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COMPARISON BETWEEN STANDARDIZED APPROXIMATED AND RIGOROUS STRUCTURAL ANALYSIS FOR ESTIMATING 2ND ORDER EFFECTS APPLIED TO STEEL STRUCTURES

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Abstract. Considering the approximated method of structural analysis estimating 2nd order effects applied to steel structures, recommended by the ABNT NBR 8800 (2008) standard, also known as Amplification Coefficients Method, and the constant search for greater structural efficiency and consequently economy, the aim of this study was to compare the numerical responses of steel structures subjected to horizontal and vertical loadings, obtained by approximated standardized analysis and by rigorous geometrically nonlinear analysis. For that, a program was developed to perform all the constituent steps of the Amplification Coefficients Method and to determine internal forces and displacements. The resulting considered models are useful for medium lateral displaceability class in which the 2nd order analysis, as a rule, can be performed by simplified or approximated method. The results from the approximated approach were compared to the results from a more rigorous nonlinear approach aided by the software SAP2000. It was observed that the internal forces obtained by approximation are similar to those with the rigorous one, concerning axial and shearing forces. Differences can be witnessed regarding bending moment results, being the standardized method conservative. Analysis of the results from structures classified with small and large lateral displaceability were also made, grounding standard classification method and following proceedures recommendations. Concluding, the normative methodology is a proper approach to nonlinear analysis in steel structures as long as classification permits.

Keywords: ABNT NBR 8800:2008, geometric nonlinearity, steel structures.

1 Introduction

1.1 Brazilian construction scenery

Iron alloys have been applied as structural building solutions before cement based composites, like reinforced concrete (Santos [1]) but, according to Fleck *et al* [2], "it was steel, made possible in industrial quantities by the Bessemer process of 1856, that gave iron its dominant role in structural design".

In Brazil, the construction sector took a different turn. The use of steel is more often as a component in reinforced concrete or in large scale projects such as industrial plants, bridges, railways and skyscrapers. Because of cultural and economic reasons, the reinforced concrete structural system is more common on construction sites and also as a subject of scientific studies.

Great evolution of concrete structures materials has been witnessed, achieving higher resistance, less cost, at first sight, and standardization of product. But the need for more efficient and slender elements, to fit architectural design styles, makes steel profiles a great alternative to concrete.

Regarding financial costs, steel can be a cheaper alternative in some sort of building projects, mainly because of time reduction of execution and material waste, as shown by Paz *et al* [3] in a particular case.

Motivated by the increasing volume of knowledge generated through academic research, showing the benefits of steel profile as structural system, the material has become more constant in modern engineering projects, demanding structural design tools to achieve even higher efficiency and smooth the resistance to innovation.

1.2 Structural design

During a project, many design phases take place. After architectural aspects of the building have been decided, the structural engineer is responsible for elements disposal, respecting both the architecture features, safety codes as well as economy. Proceeding with the structural design phase, a model of the building is conceived, in a sufficiently representative degree, to be the input on a dimensioning routine.

Between modeling and dimensioning is the analysis phase, in which structure's response to external forces and loads may be determined through different methods. The result of this step is extremely important since the variables calculated, such as internal forces, reactions and displacements, are the parameters for section sizing, with a profile as a result.

Given the fact that cold-drawn steel profile industry adopts a table of fixed section dimensions, for production reasons, small insufficiency of a certain profile can lead to oversizing, that impacts the financial aspect of a project. In some cases, the adequacy of a profile is driven by the analysis method choice and inherent loss of precision and profit are associated.

1.3 Structural analysis

Different considerations are made before the analysis itself regarding material properties and structural equilibrium. If an element presents small enough strain, elastic behavior takes place. When greater strains are allowed, plasticity is accounted as a nonlinear factor.

Another nonlinear factor may be a consequence of structure equilibrium condition. A linear behavior is assumed when internal forces and displacements are calculated in the undeformed structure (1st order), and nonlinear when the equilibrium state is reached in the deformed one (2nd order).

Depending on design codes, limits to simplifications are applied to guarantee a sufficient degree of safety. The standard NBR 8800 (2008) from the Brazilian Association of Technical Standards (*Associação Brasileira de Normas Técnicas* – ABNT) [4] defines a classification method based on 1st order analysis to establish the analysis approach.

In this paper, analysis is considered to be elastic with geometric nonlinearity. Depending on the type of structural analysis, different load-dispacement paths are determined, as Fig. 1 illustrates (Walport *et al* [5]).

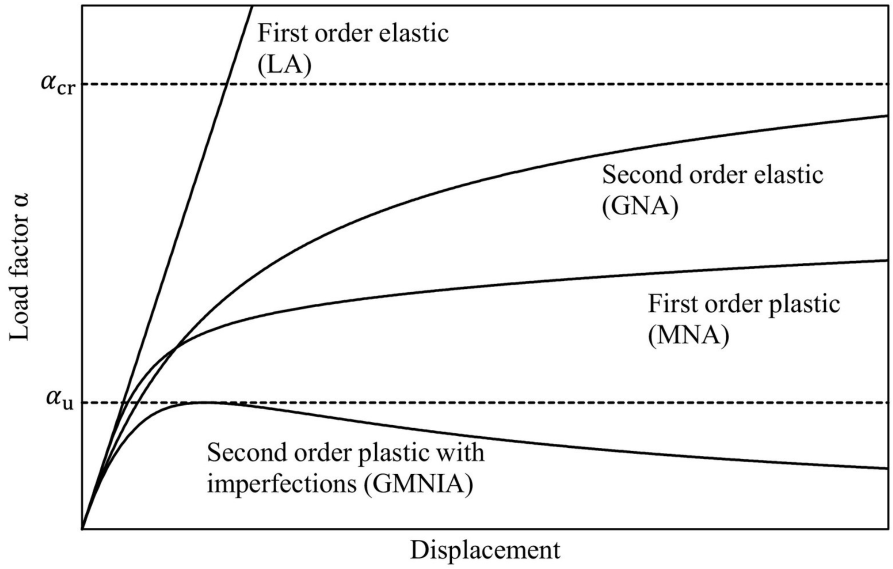


Figure 1. Load-displacement paths associated with different analysis methods. LA – linear analysis; GNA – geometrically nonlinear analysis; MNA – materially nonlinear analysis and GMNIA – geometrically and materially nonlinear analysis with imperfections. (Walport *et al* [5])

1.4 Justificative

As the use of steel profile structures expands in Brazil and the efficient design is a requirement the development of tools capable of automating standard set routines and validation of those methods are needed. The least resistance path to increase the usage of a different system is to quantify advantages and disadvantages associated with innovation and authenticate the efficiency of a method.

1.5 Objective

The main objective of this paper is to compare internal forces results obtained through the approximated 2nd order analysis method provided by the standard NBR 8800 (ABNT [4]) with the obtained with SAP2000, a software capable of evaluating nonlinearity in a rigorous way, regarding geometric nonlinearity.

A specific objective is to automate the standard Amplification Coefficients Method using a computational routine for all of it's steps with little input data being necessary.

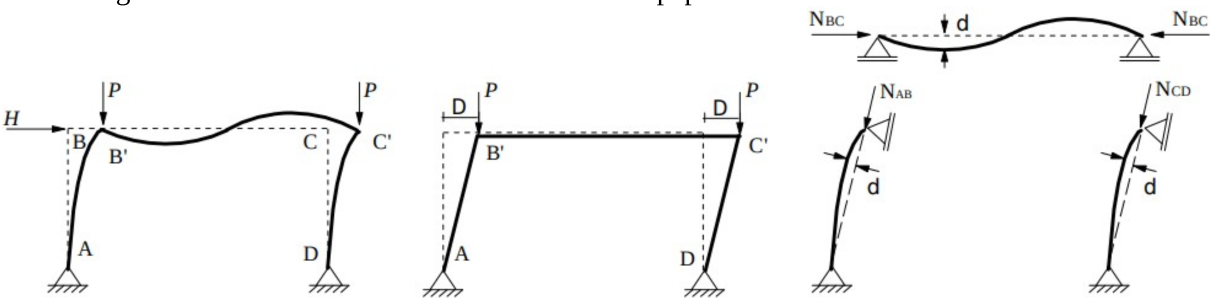
2 Elastic second order analysis

In elastic second order analysis perfectly elastic body is assumed regarding material response to stresses. The structure stiffness due to material properties is considered constant, being nonlinearities caused only by geometrical nonlinearity.

