NONLINEAR MODELING OF TWO LINKS OF A THREE DEGREES OF FREEDOM ROBOT

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Abstract. The objective of this paper is the nonlinear modeling of two links of a manipulator robot with three degrees of freedom (3 DOF).The manipulator robot consists of two rotational joints and a prismatic one. A gearmotor with frequency inverter and two electropneumatic valves are used to drive the robot. There are dynamic systems that can't be represented by linear models; in these cases, the use of nonlinear models is indispensable. Robot Models, known as white box models, that are coupled and presenting nonlinear equations can be obtained using Newton-Euler or Lagrange equations. In this work a nonlinear model, known as black box model, will be obtained using data collected from links 1 and 3 of the robot and considering the dynamic coupling between the links of the robot. In the models obtained the nonlinearities will be in the inputs, and the parameters of the obtained models are linear. A parametric model described by an equation the difference with nonlinearity at the input, is called the Volterra parametric model. Finally, the results obtained will be presented through the generated model.

Keywords: Robotics, Nonlinear system, Identification.

1 Introction

The aim of this work is the non-linear modeling of two links of a robot manipulator with three degrees of freedom (3 DOF). The manipulator robot consists of two rotational joints and a prismatic one. A gearmotor with frequency inverter and two electropneumatic valves are used to drive the robot. There are dynamic regimes that can not be represented by linear models; in such cases the use of non-linear models is indispensable. Robot Models, known as white-box models (Craig [1], Spong and Vidyasagar [2]), which are coupled and nonlinear equations can be obtained using Newton-Euler or Lagrange. In this paper nonlinear models (Riul and Montenegro [3], Riul et al. [4]), known as black box model, are obtained using data collected on the links of the robot 1 and 3 and considering the dynamic coupling between them. In the models to be obtained will be considered non-linearities of input, being considered linear the parameters of the robot. A parametric model described by a difference equation, with input nonlinearity is called the Volterra parametric model. Finally, results obtained through the generated models will be presented.

2 System description

The 3 GDL manipulator robot shown in Fig. 1 is composed of two rotational joints and a prismatic one. The rotational joint 1 is driven by a gearmotor fed by a frequency inverter and drives the link 1 of the robot. This link is a cylindrical column and has a maximum angular displacement of 160 °, measured through a potentiometer. The movement of the gasket 1 is transmitted to the link 1 through two pulleys and a toothed belt. The rotational joint 2 is driven by an electro-pneumatic system composed of an electro-pneumatic valve and a pneumatic cylinder and drives the link 2 of the robot. This link is a U-profile and has a maximum angular displacement of 45 °, measured through a potentiometer. The movement of the gasket 2 is transmitted to the link 2 through the displacement of the piston of the pneumatic cylinder. The prismatic joint 3 is driven by an electro-pneumatic valve and drives the robot link 3 which is the piston rod of a 500 mm stroke piston, of a pear-shaped cylinder fixed inside the U-shaped profile (link 2). The linear displacement of the piston rod is measured through a potentiometric ruler. A PC computer is used to send drive control to the gearmotor via the frequency inverter and to the two electropneumatic valves and to receive signals from the potentiometric sensors. The communication of the robot with the computer is performed through two NI USB-6009 data entry and exit cards, using a computer program on the LabView and Matlab platforms. Considering the voltage characteristics and maximum current capacity of the input and output plates, a power amplifier was inserted to serve as the supply source for the manipulator robot drive elements.

Figure 1. Robot manipulator of 3 GDL

3 Non-linear robot identification

The identification of systems is an area of knowledge that studies alternative techniques of mathematical modeling (Isermann et al. [5], Aström and Wittenmark, [6], Rúbio and Sanchez, [7], Coelho and Coelho, [8]). One of the characteristics of these techniques is that little or no previous knowledge of the system is necessary and, consequently, such methods are referred to as black box modeling or empirical modeling (Aguirre, [9]).

