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COMPARING AN ANALYTICAL AND A NUMERICAL MODEL FOR EARTH-AIR HEAT EXCHANGERS IN THE BRAZILIAN SOUTH REGION

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Abstract. The Earth-Air Heat Exchangers (EAHE) represent a promising option to reduce the heating/cooling load of buildings. Unlike traditional air conditioning systems, the EAHE employ a renewable source of energy and they can operate using little electric power. Basically, EAHE use buried ducts, where the air is blown by fans to exchange heat with the soil. Since the first layers of the ground are warmer than the outside air in the winter, and colder in the summer, the Earth can be used as heat source or sink. Therefore, the air leaves the ducts at milder temperatures. Recent research has shown that these devices can work properly in the south region of Brazil, where prevails a subtropical climate. This work aims to compare an analytical model with a numerical one, analyzing their results with experimental data from an EAHE installation in the city of Viamão, located in the Brazilian state of Rio Grande do Sul. As it is shown ahead, the analytical model is not only more simple, but also more accurate than the numerical model.

Keywords: Earth-Air Heat Exchangers (EAHE), Computer Modeling, Computer Simulations.

1 Introduction

With the climate changes directly affecting the world population nowadays, it is necessary the emergence of new techniques to minimize the damage caused by these transformations. The greenhouse effect, the melting of the polar ice caps and other problems, caused mostly by human influence, contribute, either to the temperature increases in summer or to their drops in winter.

According to Galileo Magazine [1], these warming and cooling events have happened gradually over thousands of years. However, the fast warming on Earth over the last years is unquestionably a human effect, since it is due to the potent greenhouse gases produced by burning coal and other fuels that have been accumulating in the atmosphere.

Besides, Brum [2] highlights the human needs of protection from climatic diversity and the search for satisfactory conditions of well-being and physical comfort. Hence, climate has always been an important aspect for housing projects. On the other hand, a current major concern is to achieve thermal comfort through sustainable solutions and minimizing energy consumption.

The southern region of Brazil has a subtropical climate, i.e., the four seasons of the year are well defined. Hence, one can observe very low temperatures in the winter, and very high ones in the summer (relative to rest of the country). According to Silva et al. [3], due to the high humidity, some hot summer days are suffocating and unsuitable for many activities. In particular, this happens for instance in the south Brazilian city of Pelotas, which causes numerous inconveniences to the population.

Thus, this paper aims to compare the results of an analytical and a numerical model for Earth-air heat exchangers (EAHE). They are tested against experimental data, obtained from an installation in the city of Viamão, located in the south Brazilian state of Rio Grande do Sul. Basically, EAHE consist of the installation of buried ducts connected to both the external and internal environment of a building. This allows to blow the air in the ducts, helping to cool the internal environment in the summer and warm it in the winter, at low power consumption. This process is due to the thermal exchange of the air flowing in the pipes with the ground, which acts as a heat source or sink. As shown below, the analytical model can provide very accurate results compared to experimental data and numerical results.

2 Analytical Model for EAHE

The heat transport fluid in EAHE is the air. As seen in Bisoniya [4], if the duct wall contact with the ground is considered perfect and the ground conductivity is sufficiently high, compared to the surface resistance, then the wall temperature inside the pipe can be considered constant. This is a simplifying hypothesis used to make the analytical model for EAHE.

The efficiency of EAHE is expressed by Eq. (1), where NTU is a dimensionless quantity, Number of Transfer Units (NTU). Hence,

$$\varepsilon = 1 - e^{-NTU}. \tag{1}$$

The NTU depends on h, which represents the convective heat transfer coefficient; A_s, the surface area of the duct; $\dot{m}_a$, the mass flow of air; and c_p^a, the specific heat of air, as follows:

$$NTU = \frac{hA_s}{\dot{m}_a c_p^a}. \tag{2}$$

The mass air flow is given by the formula:

$$\dot{m}_a = \rho_a A_t v_a, \tag{3}$$

where ρ_a is the air density, A_t is the cross-sectional area of the duct, and v_a is the average air velocity within the duct.