Paraphrasing Souza [1], second order effects in buildings derives from vertical load association with horizontal displacement, generating additional internal forces and enhancing displacements. The horizontal displacement may be a consequence of horizontal external forces or geometric imperfections.

On framed structures second order global (P-Δ) [Fig. 2.a] and local (P-δ) [Fig. 2.b] effects must be considered, along with initial geometric imperfections and joints stiffness behavior, if necessary, when evaluating the stability of the entire structure and it's individual elements.

The ABNT NBR 8800 [4] establishes that analysis methods in which the previous variables are considered, directly or indirectly, due to material or geometric nonlinearities can be used to calculate reactions, internal forces and displacements, such as the Amplification Coefficients Method described on appendix D of the same document, sufficient under certain conditions. The standardized method of calculating 2nd order effects is described further in this paper.



a) Deformed structure b) P-D effect c)P-d effect

Figure 2. Second order effects on plane frames

(Silvestre and Camotim (2007, *apud* RIBEIRO [6], 2016, p. 7))

The 2nd order effects in a structure might be analyzed employing three different classes of methods, rigorous, approximated or simplified as long as standard requirements are met.

Rigorous methods rely on iterative and incremental procedures. Iteration is necessary to achieve equilibrium for a given load. The use of external forces incremental steps is a way to represent the equilibrium path.

In some simplified methods, the nonlinearity is divided in a sequence of linear events, and equilibrium is evaluated through the strain, Ormonte (*apud* SOUZA [1]). Simplified methods, *e.g.* Alpha parameter and Gamma-Z coefficient, are widely used in concrete structures design, as it is presented in the standard NBR 6118 from ABNT [7].

Approximation methods are defined by the amplification of results obtained through linear 1st order analysis by coefficients, these coefficients are also determined through 1st order analysis. The method differs from those presented before due to equilibrium stage. In approximations, the equilibrium occurs with undeformed structure. The ABNT [4] code for steel structures design describes the Amplification Coefficients Method (also known as B_1-B_2 method), an approximated method, main study of this paper.

3 Aspects of the NBR 8800 standard (ABNT [4])

3.1 Sensitivity to horizontal displacement classification

Building structures may be classified among three different magnitude of displaceability, small, medium or large. To define in which class a structure fits, a parameter B_2 is calculated to each floor. The coefficient is an approximated way to represent the relation between second and first order global displacements and is given by Eq. 1.

$$B_2 = \frac{1}{1 - \frac{1}{R_m} \frac{\Delta_h}{h} \frac{\sum N_{Sd}}{\sum H_{Sd}}} \quad (1)$$

In which:

R_m: adjustment coefficient equals to 0,85 where the horizontal forces are resisted only by their elements stiffness on framed structures and 1 on remaining conditions;

Δ_h: relative displacement between superior and inferior levels of each floor obtained with 1st order analysis;

h: height of vertical element (floor height);

ΣN_{Sd}: sum of axial design forces on vertical elements (columns) on floor level;

ΣH_{Sd}: shear force at the floor level due to horizontal design forces.

If the B_2 parameter is less than or equal to 1.1 in every floor level, the structure is classified with small displaceability. In this case 2nd order effects may be ignored as long as no element is requested by axial force greater than 50% of section yield limit, the imperfections are accounted in the analysis and local second order effects are considered when calculating internal forces on each element.

If B_2 is less than or equal to 1.4, but greater than 1.1, it is classified as medium displaceability. In this case nonlinearity effects must be considered, at least, geometrically. In this cases methods like B_1-B_2 is applicable. Considerations about material and geometry imperfections must be accounted, at least on dimensioning phase, and B_2 have to be recalculated with 20% reduction of structure stiffness (0.8 x E).

When B_2 is greater than 1.4, simplified and approximated analysis methods are not sufficient anymore, so rigorous analysis must take place, using iterative and incremental (optional) computational tools. Material nonlinearity must also be accounted.

3.2 Amplification Coefficients Method

If approximated analysis methods apply, due to classification of displaceability ($B_2 \leq 1.4$), Amplification Coefficients Method can be used to perform 2nd order analysis. The method consists in approaching the nonlinear problem as a sum of amplified results determined by linear analysis on auxiliary structures, the “Nt structure” (no translation structure – Fig. 3.b)) and the “Lt structure” (lateral translation – Fig 3.c)). The first one is acquired by adding virtual lateral displacement restraints to each floor of the original model (Fig. 3.a)) and, after reaction forces in these virtual restraints are determined by linear analysis, the Lt structure can be obtained. The Lt structure consists of the original structure, unloaded, with the horizontal reaction forces from virtual restraints in “Nt structure” analysis applied in the respective joint, but in opposite direction.

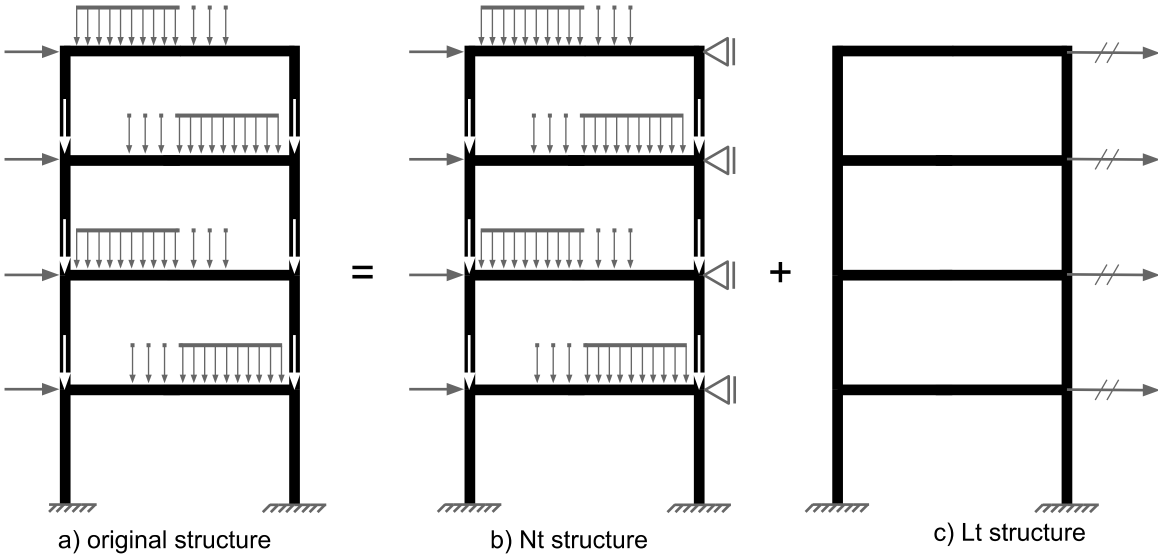


Figure 3. Generic structure model concept for Amplification Coefficients Method

The first order analysis results of bending moments and axial forces are, then, amplified by coefficients, given by Eqs. (2) and (3), to account second order effects on these structure internal forces. Shear forces are considered to be the same as in linear analysis, in spite of second order effects (Eq. 4).

$$M_{Sd} = B_1 M_{Nt} + B_2 M_{Lt} \tag{2}$$

$$N_{Sd} = N_{Nt} + B_2 N_{Lt} \tag{3}$$

$$V_{Sd} = V_{Nt} + V_{Lt} \tag{4}$$

In which:
B_1 and B_2 : are amplification coefficients;
M_{Nt}, N_{Nt} and V_{nt}: design bending moment, axial and shear forces, respectively, obtained by elastic first order analysis with virtual joint restraints, on each floor level of the structure, to lateral displacement (Fig. 3.b);
M_{Lt}, N_{Lt} and V_{lt}: design bending moment, axial and shear forces, respectively, obtained by elastic first order analysis regarding, only, horizontal displacement effects of structure joints (effect of virtual restraints to horizontal displacement reaction force applied to the same joint) (Fig. 3.c).