Nonlinear mathematical models of systems can be obtained using the Volterra series (Isermann et al., [5]). The Volterra parametric model Eq. (1) is suitable for parameter estimation based on the input and output signals of a SISO system, with non-linearity at the input.

$$\nu(k) = -\sum_{i=1}^{m} a_i \nu(k-i) + \sum_{i=1}^{m} b_i u(k-d-i) + \sum_{\beta=0}^{h} \sum_{i=1}^{m} b_{2\beta i} u(k-d-i) u(k-d-i-\beta) + \ldots.$$
$$\sum_{\beta_1=0}^{h} \sum_{\beta_2=\beta_1}^{h} \ldots \sum_{\beta_{p-1}=\beta_{p-2}}^{h} \sum_{i=1}^{m} b_{p\beta_1 \ldots \beta_{p-1} i} u(k-d-i) \prod_{\xi=1}^{p-1} u(k-d-i-\beta_\xi) + c_{ss} \tag{1}$$

where:

m - order of the system model;

d - transport delay;

p - degree of non-linearity of the system model;
h - horizon;
css - DC level.

Using Equation (1) with css = 0, the degree of non-linearity at the input, with the parameters of Table 1, we arrive at the models shown in Eq. (2), (3), (4) and (5) .

Table 1. Parameters for degree of non-linearity of input

	p	m	d	h
1	2	1	0	1
2	2	2	0	1
3	3	1	0	1
4	3	2	0	1

$$\nu(k) = \begin{bmatrix} -a_1 & b_1 & b_{201} & b_{211} \end{bmatrix} \begin{bmatrix} \nu(k-1) \\ u(k-1) \\ u^2(k-1) \\ u(k-1)u(k-2) \end{bmatrix} \tag{2}$$

$$\nu(k) = \begin{bmatrix} -a_1 & -a_2 & b_1 & b_2 & b_{201} & b_{211} & b_{202} & b_{212} \end{bmatrix} \begin{bmatrix} \nu(k-1) \\ \nu(k-2) \\ u(k-1) \\ u(k-2) \\ u^2(k-1) \\ u(k-1)u(k-2) \\ u^2(k-2) \\ u(k-2)u(k-3) \end{bmatrix} \tag{3}$$

$$\nu(k) = \begin{bmatrix} -a_1 & b_1 & b_{201} & b_{211} & \mathrm{b}_{3001} & \mathrm{b}_{3011} & \mathrm{b}_{3111} \end{bmatrix} \begin{bmatrix} \nu(k-1) \\ u(k-1) \\ u^2(k-1) \\ u(k-1)u(k-2) \\ u^3(k-1) \\ u^2(k-1)u(k-2) \\ u(k-1)u^2(k-2) \end{bmatrix} \tag{4}$$

$$\nu(k) = \left[-a_1 \ \ -a_2 \ b_1 \ b_2 \ b_{201} \ b_{211} \ b_{202} \ b_{212} \ b_{3001} \ b_{3011} \ b_{3111} \ b_{3002} \ b_{3012} \ b_{3112}\right]. \begin{bmatrix} \nu(k-1) \\ \nu(k-2) \\ u(k-1) \\ u(k-2) \\ u^2(k-1) \\ u(k-1)u(k-2) \\ u^2(k-2) \\ u(k-2)u(k-3) \\ u^3(k-1) \\ u^2(k-1)u(k-2) \\ u(k-1)u^2(k-2) \\ u^3(k-2) \\ u^2(k-2)u(k-3) \\ u(k-2)u^2(k-3) \end{bmatrix} \tag{5}$$

Equations (2), (3), (4) and (5) are rewritten according to Eq. (6) for a two-degree MIMO system with excitations at links 1 and 3; u1 (k) and u3 (k) and responses; $\theta_1(k)$ e $r_3(k)$.

$$\left[\nu(k)\ \right] = \left[\psi(k)\right]\left[\varphi(k)\right] \tag{6}$$

where:

$\left[\nu(k)\ \right]-$ output vector;
$\left[\psi(k)\right]-$ matrix of parameters;
$\left[\varphi(k)\right]-$ vector of measures.