The convective heat transfer coefficient, h, depends on the Nusselt number, Nu; the thermal conductivity of the air, k_a; and the diameter of the duct, D. That is:

$$h = \frac{Nu\, k_a}{D}. \tag{4}$$

The Nusselt number for turbulent airflow within a duct with smooth surfaces is given by the following equation:

$$Nu = \frac{\frac{f}{8}(Re-1000)P_r}{1+12.7\sqrt{\frac{f}{8}}\left(P_r^{\frac{2}{3}}-1\right)}. \tag{5}$$

Here, the Reynolds number (Re) and the Prandtl number (P_r) are given by Eq. (6) and Eq. (7), respectively. The parameter f, which is the friction factor for smooth ducts, is given by Eq. (8), as follows:

$$Re = \frac{\rho_a v_a D}{\mu_a}, \tag{6}$$

where μ_a is the dynamic viscosity of the air;

$$P_r = \frac{\mu_a\, c_p^a}{k_a}, \tag{7}$$

and

$$f = \frac{1}{[0{,}79\, ln\,(Re)-1{,}64]^2}. \tag{8}$$

Finally, the EAHE efficiency can also be defined by:

$$\varepsilon = \frac{T_a^o - T_a^i}{T_s - T_a^i}, \tag{9}$$

where T_a^o and T_a^i are, respectively, the air temperatures at the outlet and inlet of the duct, while T_s is the soil temperature, in the vicinity of the duct walls. This gives an analytical model for the air temperature at the EAHE outlet, i.e.:

$$T_a^o = T_a^i + \varepsilon(T_s - T_a^i), \tag{10}$$

and we compute ε by the formula given in the Eq. (1).

In general, the air temperature can be obtained by meteorological reports from trusted sources, while the soil temperature is usually hard to know. On the other hand, it can be estimated from the air temperature, at time t and depth z, by the following formula given by Ozgener [5]:

$$T_s(t,z) = T_m + A\, sin\left[\frac{2\pi}{P}(t-t_o) - \gamma z - \frac{\pi}{2}\right], \tag{11}$$

where T_m is the mean temperature of the air close to the soil surface; t_o is the time lag needed for the temperature of the air reach T_m. In Eq. (11), A is the amplitude of the temperature wave at the depth z, which is given by:

$$A = A_o \exp\,(-\gamma z), \tag{12}$$

where A_o is the amplitude of the temperature wave at the soil surface (at $z = 0$). Finally:

$$\gamma = \sqrt{\frac{\pi}{\alpha_s\, P}}, \tag{13}$$

where P is the period of the oscillation and α_s is the thermal diffusivity of the soil.

3 Numerical Model for EAHE

Based on the experimental data of Vaz [6], relative to the installation of an EAHE in the aforementioned city of Viamão, Brum et al. [7] developed a numerical model, called Reduced Model, because, as in the Fig. 1, it considers only the horizontal part of the duct that is buried in the ground. This is done to reduce the computational costs involved in the simulation of this type of problem, as it takes hours to run one year of operation for an EAHE.

In spite of the name, the Reduced Model is a very complete three-dimensional model, which can describe most of the physics for a general EAHE system, adopting simple or complex geometries. In this model, heat conduction in the ground is described by the energy conservation equation:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i}\left(\alpha_s \frac{\partial T}{\partial x_i}\right), \tag{14}$$

which gives the temperature values T, at any time t and any spatial position x_i, assuming that the thermal diffusivity of the soil α_s is known.

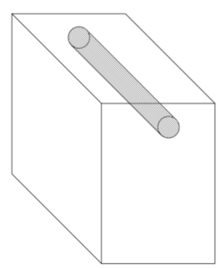

Figure 1. Representation of the soil and duct portions in the Reduced Model.