As shown in Eqs. (2) and (3), the respective internal forces results are multiplied by coefficients B_1 and B_2. The B_2 parameter amplifies global effects of second order and is calculated as in Eq. (1) presented in **3.2**. The B_1 parameter, obtained by Eq. (5) for every element, amplifies results obtained through the Lt structure (Fig. 3.c), leading to the conclusion that it acts on local second order effects.

$$B_1 = \frac{C_m}{1 - \frac{N_{Sd1}}{N_e}} \geq 1 \tag{5}$$

In which:
N_{sd1}: internal axial force (compression) obtained by first order analysis on the element;

N_e: axial force needed to cause elastic buckling. It is calculated using the real bar length and accounting initial material imperfections (stiffness reduction to 80% of original) if $B_2 > 1.1$;
C_m: coefficient of moment equivalence (Zugno [8]). If the element is requested transversally, C_m=1.0. If not, $C_m = 0.6 – 0.4 (M_1 / M_2)$, being M_1 / M_2 the relation between maximum and minimum bending moments (absolute value) obtained in first order analysis of the Nt structure. If the element is subjected to simple curvature, $(M_1 / M_2) < 0$, otherwise, for inverse curvature, $(M_1 / M_2) > 0$ according to ABNT [4]. In this paper the C_m was calculated as $0.6 + 0.4 (M_1 / M_2)$. This proposed expression considers the local coordinates system instead of the code expression which uses global coordinates.

To perform every step of the method, a computational routine was implemented in Python, and second order analysis made possible with very little input data. The routine is limited to modeling structures with floors and spams of the same size where the quantity of floors and the quantity of spams are set as parameters. All beams, as well as all columns, present the same section along the whole structure. Distributed loads can only be applied to beams, vertical forces are applied to every joint above the terrain level, and horizontal forces to joints, also above terrain level, on the left side of the structure.

4 Computational routine

The computional routine is represented as flow charts in Fig. 4 and Fig. 5. First order analysis were made using Direct Stiffness Method.

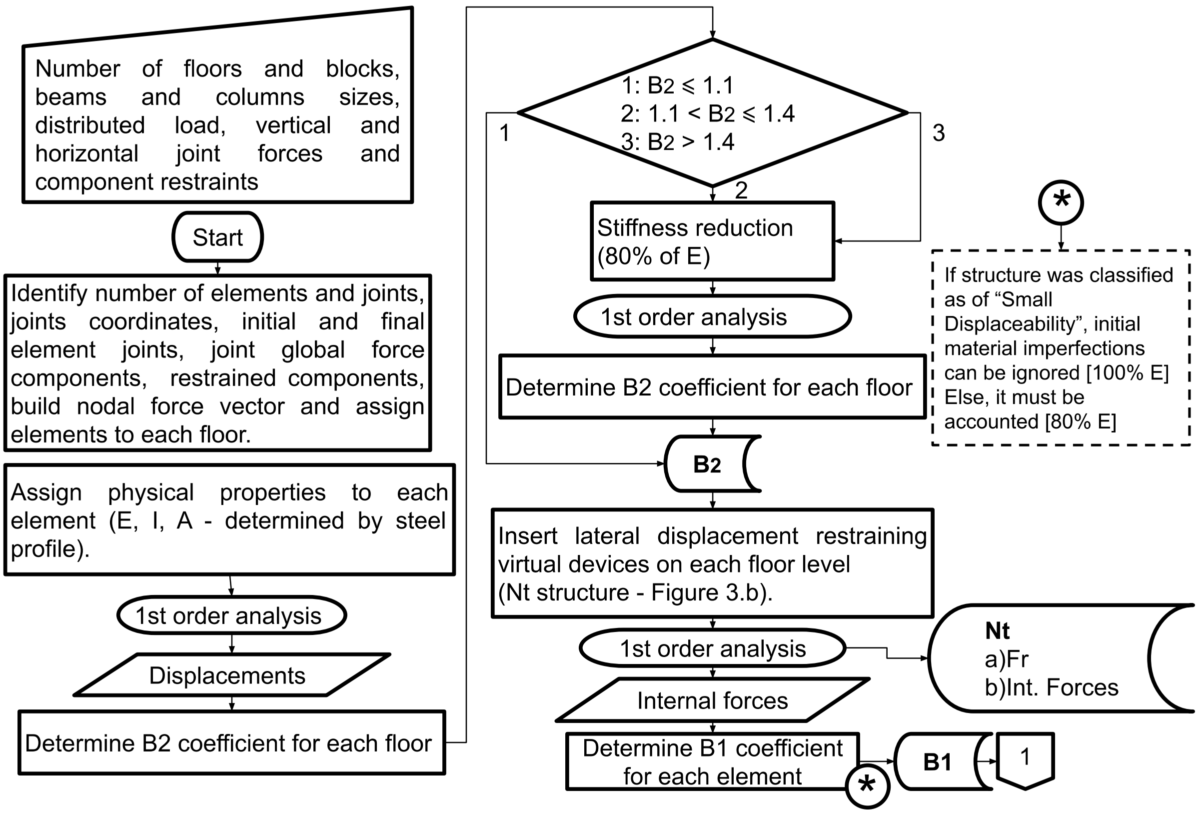


Figure 4. First part of computational routine flow chart

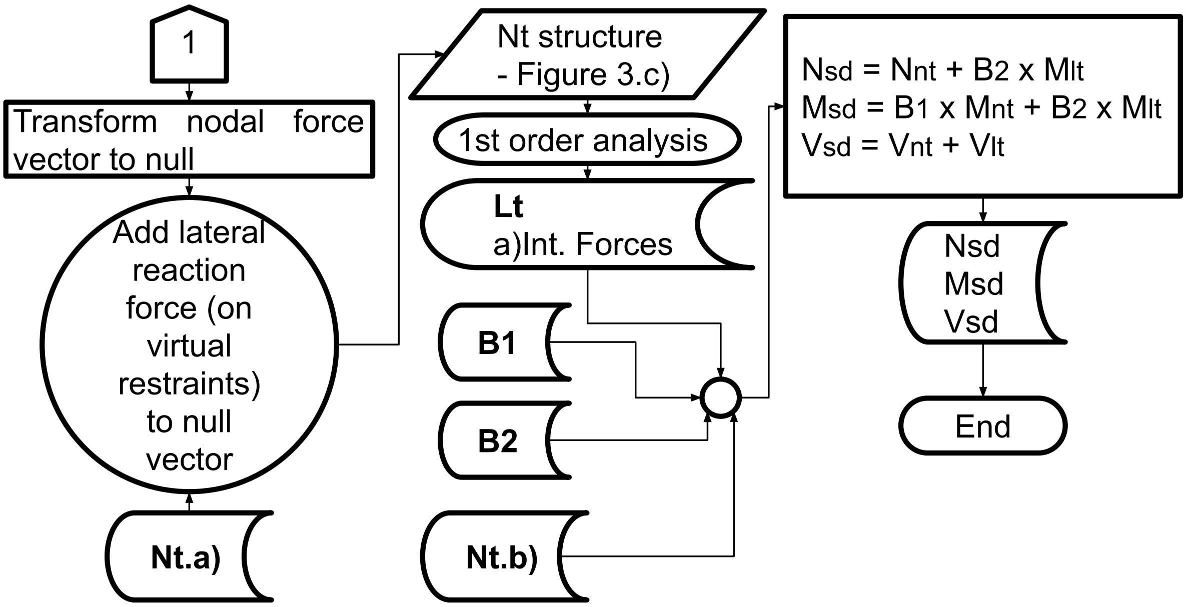


Figure 5. Second part of computational routine flow chart

5 Case studies

Three different models were studied in this paper, one of each displaceability class. Elements and joints were numbered from bottom to top and left to right.

