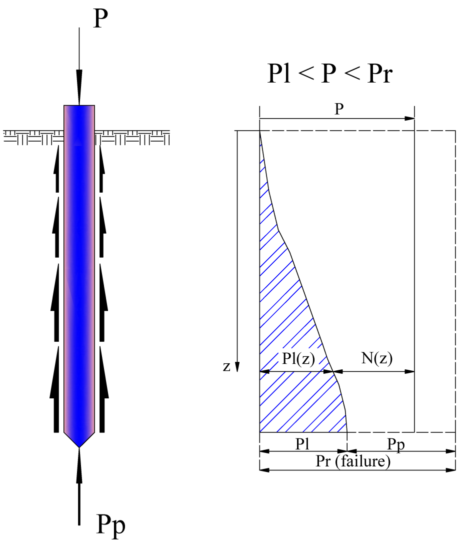

Figure 3. Diagram of transfer load

Once defined the shaft and base forces, it is possible to apply the Mindlin's equations in order to obtain settlements of a single pile or a group of piles in the same pile cap. Figure 4 represents Mindlin's half space with forces applied in a mass of soil.

The settlement r_z at any point $B(x, y, z)$ is given by Eq. (11).

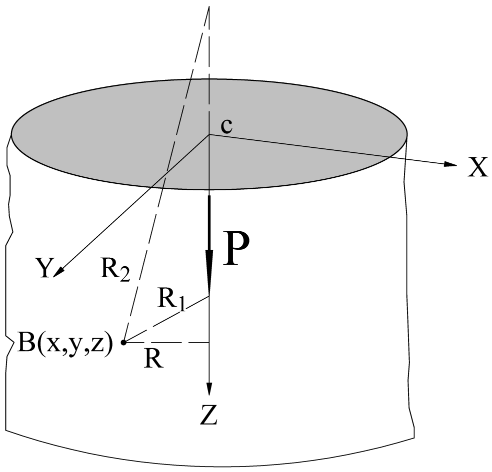

Figure 4. Semi-infinite elastic medium

$$r_z = \frac{P(1+\nu)}{8\pi E_s(1-\nu)}\left[\frac{3-4\nu}{R_1} + \frac{8(1-\nu)^2-(3-4\nu)}{R_2} + \frac{(z-c)^2}{R_1^3} + \frac{(3-4\nu)(z+c)^2-2cz}{R_2^3} + \frac{6cz(z+c)^2}{R_2^5}\right]. \tag{11}$$

Where

$$R_1 = \sqrt{R^2+(z-c)^2}.$$

$$R_2 = \sqrt{R^2+(z+c)^2}.$$

The Eq. (11) also depends on the Young's modulus (E_s) and Poisson's ratio (ν) of soil, the initial elevation of the soil layer (c) and de coordinate (z) on the pile base.

After obtained forces and settlements, it is possible to make use of the Winkler hypothesis to consider the influence of soil near the foundation by means of vertical reaction coefficients. This hypothesis establishes that the applied load is proportional to the mobilized settlement and, therefore, there is no influence between the point of application of this force with the neighbourhood. The coefficients of vertical reaction of soil along the shaft $k_v(z)$ and base k_p of pile are represented by Eq. (12) and Eq. (13), respectively. The representation of the coefficient reaction is showed in Fig. 5.