In Equation (6), the vector of measurements is used to obtain the outputs.

By using Equation (6) for the models given by Equations (2), (3), (4) and (5), omitting the discrete time k of the parameters ai and bj and renaming the parameters of the robot models, we have the models of two degrees of freedom:

- Model 1M

$$\begin{aligned}[\varphi^T(k)] = [&\theta_1(k-1) \ \ r_3(k-1) \ \ u_1(k-1) \ \ u_1(k-2) \ \ u_1(k-1)u_1(k-2) \\ &u_3(k-1) \ \ u_3(k-2) \ \ u_3(k-1)u_3(k-2)]\end{aligned} \tag{7}$$

$$\begin{bmatrix} \psi_1(k) \\ \psi_3(k) \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ -a_3 & -a_4 & b_7 & b_8 & b_9 & b_{10} & b_{11} & b_{12} \end{bmatrix} \tag{8}$$

$$\left[\nu(k)\ \right] = \begin{bmatrix} \theta_1(k) \\ \\ r_3(k) \end{bmatrix} \tag{9}$$

where:
$\psi_i(k)$ - parameters of links 1 and 3 of the robot;
$\varphi_i(k)$ - vector of measurements of robot links 1 and 3,
$v_i(k)$ - outputs from links 1 and 3 of the robot;
i = 1 and 3.

- Model 2M

$$\begin{aligned}[\varphi^T(k)] = [&\theta_1(k-1)\ \ \theta_1(k-2)\ \ r_3(k-1)\ \ r_3(k-2)\ \ u_1(k-1)\ \ u_1(k-2)\ \ u_1^2(k-1)\\ &u_1(k-1)u_1(k-2)\ \ u_1^2(k-2)\ \ u_1(k-2)u_1(k-3)\ \ u_3(k-1)\ \ u_3(k-2)\\ &u_3^2(k-1)\ \ u_3(k-1)u_3(k-2)\ \ u_3^2(k-2)\ \ u_3(k-2)u_3(k-3)]\end{aligned} \tag{10}$$

$$\begin{bmatrix}\psi_1(k)\\ \psi_3(k)\end{bmatrix} = \begin{bmatrix}-a_1 & -a_2 & -a_3 & -a_4 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 & b_9 & b_{10} & b_{11} & b_{12}\\ -a_5 & -a_6 & -a_7 & -a_8 & b_{13} & b_{14} & b_{15} & b_{16} & b_{17} & b_{18} & b_{19} & b_{20} & b_{21} & b_{22} & b_{23} & b_{24}\end{bmatrix} \tag{11}$$

- Model 3M

$$\begin{aligned}[\varphi^T(k)] = [&\theta_1(k-1)\ \ r_3(k-1)\ \ u_1(k-1)\ \ u_1^2(k-1)\ \ u_1(k-1)u_1(k-2)\ \ u_1^3(k-1)\\ &u_1^2(k-1)u_1(k-2)\ \ u_1(k-1)u_1^2(k-2)\ \ u_3(k-1)\\ &u_3^2(k-1)\ \ u_3(k-1)u_3(k-2)\ \ u_3^3(k-1)\ \ u_3^2(k-1)u_3(k-2)\\ &u_3(k-1)u_3^2(k-2)]\end{aligned} \tag{12}$$

$$\begin{bmatrix}\psi_1(k)\\ \psi_3(k)\end{bmatrix} = \begin{bmatrix}-a_1 & \text{-}a_2 & b_1 & b_2 & b_3 & \mathrm{b}_4 & \mathrm{b}_5 & \mathrm{b}_6 & \mathrm{b}_7 & \mathrm{b}_8 & b_9 & \mathrm{b}_{10} & \mathrm{b}_{11} & \mathrm{b}_{12}\\ -a_3 & \text{-}a_4 & b_{13} & b_{14} & b_{15} & \mathrm{b}_{16} & \mathrm{b}_{17} & \mathrm{b}_{18} & \mathrm{b}_{19} & \mathrm{b}_{20} & b_{21} & \mathrm{b}_{22} & \mathrm{b}_{23} & \mathrm{b}_{24}\end{bmatrix} \tag{13}$$