5.1 Small displaceability model

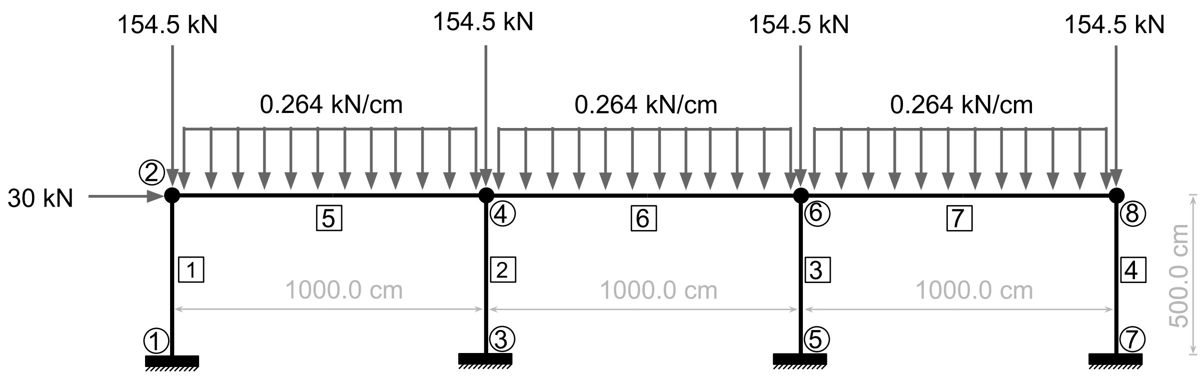


Figure 6. Small diplaceability model

Physical properties of this model (Fig. 6), such as elastic modulus, section area and moment of inertia, and the amplification coefficients of each floor (B_2) and element (B_1) are presented in Table 1.

Table 1. Small displaceability model characteristics

Floor	Elements	Profile	A (cm²)	I (cm⁴)	E (kN/cm²)	B1	B2	Class
1st	1	HP 250 x 62.0	79.6	8728	20000	1	1.087	Small Displaceability
	2	HP 250 x 62.0	79.6	8728	20000	1		
	3	HP 250 x 62.0	79.6	8728	20000	1		
	4	HP 250 x 62.0	79.6	8728	20000	1		
	5	W 410 x 38.8	50.3	12777	20000	1.025		
	6	W 410 x 38.8	50.3	12777	20000	1.019		
	7	W 410 x 38.8	50.3	12777	20000	1.018		

Profiles used in the model are avaible on a profile gauge table from Gerdau [9].

5.2 Medium displaceability model

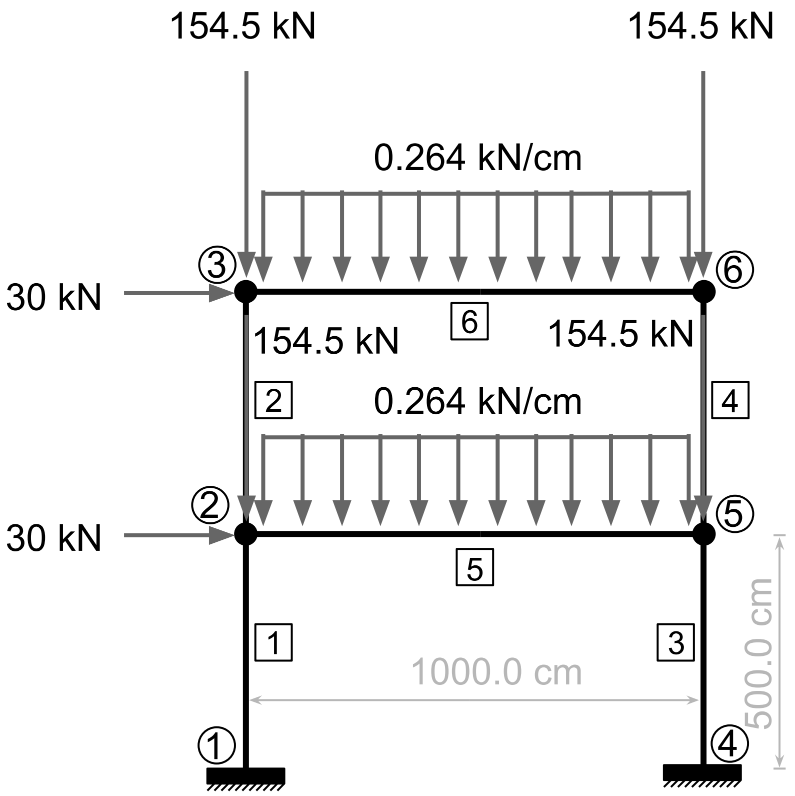


Figure 7. Medium displaceability model

Physical properties of this model (Fig. 7), such as elastic modulus, section area and inertia moment, and the amplification coefficients of each floor (B_2) and element (B_1) are presented in Table 2.

Table 2. Medium displaceability model characteristics

Floor	Elements	Profile	A (cm²)	I (cm⁴)	E (kN/cm²)	B1	B2	Class	B2 (80% of E)
1st	1	HP 250 x 62.0	79.6	8728	20000	1	1.165	Medium Displaceability	1.216
	3	HP 250 x 62.0	79.6	8728	20000	1			
	5	W 410 x 38.8	50.3	12777	20000	1.013			
2nd	2	HP 250 x 62.0	79.6	8728	20000	1	1.161		1.210
	4	HP 250 x 62.0	79.6	8728	20000	1			
	6	W 410 x 38.8	50.3	12777	20000	1.039			

5.3 Large displaceability model

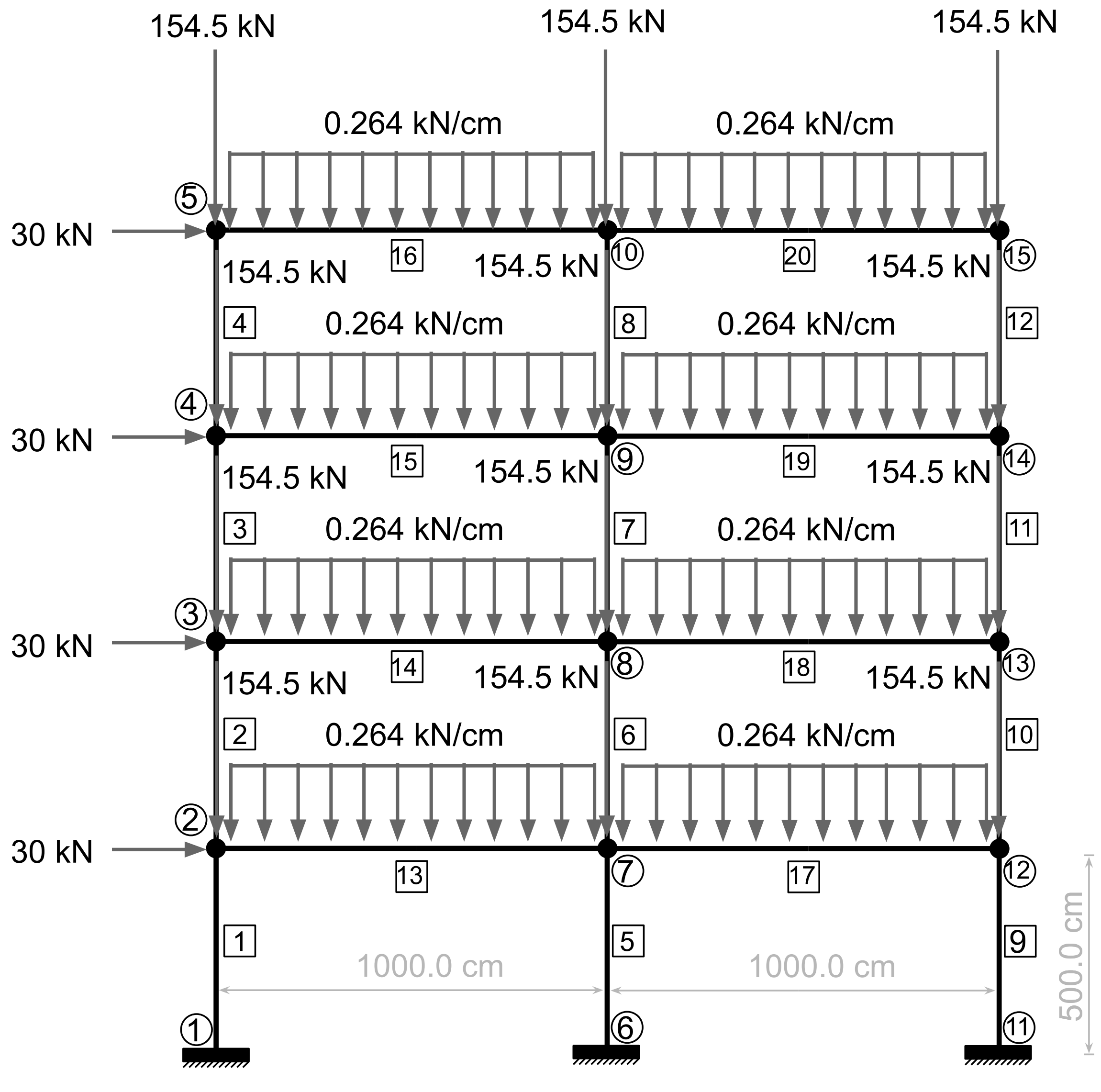


Figure 8. Large displaceability model

Physical properties of this model (Fig. 8), such as elastic modulus, section area and inertia moment, and the amplification coefficients of each floor (B_2) and element (B_1) are presented in Table 3.