As for the air flow in the ducts, it is assumed to be incompressible and turbulent. It is governed by the average conservation equations of mass, momentum and energy:

$$\frac{\partial \bar{v}_{\mathrm{i}}}{\partial x_{\mathrm{i}}} = 0, \tag{15}$$

$$\frac{\partial \bar{v}_{\mathrm{i}}}{\partial t} + \frac{\partial(\bar{v}_{\mathrm{i}}\bar{v}_{\mathrm{j}})}{\partial x_{\mathrm{j}}} = -\frac{1}{\rho_a}\frac{\partial \bar{p}}{\partial x_{\mathrm{j}}}\delta_{\mathrm{ij}} + \frac{\partial}{\partial x_{\mathrm{j}}}\left[\nu_a\left(\frac{\partial \bar{v}_{\mathrm{i}}}{\partial x_{\mathrm{j}}} + \frac{\partial \bar{v}_{\mathrm{j}}}{\partial x_{\mathrm{i}}}\right) - \overline{v_{\mathrm{i}}'v_{\mathrm{j}}'}\right], \tag{16}$$

$$\frac{\partial \bar{T}}{\partial t} + \frac{\partial}{\partial x_{\mathrm{i}}}(\bar{v}_{\mathrm{i}}\bar{T}) = \frac{\partial}{\partial x_{\mathrm{i}}}\left(\alpha_a \frac{\partial \bar{T}}{\partial x_{\mathrm{i}}} - \overline{v_{\mathrm{i}}'T'}\right). \tag{17}$$

Here, v, ν_a and α_a are, respectively, the velocity, kinematic viscosity and thermal diffusivity of the air. Finally, δ_{ij} is the Kronecker delta.

To close the equations, it is necessary to adopt some turbulence model. As described more detailed in Brum [2] and Brum et al. [7], the Reynolds-averaged Navier Stokes model (RANS) was adopted in the Reduced Model. The equations were numerically solved using the Fluent software, which is based on finite volume methods.

4 Results and discussions

In the experimental work of Vaz [6], the buried horizontal part of the duct had a length of 25.77m and a diameter of 0.11m. Thus, in the numerical simulation with the Reduced Model, it was considered a soil portion with the following dimensions: 25.77m, 5m and 15m of length, width, and depth, respectively. The air velocity at the inlet of the tube was 3.3m/s. The physical conditions for the local clay soil as well as for the air are shown in Table 1.

The air temperature T_a (in ^{o}C) was fitted by least squares and the following temporal function was obtained to give its values for each day t of the year:

$$T_a^i(t) = 20.49 + 5.66 sin\left(\frac{2\pi}{365}t - 5.30\right). \tag{18}$$

As for the soil temperature, at the depth z=1.6 where the duct was buried, it is given by:

$$T_s(t) = 20.49 + 3.03 sin\left(\frac{2\pi}{365}t - 5{,}92\right). \tag{19}$$

Using the thermo-physical properties of the soil and air, presented in Table 1, together with the functions given in Eq. (18) and (19), the following analytical function was obtained to describe the temperature at the outlet of the EAHE ducts:

$$T_a^o(t) = 20.49 + 0.23 sin\left(\frac{2\pi}{365}t - 5.30\right) + 2.91 sin\left(\frac{2\pi}{365}t - 5.92\right). \tag{20}$$

Table 1. Thermo-physical properties of the soil and air

Property	Air	Clay	Unit
ρ	1,16	1800	kg/m³
K	0,0242	2,1	w/mK
c_p	1010	1780	J/kgK

The Fig. 2 presents a comparison between the experimental data of Vaz [6], the numerical data of Brum [2] and the results obtained with the analytical function of Eq. (17). As one can see, a good agreement can be noted between the data presented.