- Model 4M

$$\begin{aligned}[\varphi^T(k)] = [&\theta_1(k-1)\ \ \theta_1(k-2)\ \ r_3(k-1)\ \ r_3(k-2)\ \ u_1(k-1)\ \ u_1(k-2)\ \ u_1^2(k-1)\\ &u_1(k-1)u_1(k-2)\ \ u_1^2(k-2)\ \ u_1(k-2)u_1(k-3)\ \ u_1^3(k-1)\ \ u_1^2(k-1)u_1(k-2)\\ &u_1(k-1)u_1^2(k-2)\ \ u_1^3(k-2)\ \ u_1^2(k-2)u_1(k-3)\ \ u_1(k-2)u_1^2(k-3)\\ &u_3(k-1)\ \ u_3(k-2)\ \ u_3^2(k-1)\ \ u_3(k-1)u_3(k-2)\ \ u_3^2(k-2)\ \ u_3(k-2)u_3(k-3)\\ &u_3^3(k-1)\ \ u_3^2(k-1)u_3(k-2)\ \ u_3(k-1)u_3^2(k-2)\ \ u_3^3(k-2)\ \ u_3^2(k-2)u_3(k-3)\\ &u_3(k-2)u_3^2(k-3)]\end{aligned} \tag{14}$$

$$\begin{bmatrix}\psi_1(k)\\ \psi_3(k)\end{bmatrix} = \begin{bmatrix}-a_1 & -a_2 & -a_3 & -a_4 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & \ldots\ldots\ldots\ldots b_{23} & b_{24}\\ -a_5 & -a_6 & -a_7 & -a_8 & b_{25} & b_{26} & b_{27} & b_{28} & b_{29} & b_{30} & b_{31} & \ldots\ldots\ldots\ldots b_{47} & b_{48}\end{bmatrix} \tag{15}$$

The estimation of the parameters of the nonlinear models of the manipulator robot links under analysis, described above, is performed by the Recursive Least Squares (MQR) algorithm, given by Equations (16), (17), (18) and (19) (Aguirre, [9]; Coelho and Coelho, [8]).

$$\varepsilon(k+1) = y(k+1) - \varphi^T(k+1)\psi(k) \tag{16}$$

$$K(k+1)=\frac{p(k)\varphi(k+1)}{\lambda+\varphi^{T}(k+1)p(k)\varphi(k+1)} \tag{17}$$

$$\psi(k+1)=\psi(k)+K(k+1)\varepsilon(k+1) \tag{18}$$

$$p(k+1)=\frac{1}{\lambda}\left\{p(k)-\frac{p(k)\varphi(k+1)\varphi^{T}(k+1)p(k)}{\lambda+\varphi^{T}(k+1)p(k)\varphi(k+1)}\right\} \tag{19}$$

The quality of the estimated models can be verified using several techniques; among them, one can investigate the magnitude of certain performance indices. The models determined in this paper will be evaluated through the sum of the quadratic error (SEQ), given by Eq. (20) and the multiple correlation coefficient (R^2), given by Eq. (21) (Coelho and Coelho, [8]).

$$SEQ_i=\sum_{k=1}^{N}\left[\nu_i(k)-\hat{\nu}_i(k)\right]^2 \tag{20}$$

$$R_i^2=1-\frac{\sum_{k=1}^{N}\left[\nu_i(k)-\hat{\nu}_i(k)\right]^2}{\sum_{k=1}^{N}\left[\nu_i(k)-\bar{\nu}_i\right]^2} \tag{21}$$

where:

$\hat{\nu}_i(k)$ e $\bar{\nu}_i(k)$ - estimated output of the link i and average of the real output of the link i;
i = 1, 3 - links 1 and 3 of the robot;
$\varepsilon_i(k+1)=\nu_i(k)-\hat{\nu}_i(k)$ - link prediction error i.