Table 3. Large displaceability model characteristics

Floor	Elements	Profile	A (cm²)	I (cm⁴)	E (kN/cm²)	B1	B2	Class	B2 (80% of E)
1st	1	HP 250 x 62.0	79.6	8728	20000	1	1.497	Big Displaceability	1.709
	5	HP 250 x 62.0	79.6	8728	20000	1			
	9	HP 250 x 62.0	79.6	8728	20000	1			
	13	W 410 x 38.8	50.3	12777	20000	1			
	17	W 410 x 38.8	50.3	12777	20000	1.003			
2nd	2	HP 250 x 62.0	79.6	8728	20000	1	1.692		2.047
	6	HP 250 x 62.0	79.6	8728	20000	1			
	10	HP 250 x 62.0	79.6	8728	20000	1			
	14	W 410 x 38.8	50.3	12777	20000	1.014			
	18	W 410 x 38.8	50.3	12777	20000	1.006			
3rd	3	HP 250 x 62.0	79.6	8728	20000	1	1.406		1.566
	7	HP 250 x 62.0	79.6	8728	20000	1			
	11	HP 250 x 62.0	79.6	8728	20000	1			
	15	W 410 x 38.8	50.3	12777	20000	1.003			
	19	W 410 x 38.8	50.3	12777	20000	1.004			
4th	4	HP 250 x 62.0	79.6	8728	20000	1	1.193		1.253
	8	HP 250 x 62.0	79.6	8728	20000	1			
	12	HP 250 x 62.0	79.6	8728	20000	1			
	16	W 410 x 38.8	50.3	12777	20000	1.037			
	20	W 410 x 38.8	50.3	12777	20000	1.029			

6 Results

The internal forces results for each analysis method (1^{st} order, approximated and rigorous 2^{nd} order) are presented in Tables 4, 6 and 8 for each example of displaceability. The shear force is not calculated in $B_1 – B_2$ method since it is the same obtained by 1^{st} order analysis (Eq. 4 shows that coefficients are not applied). The comparison between results from $B_1 – B_2$ and 1^{st} order analysis methods and between results from SAP 2000 and $B_1 – B_2$ method are show in Tables 5, 7 and 9 for each example of displaceability.

The SAP 2000® software (CSi) were used to perform a more rigorous analysis on plane frame structures. Analysis of second order were made with the P-Delta method in one stage. The load cases were the same as in the models analyzed with the computational routine implemented.

To illustrate the difference among the methods, graphics have been made taking 1^{st} order analysis as parameter. Since the approximated method is indicated to structures in medium displaceability class, all three internal forces are represented in Figs. 9, 10 and 11, axial force, shear force and bending moment, respectively, for each element. To verify the $B_1 – B_2$ method in other classes, of small and large displaceabilities, graphics of bending moment comparison are also presented (Figs. 12 and 13). The linear parameter is highlighted in each graphic by a red continuous line.

Table 4. Internal forces results obtained with different analysis methods in small displaceability model

Element	Joint	1st Order Analysis – Original Structure			2nd Order Analysis (B1 – B2 Method)		2nd Order Analysis (SAP 2000)		
		N (kN)	V (kN)	M (kN.cm)	N (kN)	M (kN.cm)	N (kN)	V(kN)	M (kN.cm)
1	1	-272.035	-32.326	4379.132	-271.864	4235.907	-271.834	-32.102	4162.430
	2	-272.035	-32.326	-11783.858	-271.864	-11690.495	-271.834	-32.102	-11667.550
2	3	-431.290	15.548	-3513.376	-431.337	-3680.462	-431.382	15.441	-3660.070
	4	-431.290	15.548	4260.713	-431.337	4399.825	-431.382	15.441	4383.860
3	5	-429.897	1.228	-1061.755	-429.840	-1232.953	-429.878	1.229	-1218.830
	6	-429.897	1.228	-447.727	-429.840	-304.909	-429.878	1.229	-302.560
4	7	-276.778	45.550	-8385.473	-276.958	-8538.476	-276.906	45.420	-8450.320
	8	-276.778	45.550	14389.401	-276.958	14488.690	-276.906	45.420	14440.920
5	2	-62.326	117.535	-11783.858	-61.853	-12016.333	-62.100	117.340	-11667.650
	4	-62.326	-146.465	-26249.202	-61.853	-26969.076	-62.100	-146.666	-26330.310
6	4	-46.778	130.325	-21988.490	-45.692	-22355.701	-46.655	130.216	-21946.350
	6	-46.778	-133.675	-23663.399	-45.692	-24159.142	-46.655	-133.784	-23730.380
7	6	-45.550	141.722	-24111.125	-43.836	-24490.954	-45.422	141.594	-24032.960
	8	-45.550	-122.278	-14389.401	-43.836	-14732.309	-45.422	-122.406	-14440.940

Table 5. Results comparison for small displaceability model

Element	Joint	$[\frac{(B_1-B_2)}{1_{st}\,order}-1]\times 100\%$		$[\frac{SAP\,2000}{(B_1-B_2)}-1]\times 100\%$		
		N	M	N	V	M
1	1	-0.06%	-3.27%	-0.01%	-0.69%	-1.73%
	2	-0.06%	-0.79%	-0.01%	-0.69%	-0.20%
2	3	0.01%	4.76%	0.01%	-0.69%	-0.55%
	4	0.01%	3.27%	0.01%	-0.69%	-0.36%
3	5	-0.01%	16.12%	0.01%	0.08%	-1.15%
	6	-0.01%	-31.90%	0.01%	0.08%	-0.77%
4	7	0.07%	1.82%	-0.02%	-0.28%	-1.03%
	8	0.07%	0.69%	-0.02%	-0.28%	-0.33%
5	2	-0.76%	1.97%	0.40%	-0.17%	-2.90%
	4	-0.76%	2.74%	0.40%	0.14%	-2.37%
6	4	-2.32%	1.67%	2.11%	-0.08%	-1.83%
	6	-2.32%	2.09%	2.11%	0.08%	-1.77%
7	6	-3.76%	1.58%	3.62%	-0.09%	-1.87%
	8	-3.76%	2.38%	3.62%	0.10%	-1.98%

Table 6. Internal forces results obtained with different analysis methods in medium displaceability model

Element	Joint	1st Order Analysis			2nd Order Analysis (B1 – B2 Method)		2nd Order Analysis (SAP 2000)		
		Original Structure							
		N (kN)	V (kN)	M (kN.cm)	N (kN)	M (kN.cm)	N (kN)	V(kN)	M (kN.cm)
1	1	-546.860	10.509	-6163.581	-541.283	-8202.442	-543.027	10.820	-7465.260
	2	-546.860	10.509	-908.871	-541.283	303.672	-543.027	10.820	39.160
2	2	-277.260	-45.159	10289.407	-275.363	9707.408	-275.968	-45.077	9872.520
	3	-277.260	-45.159	-12290.146	-275.363	-11340.969	-275.968	-45.077	11639.490
3	4	-599.140	49.491	-12696.073	-604.717	-14745.817	-602.973	49.015	-13984.260
	5	-599.140	49.491	12049.217	-604.717	13269.567	-602.973	49.015	12864.850
4	5	-295.740	75.159	-16049.260	-297.637	-16625.859	-297.032	75.008	-16411.020
	6	-295.740	75.159	21530.293	-297.637	22478.450	-297.032	75.008	22167.380
5	2	25.669	115.100	-11198.278	29.020	-9639.385	25.939	112.559	-9833.570
	5	25.669	-148.900	-28098.477	29.020	-30165.859	25.939	-151.441	-29275.300
6	3	-75.159	122.760	-12290.146	-72.097	-11680.008	-75.045	121.469	-11638.860
	6	-75.159	-141.240	-21530.293	-72.097	-23136.568	-75.045	-142.513	-22168.340