$$k_v(z) = \frac{P_l(z)}{r_z}. \tag{12}$$

$$k_p = \frac{P_p}{r_z}. \tag{13}$$

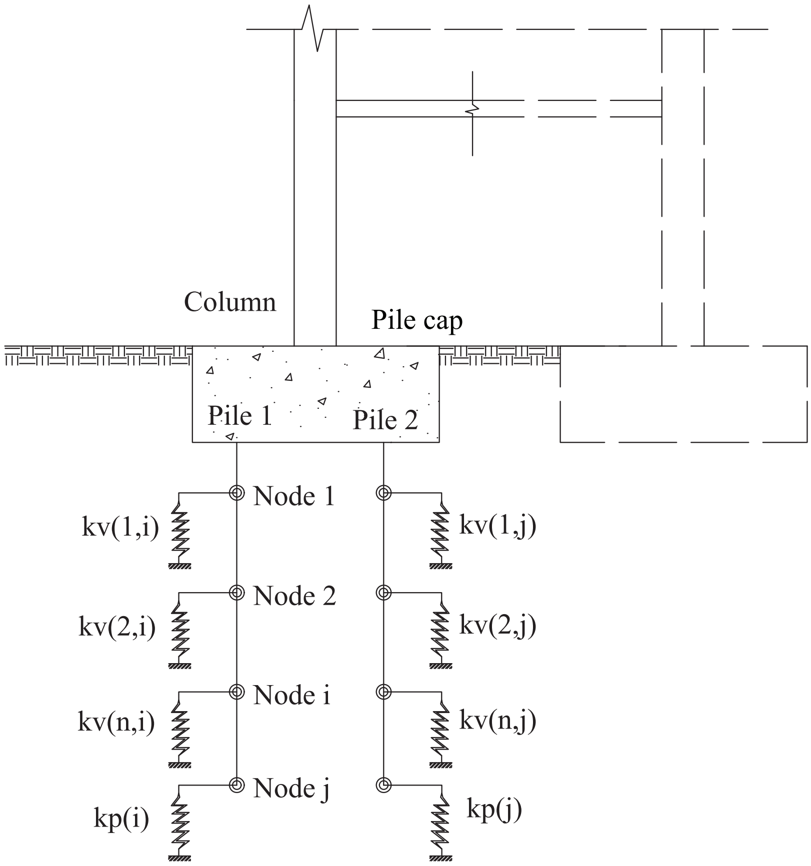

Figure 5. Spring coefficient reaction

The coefficients can be physically interpreted as a group of equivalent springs along the shaft and base of each pile simulating the soil-pile interaction stiffness according to the load transfer distribution diagram, allowing their use in more complex sets as a building defined in FEM, for example. The procedure for calculating settlements is important since they directly influence the spring stiffness coefficient obtained in the proposed method.

3 Numerical analysis

3.1 Single pile compared with analytical model

The results presented in this item compare the settlement response of analytical model proposed by [5] which is based on the theory of elasticity and influence factors to evaluate the behavior of an axially load pile. In this example, we consider a drilled pile with geometrical characteristics and factor F_1 and F_2 obtained in [6] as well as Young's modulus of the pile are shown in Table 1.

Table 1. Pile data

Diameter (cm)	Length (m)	F_1	F_2	E_p (MPa)
70.0	21.0	2.0	4.0	2.8×10^4

For the soil, it is considered a clay layer with constant $N_{SPT} = 8$, the values of K, α, E_s and ν are estimate from SPT correlation obtained in [4] and presented in Table 2.

Table 2. Soil data

Layer	K (MPa)	$\alpha(\%)$	E_s (MPa)	Poisson ratio (ν)
1	0.2	6	40.8	0.5

Thus, a group of loads are applied, which values are lower than bearing capacity and greater than lateral resistance of pile, so that both the shaft and base are mobilized. The settlements obtained for different stages of load are shown in Table 3.

Table 3. Settlements obtained via analytical model [5] and the present model

Load (kN)	Present model (mm)	[5] - analytical (mm)	Relative Difference $(\%)$
1250.0	2.88	2.19	32.0
1400.0	3.25	2.45	33.0
1521.0	3.35	2.66	26.0

The relative difference between the analytical and numerical model responses is due to the different force distribution along the shaft, which in the Fig. 6 can be seen this difference. In consequence of the unknowing form of the distribution of the analytical model is considered by [5], it was not possible to simulate the previously mentioned model [5].

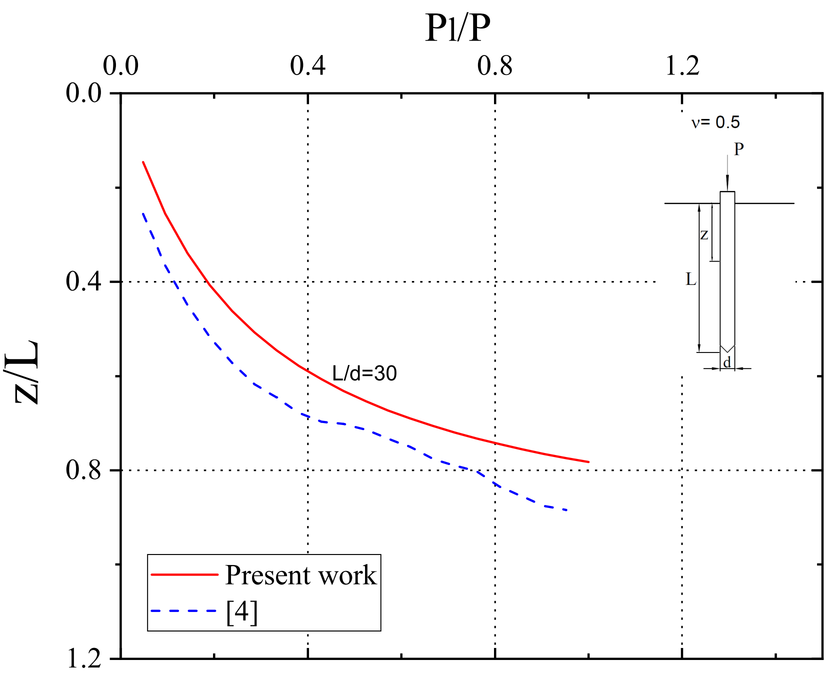

Figure 6. Force distribution along the shaft

3.2 Single pile numerical comparison

The following results compare the present formulation with numerical models proposed by Ottavini [7] using FEM and by [3], that modeled via BEM. Both formulations were analyzed via the simulation of the group soil and the piles with three-dimensional elements. They also evaluated the effects of considering different depths of the rigid base layer and how the parameter $\lambda = E_p/E_s$ can influence the settlements of each case. The values of Table 4 and Table 5 are used to define geometrical and physical characteristics of the pile and soil, respectively.