When the value of the multiple correlation coefficient, R^2 is equal to unity, it indicates an exact fit of the model for the measured data of the system and for R^2 between 0.9 and 1.0; the model may be considered sufficient for many practical applications. Lowest value of the sum of the quadratic error, SEQ, for the test data set indicates the best model.

The nonlinear mathematical models of the robot manipulator links under study are obtained through parametric identification. The data that make up the measurement vector are the excitations sent from the computer to the joints 1 and 3 of the robot; ; $u_1(k),\ u_3(k)$, and the responses obtained, which are the angular and linear positions of links 1 and 3; $\nu_1(k)=\theta_1(k),\ \nu_3(k)=r_3(k)$. With the solution of Eq. (18), the estimated parameters $\psi_i(k)$ of each of the models 1M, 2M, 3M, 4M, of links 1 and 3 of the manipulator robot are obtained; and with the solution of Eq. (6), the estimated outputs $\nu_1(k)=\theta_1(k)$ e $\nu_3(k)=r_3(k)$ are obtained.

4 Results

The results of the obtained models are shown in Table 2 and Figures 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11. Table 2 shows the performance indexes obtained for the two robot links; and it is observed that model 2 is the best. Figure 2 shows the real input and output signals of the manipulator robot link 1, and as the motor used was alternating current, the reversal of the direction of rotation of the link 1 was obtained through the inversion signal. Figure 3 shows the real input and output signals of link 3. Figure 4 and 5 show the real and estimated outputs of links 1 and 3 of the robot; and the estimated outputs were obtained using model 1. Figures 6 and 7, 8 and 9, 10 and 11 show the outputs obtained using models 2, 3 and 4 respectively.

Table 2. Performance Indices of Models

Model	link	p	m	R^2	SEQ
1	1 3	2 2	1 1	0,997 0,987	5,204 201,677
2	1 3	2 2	2 2	0,997 0,969	4,421 48,652
3	1 3	3 3	1 1	0,997 0,991	4,952 140,830
4	1 3	3 3	2 2	0,997 0,993	4,615 117,962