Table 7. Results comparison for medium displaceability model

Element	Joint	$[\frac{(B_1-B_2)}{1_{st}\,order}-1]\times 100\%$		$[\frac{SAP2000}{(B_1-B_2)}-1]\times 100\%$		
		N	M	N	V	M
1	1	-1.02%	33.08%	0.32%	2.96%	-8.99%
	2	-1.02%	-66.59%	0.32%	2.96%	-87.10%
2	2	-0.68%	-5.66%	0.22%	-0.18%	1.70%
	3	-0.68%	-7.72%	0.22%	-0.18%	2.63%
3	4	0.93%	16.14%	-0.29%	-0.96%	-5.16%
	5	0.93%	10.13%	-0.29%	-0.96%	-3.05%
4	5	0.64%	3.59%	-0.20%	-0.20%	-1.29%
	6	0.64%	4.40%	-0.20%	-0.20%	-1.38%
5	2	13.06%	-13.92%	-10.62%	-2.21%	2.01%
	5	13.06%	7.36%	-10.62%	1.71%	-2.95%
6	3	-4.07%	-4.96%	4.09%	-1.05%	-0.35%
	6	-4.07%	7.46%	4.09%	0.90%	-4.18%

Table 8. Internal forces results obtained with differente analysis method in large displaceability model

Element	Joint	1st Order Analysis Original Structure			2nd Order Analysis (B1 – B2 Method)		2nd Order Analysis (SAP 2000)		
		N (kN)	V (kN)	M (kN.cm)	N (kN)	M (kN.cm)	N (kN)	V(kN)	M (kN.cm)
1	1	-1061.429	14.936	-8447.535	-1021.972	-16898.702	-1035.987	17.308	-13559.560
	2	-1061.429	14.936	-979.437	-1021.972	3329.715	-1035.987	17.308	2132.930
2	2	-803.770	-15.205	4614.003	-768.029	-1003.288	-789.851	-14.765	-1486.430
	3	-803.770	-15.205	-2988.495	-768.029	3720.902	-789.851	-14.765	-822.470
3	3	-541.022	-17.754	-12696.073	-532.006	3784.930	-535.886	-17.831	4717.050
	4	-541.022	-17.754	-3306.015	-532.006	-481.143	-535.886	-17.831	-1229.260
4	4	-270.137	-42.265	9675.710	-268.951	9444.621	-268.890	-42.480	9685.050
	5	-270.137	-42.265	-11456.550	-268.951	-10815.537	-268.890	-42.480	-10812.950
5	6	-1731.025	47.493	-13873.795	-1731.001	-23679.211	-1732.287	45.202	-19644.370
	7	-1731.025	47.493	9872.495	-1731.001	16853.370	-1732.287	45.202	14729.590
6	7	-1298.135	42.545	-10398.305	-1298.150	-21219.439	-1298.904	43.380	-15978.400
	8	-1298.135	42.545	10874.014	-1298.150	22175.568	-1298.904	43.380	16721.510
7	8	-868.499	28.116	-6380.440	-868.504	-10039.797	-868.994	29.081	-8669.860
	9	-868.499	28.116	7677.381	-868.504	12081.273	-868.994	29.081	10704.700
8	9	-441.465	15.196	-3116.405	-441.468	-3857.425	-441.713	15.978	-3660.290
	10	-441.465	15.196	4481.669	-441.468	5571.789	-441.713	15.978	5517.670
9	11	-1173.547	57.571	-15560.633	-1213.026	-24084.890	-1197.726	57.308	-20761.580
	12	-1173.547	57.571	13224.979	-1213.026	17591.689	-1197.726	57.308	16037.350
10	12	-872.595	62.660	-15391.429	-908.321	-20957.234	-885.745	61.211	-18453.430
	13	-872.595	62.660	15938.751	-908.321	22621.776	-885.745	61.211	19645.370
11	13	-573.479	49.638	-11733.146	-582.490	-13531.529	-578.121	48.700	-12506.140
	14	-573.479	49.638	13085.844	-582.490	15921.227	-578.121	48.700	15072.050
12	14	-279.898	57.068	-11798.590	-281.081	-12025.509	-280.896	56.496	-11699.210
	15	-279.898	57.068	16735.596	-281.081	17374.353	-280.896	56.496	17286.440
13	2	0.141	103.158	-5593.440	8.972	2517.349	2.073	91.636	645.960
	7	0.141	-160.842	-34435.215	8.972	-41584.569	2.073	-172.364	-36418.190
14	3	-27.451	108.248	-8559.292	-19.866	1205.561	-26.903	99.465	3893.900
	8	-27.451	-155.752	-32310.963	-19.866	-41675.696	-26.903	-164.535	-36418.730
15	4	-5.489	116.385	-12981.725	-0.159	-9692.790	-5.340	112.495	-10913.830
	9	-5.489	-147.615	-28596.946	-0.159	-31688.652	-5.340	-151.505	-30416.070
16	5	-72.265	115.637	-11456.550	-70.520	-11335.048	-72.479	114.390	-10812.880
	10	-72.265	-148.363	-27819.315	-70.520	-29318.364	-72.479	-149.610	-28382.860
17	7	5.089	117.548	-14164.416	17.539	-7066.794	3.899	106.518	-9009.120
	12	5.089	-146.452	-28616.408	17.539	-36794.425	3.899	-157.482	-34489.960
18	8	-13.022	119.385	-15056.509	8.959	-6173.856	-12.549	110.876	-11024.840
	13	-13.022	-144.615	-27671.897	8.959	-37801.199	-12.549	-153.124	-32152.400
19	9	7.430	124.919	-17803.160	20.716	-14859.860	7.783	121.275	-16050.250
	14	7.430	-139.081	-24884.434	20.716	-28296.243	7.783	-142.725	-26772.800
20	10	-57.068	138.602	-23337.646	-51.663	-23534.983	-56.498	137.604	-22865.060
	15	-57.068	-125.398	-16735.596	-51.663	-17788.135	-56.498	-126.396	-17286.520

Table 9. Results comparison for large displaceability model

Element	Joint	$[\frac{(B_1 - B_2)}{1_{st}\, order} - 1] \times 100\%$		$[\frac{SAP\,2000}{(B_1 - B_2)} - 1] \times 100\%$		
		N	M	N	V	M
1	1	-3.72%	100.04%	1.37%	15.88%	-19.76%
	2	-3.72%	239.96%	1.37%	15.88%	-35.94%
2	2	-4.45%	-78.26%	2.84%	-2.89%	48.16%
	3	-4.45%	24.51%	2.84%	-2.89%	-77.90%
3	3	-1.67%	-70.19%	0.73%	0.44%	24.63%
	4	-1.67%	-85.45%	0.73%	0.44%	155.49%
4	4	-0.44%	-2.39%	-0.02%	0.51%	2.55%
	5	-0.44%	-5.60%	-0.02%	0.51%	-0.02%
5	6	0.00%	70.68%	0.07%	-4.82%	-17.04%
	7	0.00%	70.71%	0.07%	-4.82%	-12.60%
6	7	0.00%	104.07%	0.06%	1.96%	-24.70%
	8	0.00%	103.93%	0.06%	1.96%	-24.59%
7	8	0.00%	57.35%	0.06%	3.43%	-13.65%
	9	0.00%	57.36%	0.06%	3.43%	-11.39%
8	9	0.00%	23.78%	0.06%	5.15%	-5.11%
	10	0.00%	24.32%	0.06%	5.15%	-0.97%
9	11	3.36%	54.78%	-1.26%	-0.46%	-13.80%
	12	3.36%	33.02%	-1.26%	-0.46%	-8.84%
10	12	4.09%	36.16%	-2.49%	-2.31%	-11.95%
	13	4.09%	41.93%	-2.49%	-2.31%	-13.16%
11	13	1.57%	15.33%	-0.75%	-1.89%	-7.58%
	14	1.57%	21.67%	-0.75%	-1.89%	-5.33%
12	14	0.42%	1.92%	-0.07%	-1.00%	-2.71%
	15	0.42%	3.82%	-0.07%	-1.00%	-0.51%
13	2	6254.26%	-54.99%	-76.89%	-11.17%	-74.34%
	7	6254.26%	20.76%	-76.89%	7.16%	-12.42%
14	3	-27.63%	-85.92%	35.42%	-8.11%	222.99%
	8	-27.63%	28.98%	35.42%	5.64%	-12.61%
15	4	-97.10%	-25.34%	3254.75%	-3.34%	12.60%
	9	-97.10%	10.81%	3254.75%	2.64%	-4.02%
16	5	-2.41%	-1.06%	2.78%	-1.08%	-4.61%
	10	-2.41%	5.39%	2.78%	0.84%	-3.19%
17	7	244.63%	-50.11%	-77.77%	-9.38%	27.49%
	12	244.63%	28.58%	-77.77%	7.53%	-6.26%
18	8	-31.20%	-59.00%	40.06%	-7.13%	78.57%
	13	-31.20%	36.61%	40.06%	5.88%	-14.94%
19	9	178.80%	-16.53%	-62.43%	-2.92%	8.01%
	14	178.80%	13.71%	-62.43%	2.62%	-5.38%
20	10	-9.47%	0.85%	9.36%	-0.72%	-2.85%
	15	-9.47%	6.29%	9.36%	0.80%	-2.82%