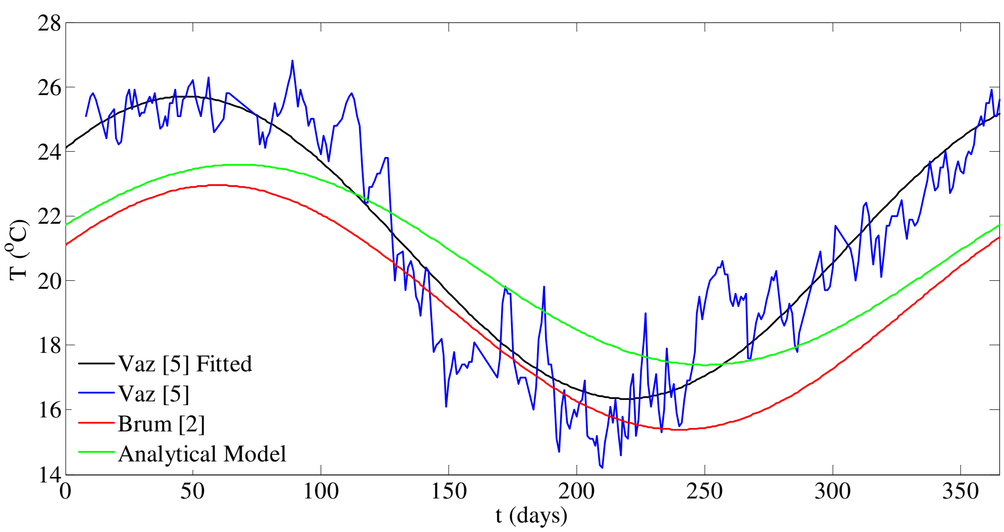

Figure 2. Comparison among the results.

All numerical and experimental results were fitted to sine based functions using the least squares method, as performed in the references Brum et al. [8]; Ramalho et al. [9]; and Brum et al. [10]. Thus, the following temporal functions (t in days):

$$T_V(t) = -4.68\, sin\left(\frac{2\pi}{365} t - 2.43\right) + 21.02, \tag{21}$$

$$T_B(t) = 3.78\, sin\left(\frac{2\pi}{365} t + 0.53\right) + 19.17, \tag{22}$$

represent the temperature (in ° C) for the results of Vaz [6] and Brum [2], respectively. The graphic of $T_V(t)$ can be seen in Fig. 2, which shows a strong correlation between experimental and adjusted data. The value of the correlation coefficient, Bulmer [11], among them is 0.94.

With these functions, it is possible to estimate model errors. The root mean square value (RMS) of the difference between the numerical model results and the fitted experimental data values is:

$$\sqrt{\frac{\int_0^{365}[\mathrm{T_V(t)-T_B(t)}]^2 \mathrm{dt}}{365}} = 2{,}37^{o}\mathrm{C}. \tag{23}$$

The difference between the fitted experimental values and those provided by the analytical model is:

$$\sqrt{\frac{\int_0^{365}[\mathrm{T_V(t)-T_{ar}^{s}(t)}]^2 \mathrm{dt}}{365}} = 1{,}94^{o}\mathrm{C}. \tag{24}$$

From Fig. 2, it is already possible to see that the analytical model provides results closer to the experimental data than the Reduced Model, but the numbers quantify the difference (there is a relative improvement of 22%) and it helps to evaluate both models.

5 Conclusions and future prospects

This paper presents a comparative study between a numerical and an analytical model for Earth-air heat exchangers that are verified through experimental data obtained in a facility built in the Brazilian state of Rio Grande do Sul, where a subtropical climate predominates. It can be observed that the analytical model has a greater simplicity in relation to the numerical one, being of easy implementation and low computational cost. However, it should be noted that the model has limitations, for example, it cannot model complex geometries as it can be done by the numerical model.

On the other hand, comparing with the experimental data, it is also observed that the annual root mean square error of the analytical model is relatively smaller than that presented by the numerical model. While the numerical model error was 2.37ºC, the analytical model had an error of 1.94ºC. Hence, for simple geometries, it is more accurate and efficient than the numerical model. Thus, the analytical model is an important tool for developing initial estimates of the potential for EAHE installations in a given location.

With this analytical model, it is possible to conduct case studies for different regions both in southern Brazil and elsewhere. In particular, the authors intend to use it in future works to evaluate the thermal potential of EAHE for the Pelotas region, located in the Brazilian state of Rio Grande do Sul, taking into account the different local soil types and advancing their regional research.

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