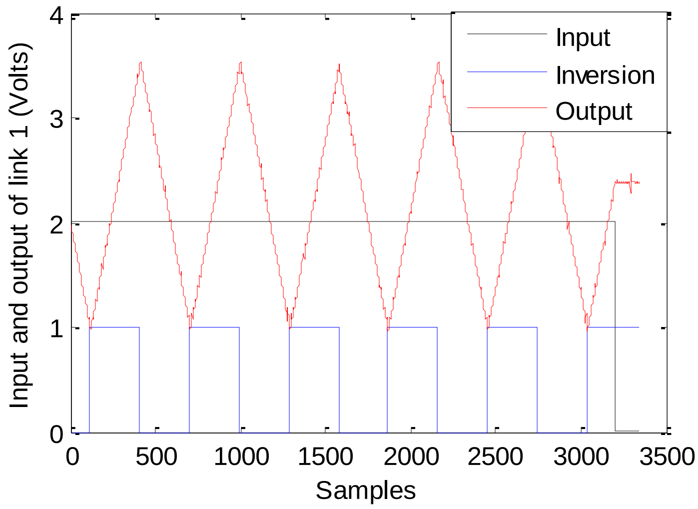

Figure 2. Input and output signals of robot link 1

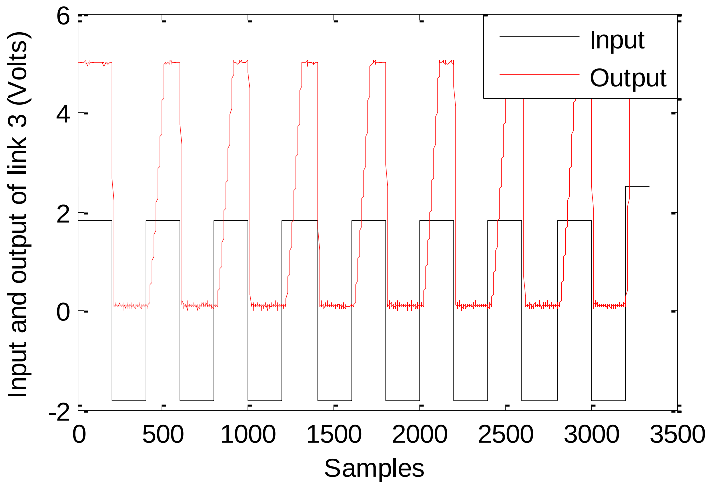

Figure 3. Input and output signals of robot link 3

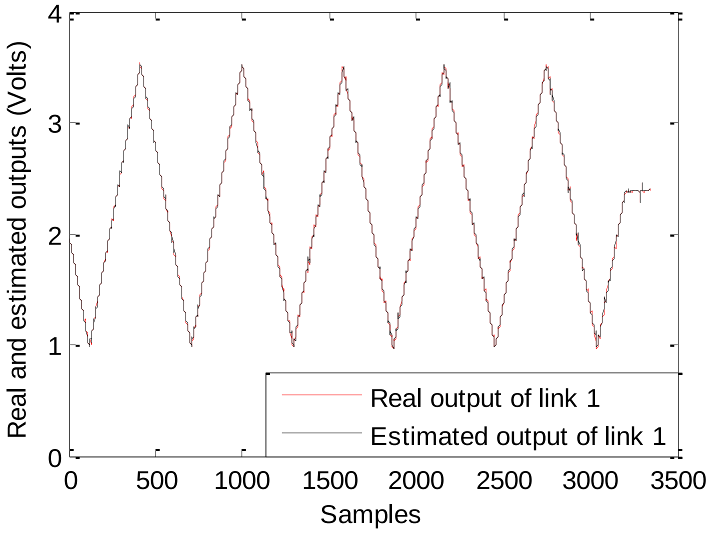

Figure 4. Real and Estimated Robot Link 1 Outputs (Model 1)

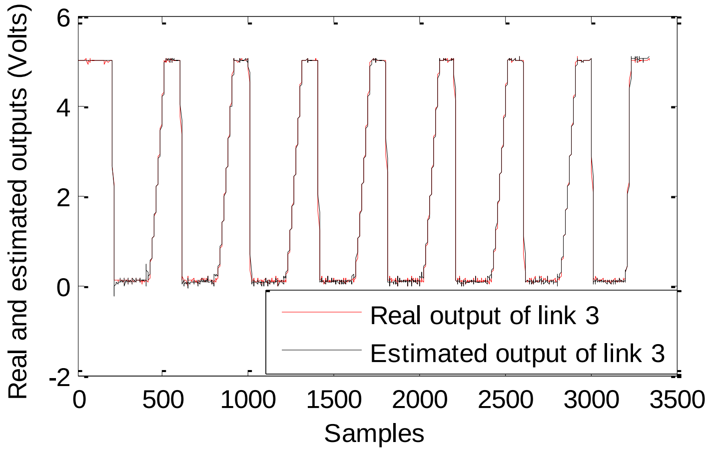

Figure 5. Real and Estimated Robot Link 3 Outputs (Model 1)