According to NBR 8800 (ABNT [4]), 1st order analysis may not be representative in structures of medium displaceability. Axial force results shows that linear analysis outcomes are not very different from both 2nd order analysis, $B_1 - B_2$ method and P-Delta, in vertical elements (Fig. 9). Columns on the left (elements 1 and 2) are slightly relieved from axial forces, as expected, since the offset of vertical loads, only accounted if analysis occurs in deformed structure, giving rise to an additional moment that tends to pull these elements. On the other hand, columns on the right (elements 3 and 4) are more requested to maintain equilibrium as other elements are relieved.

The similar behavior of all three analysis in columns takes a shift concerning beams. In horizontal elements, the results from linear and rigorous nonlinear analysis are very close, however, the approximated analysis gives results that can achieve more than 10% of overestimation in element 5, and 4% underestimation in element 6, with relation to the rigorous method. The increase of percentage difference in beams is mostly due to the low absolute value of the internal normal forces, that even under a small difference among models, present high relative difference.

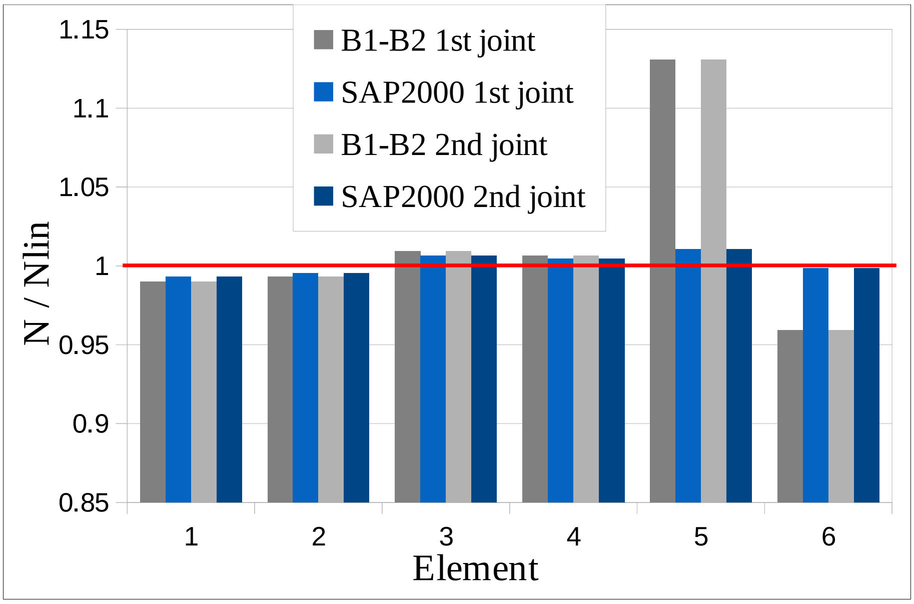


Figure 9. Absolute difference among nonlinear axial internal force results relative to linear analysis in medium displaceability structure

As Kani [10] has verified, the use of steel to increase shear strength in reinforced concrete elements leads to considerable gains as the rate of steel grows. Shearing along steel elements hardly ever happen, being more common failure of joints and bolts.

The analysis results of shear forces from approximated 2nd order method are the same as in 1st order analysis, since amplification coefficients are not applied to the responses obtained in Lt and Nt structures. The columns graph in Fig. 10 shows that the relation between shear forces from $B_1 - B_2$ method and 1st order analysis are the same.

The ratios between linear and nonlinear models in terms of shear internal forces show that approximated results does not surmount 3% for the considered example. The irrelevance of possible underestimations provided by first order analysis comes from small differences with relation to the rigorous analysis combined with the great capacity of steel structures to resist shearing forces.

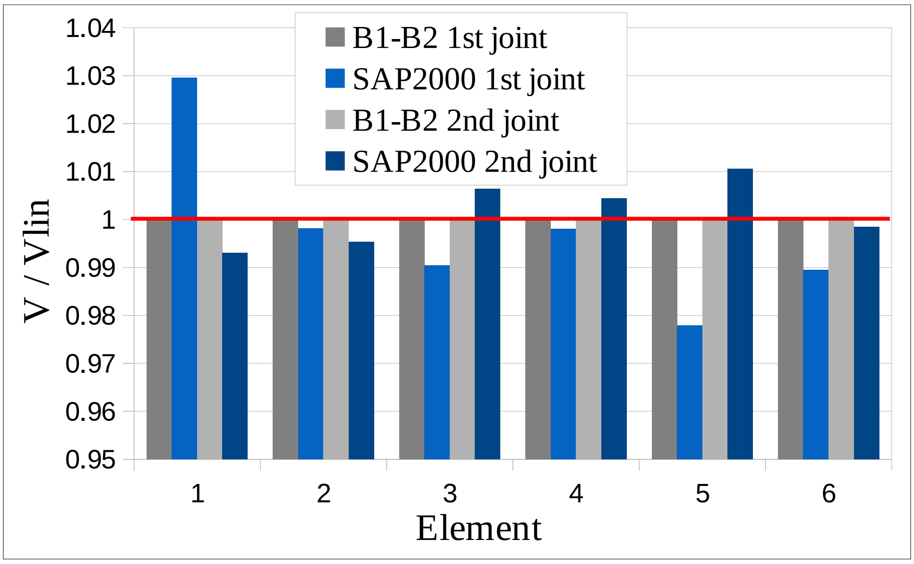


Figure 10. Absolute difference among nonlinear shear internal force results relative to linear analysis in medium displaceability structure

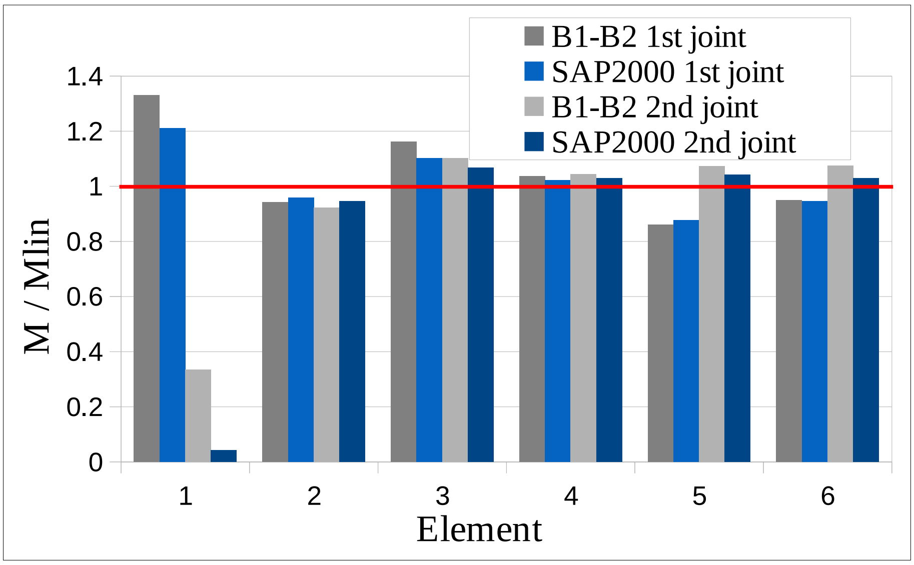


Figure 11. Absolute difference among nonlinear bending moment results relative to linear analysis in medium displaceability structure

The main necessity of regulating the type of structural analysis has to do with bending moment results. Considering the bending moment of the medium displaceabilty model, the results from linear analysis are smaller than from both of 2nd order, regarding only extremum values. If P-Delta is considered to be the true response of the structure, linear analysis understimates bending moment results in about 20%. Safety issues can occur in such large gap, shown in Fig. 11.