Table 4. Pile data for the FEM and BEM analysis

Diameter (cm)	Length (m)	F_1	F_2	E_p (MPa)
100.0	40.0	2.0	4.0	2.0×10^2

Table 5. Soil data for the present formulation

Layer	K (MPa)	$\alpha(\%)$	E_s (MPa)	Poisson ratio (ν)
1	0.2	6	-	0.45

The displacement response for all analyses are depicted in the Fig. 7, where [3] and [7] showed them for different relations between layer thickness H, pile length L and λ ratio. The relative difference among the present formulation, results of [3] for H/L = infinity and for [7] with $H/L = 4$ at $\lambda = 2000$ 20% and 22%, respectively.

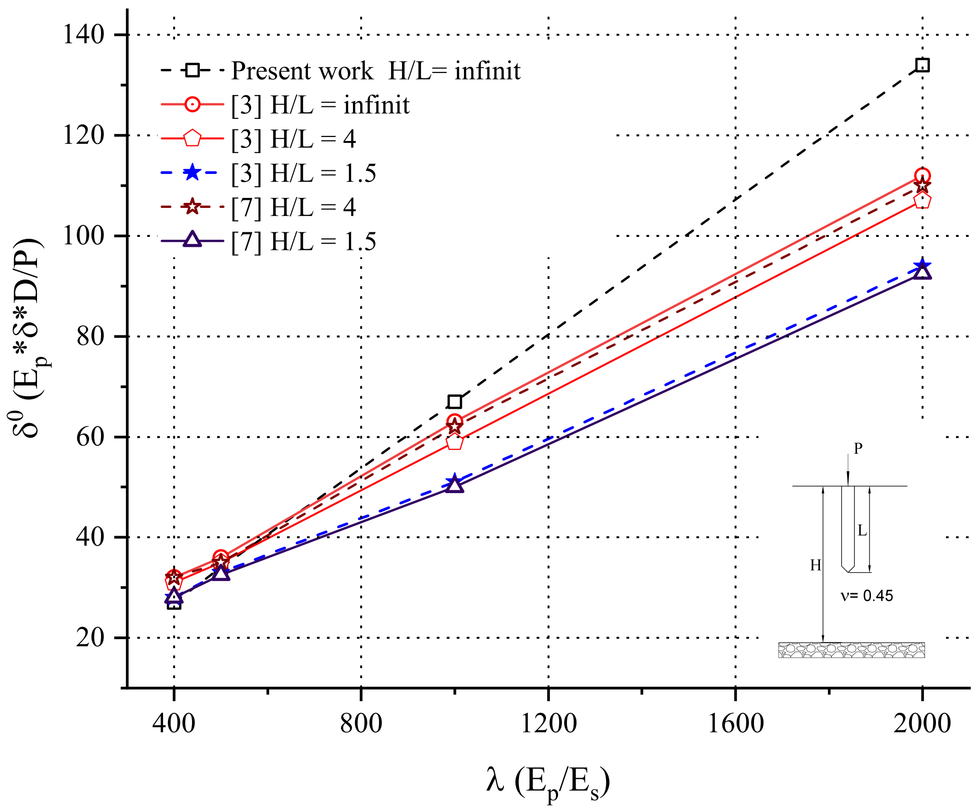

Figure 7. Settlements of soil

To consider the finite layer of soil in the present formulation, must be inserted in the future, the procedure proposed by Streibrenner [8], which considers a difference in settlements between the application points and the bedrock layer. As described in [5], this simplified model generates maximum errors of 15% for bedrock layer at distance of less than 5 meters.

Conclusions

For the classical soil-structure interaction problem, several complex methods are proposed in the literature, such as FEM or BEM, Fourier Series, which show good agreement with semi-analytical responses and form some experimental cases. However, the present paper presented a simple model for analyzing this type of problem. The model with equivalent spring, as already known, presents two major problems: the non-consideration of the continuity of the environment and the difficulty in relating soil parameters with the coefficient itself. However, with the present model, it was shown that the responses are satisfactory when compared to the more complex models, highlighting the use of force transfer process in the shaft and base that is associated with an experimentally developed model calibrated with soil parameters and SPT report.

Thus, in consequence of the responses obtained via the present formulation, and compared with others models, it is concluded that the present Winkler model associated with the Aoki-Velloso process and the Mindlin's equation is applicable and the difference of the present results are acceptable considering the particularities of each model (analytical and numerical). The main advantage of the proposed model is the short processing time and low computational cost. Therefore, it is intended to couple the soil-structure interaction to a building on based FEM that contemplates the fluid-structure interaction leading to high processing time, so that it is possible to simulate approximately, in one model, all the physical interactions that a multi-story building can undergo.

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