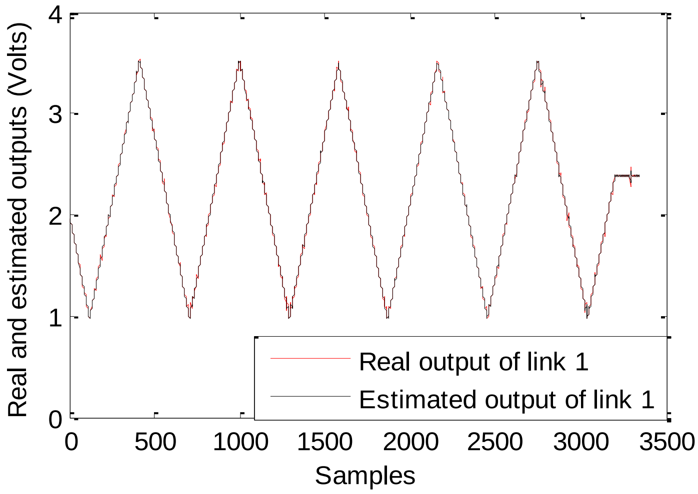

Figure 6. Real and Estimated Robot Link 1 Outputs (Model 2)

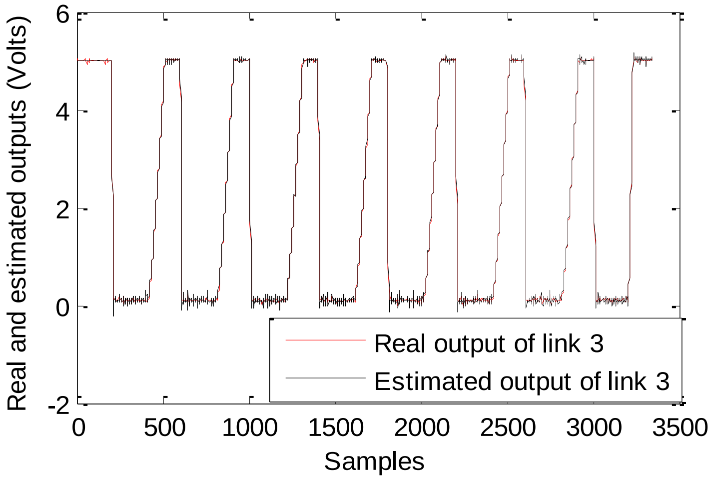

Figure 7. Real and Estimated Robot Link 3 Outputs (Model 2)

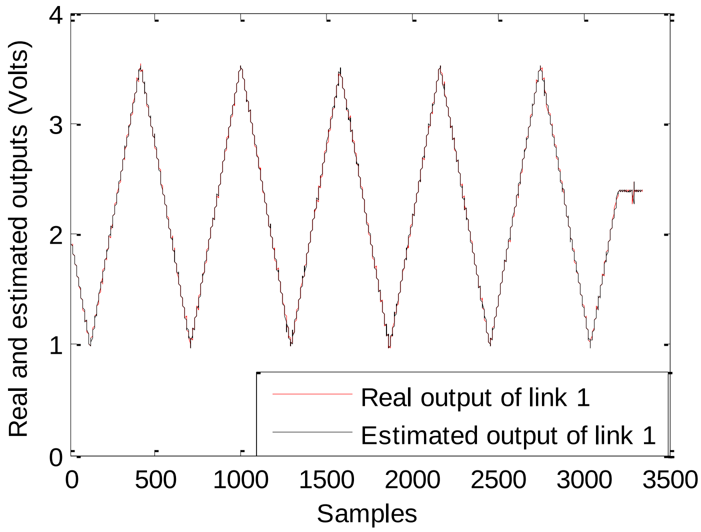

Figura 8. Real and Estimated Robot Link 1 Outputs (Model 3)