Also, taking P-Delta as the reference, the standard method shows good approximations with errors less than 5.2%. An exception was element 1, at node 2, which presented a huge distance between results that may induce to oversizing mainly if considered linear approach but also for the approximated $B_1 – B_2$ method. Efficiency-wise, approximations tend to overestimate the results up to 8.99%, considering only the biggest values for each element, so economical impacts may occur due to oversizing.

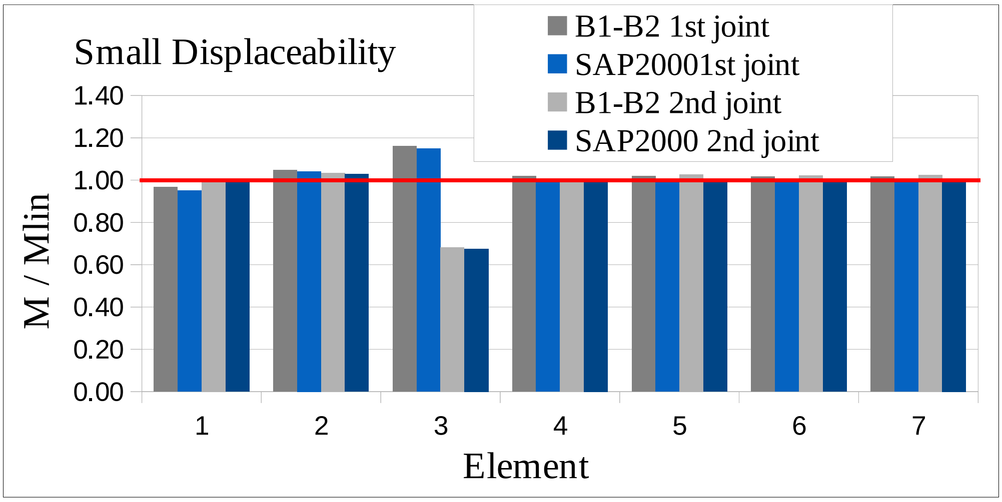


Figure 12. Absolute difference among nonlinear bending moment results relative to linear analysis in small displaceability structure

Comparing the results among different analysis methods a sign of misleading procedure by the standard is shown in Fig. 12, element 3. In this case the magnitude of solicitation is relatively small, culminating in big percentual difference, but not in disqualification of the standard method. Second order effects can be ignored if the structure is of small displaceability and if conditions described in subtopic **3.1** the analysis can be made with linear approach, so 2^{nd} order effects are not considered.

When it comes to large displaceability structures, standard prohibites the use of approximated method and, without specifying which, imposes the use of rigorous analysis methods, accounting material and geometric nonlinearities. For academic purposes, comparisons among methods have been made in large displaceability structure.

First examination shows that linear analysis is not sufficient to calculate the responses of the structure to loads, as expected. Another general feature is the fact that approximated method tends to overestimate the maximum bending moment in each element. The great order of magnifying results lead to unnecessary safety measures at the cost of massive efficiency reduction.

In Fig. 13 the discrepancy of results is illustrated in a columns graphic. The difference between the approximated method and P-Delta can achieve over 50%, in element 1. Differences among 1^{st} order and 2^{nd} order methods results are also explicit, achieving in most cases over 20% higher values when accounting second order effects.

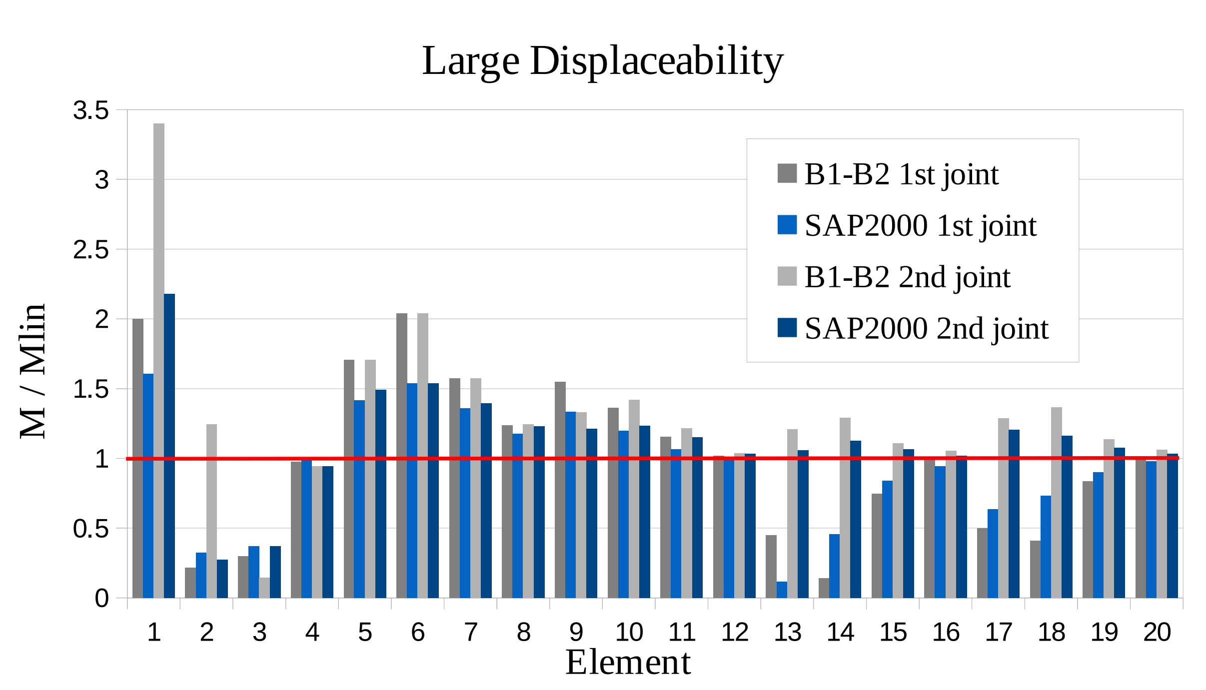


Figure 13. Absolute difference among nonlinear bending moment results relative to linear analysis in large displaceability structure

7 Final considerations

In medium displaceability structures, the linear estimation of axial and shearing forces are closer to rigorous nonlinear analysis than standardized method, but great differences in bending moment estimations, between linear and rigorous nonlinear models, justify the need of accounting second order effects. As in other classes, the second order effects are more explicit in bending moment results comparison, but between the two nonlinear analysis, results are almost the same, showing that the standard is both safe inclined and efficient.

Concerning small and large displaceability structures, the standard seems to be correct, taking the considered examples as a reference. In the example of small displaceability, linear analysis is more consistent and is often similar to second order rigorous method results. On the other hand, in the large displaceability example, linear analysis is not representative, and differences can achieve great order. In this case, rigorous analysis is necessary to guarantee safety and efficiency of the design. This class of structure is commonly avoided, due to complexity of analysis, and alternatives to increase global stiffness are taken.

Standard classifying method and assigned analysis approaches to each class are precise and must not be neglected. Overall, the $B_1 - B_2$ method suits the necessity of simplifying 2^{nd} order effects accountance to avoid being ignored due to complexity.

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