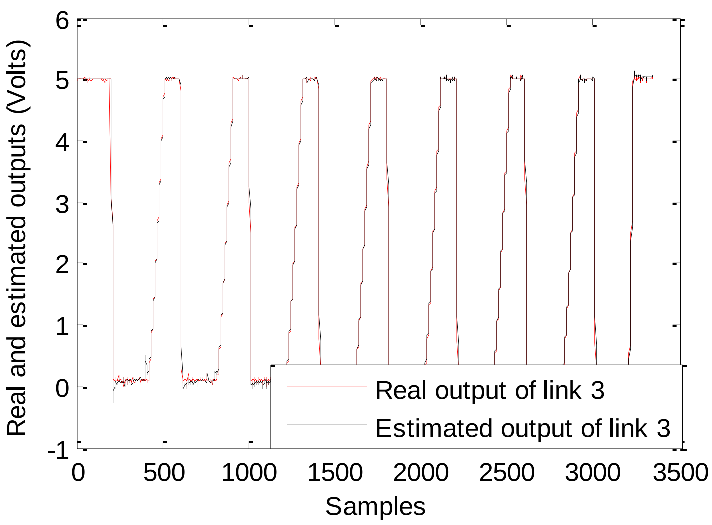

Figure 9. Real and Estimated Robot Link 3 Outputs (Model 3)

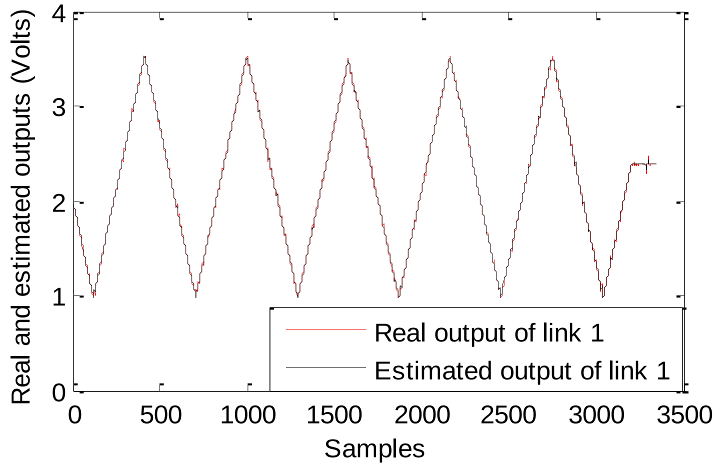

Figure 10. Real and Estimated Robot Link 1 Outputs (Model 4)

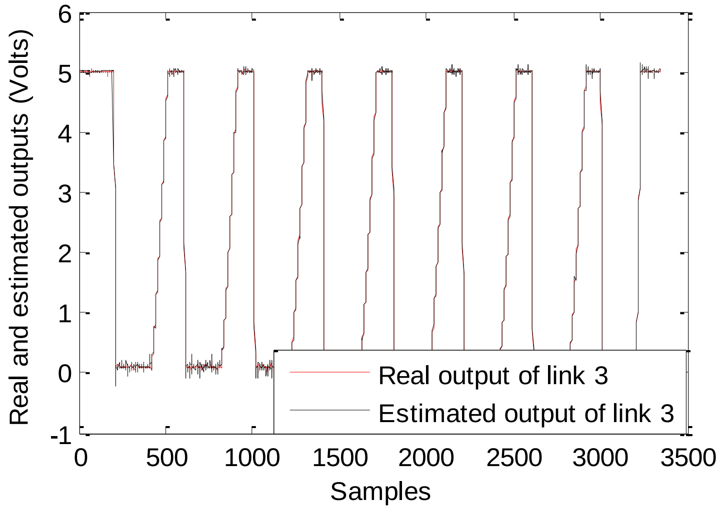

Figure 11. Real and Estimated Robot Link 3 Outputs (Model 4)

5. Conclusion

This work presented a technique of nonlinear identification of two links of a manipulator robot of three degrees of freedom. The identification of the models was performed using the recursive least squares algorithm MQR, considering the dynamics of the two links of the robot. Four nonlinear models considering non-linearity of degrees 2 and 3 were obtained. Model 2 is the most suitable for implementation in adaptive controllers, given the values of its multiple correlation coefficient and the quadratic error, since it is the model whose estimated output is closest to the real one.

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