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NUMERICAL AND EXPERIMENTAL CORRELATION OF A RECTANGULAR BEAM USING A GUYAN-SEREP MIXED METHOD

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Abstract. Structures and equipment under dynamic loading are susceptible to a vibration response, which can reduce its reliability and life span. Continuous monitoring can be a complicated and expensive task, its complexity makes it impossible to always measure vibration in the locations with higher risk of failure, especially in regions with poor accessibility. One way of estimating vibration levels of the structure in areas of interest is using numerical Finite Element method models, however, these models have the disadvantage of not always representing the real structure adequately, as they do not take into account manufacturing errors and other uncertainties. The differences between model and real structure may be minimized through calibration using experimental data, nevertheless, another difficulty arises, as the number of degrees of freedom in the finite element model are a lot bigger than the number of measured degrees of freedom, therefore, calibration cannot be performed directly. The use of reduction techniques makes the calibration, regarding the modal parameters, feasible as it allows the compatibility between model and experimental degrees of freedom, therefore allowing prediction of vibration levels at any point in the structure. In the present work we have used a mixed GUYAN-SEREP methodology for the model reduction of complex structures. This process is based on two steps: first, using GUYAN method to reduce, on the physical domain, the complete model up to a manageable number of degrees of freedom; then, secondly, using SEREP method to end the reduction, on the modal domain, ensuring the compatibilization of the degrees of freedom with the available experimental data. This methodology was applied to a rectangular beam, free-free condition, the results were compared with data obtained by an experimental modal analysis, by means of *MAC*, relative difference (*RD*) and coordinate modal assurance criteria (*COMAC*), obtaining high accuracy. Finally, a correlation of numerical modes was undertaken in relation to the experimental modes yielding improvements on the results for the criteria used.

Keywords: Model reduction, Guyan method, Serep method, Model correlation

1 Introduction

Structures and equipment that operate under dynamic loading have high damage potential, i.e. representing a threat to health and safety of operators, the environment as well as to company's productivity and profitability. These risks bring to light a concern for inspection and monitoring to prevent accidents.

Winds, hurricanes, waves, sea currents, noise, as well as mechanical systems unbalance and misalignment are examples of excitation forces capable of influencing the dynamic behavior of a system.

Controlling the excitation forces characteristics is a rather complex and sometimes impossible task. However, it would be interesting to hold the power to control since these forces can lead to responses that generate undesirable system condition, such as fatigue, stress, high vibration levels, noise, resonance conditions, among others. These types of conditions adversely affect system performance, causing operational problems and component damage.

In this context, the monitoring of the dynamic conditions of these structures and equipment is of high relevance. However, their complexity makes it not always possible to perform vibration measurements in places with higher risk of damage, due to the poor accessibility of the location, or because it contains substances toxic to human life. Furthermore, according to Qu [1], when the number of measuring points is very large, the necessary instrumentation makes the process expensive and time-consuming.

One way to estimate vibration levels in the structure regions of interest is by using finite element method models, Qu [1] and Chen *et al* [2], however, these models have the disadvantage that they do not always represent the structure properly, as they do not take into account manufacturing errors. According to Friswell *et al* [3], errors between the model and the actual structure can be minimized by performing calibration using experimental data, but one more difficulty is found, the number of degrees of freedom of the finite element model is much higher than number of degrees of freedom that can be monitored, therefore calibration cannot be performed directly.

In order to perform model calibration with experimental data, the first step was to match the number of degrees of freedom, and according to Friswell *et al* [3] and Qu [1] model reduction methodologies can be used. In this work, the GUYAN and SEREP reduction methods were used to verify their robustness when applied to models of complex structures. Two approaches are evaluated: in the first, the reduction was performed using the GUYAN method independently and in the second a combination of them was used (partial reduction by the GUYAN method and the final reduction by the SEREP).

The specified approaches are applied, as a case study, in a rectangular beam, for which a FEM model with multiple degrees of freedom was developed, and whose reduced model with the degrees of freedom compatible measured from the impact tests simulating the free body condition.

Finally, the modal parameters of the reduced model were correlated with data obtained experimentally using the *MAC*, *COMAC* and *DR* criteria, and the results showed high accuracy of the GUYAN-SEREP method in relation to the other approach tested.

2 Background

According to Friswell *et al* [3] and Qu [1], several model reduction methods can be found in the literature, but in the present work the GUYAN condensation method (in the physical space) and the SEREP method (in the modal space) were used. In addition, to verify whether the reduced model satisfactorily represents the complete model, some evaluation criteria are used, Silva *et al* [4] and Qu [1] such as Modal Assurance Criteria (*MAC*), Co-ordinate Modal Assurance Criterion (*COMAC*) and Relative Difference Between Modes (*DR*), these criteria are also used in the correlation between numerical and experimental results.

2.1 Guyan reduction method applied in dynamic analysis

Guyan's reduction or condensation method was developed for static problems, but its application can be extended to the dynamic analysis of structures and equipment, Qu [1], therefore, it has the following:

$$KX = F \tag{1}$$

Where K, X and F represent the stiffness matrix, displacement vector and force vector of the complete model, respectively. Since total degrees of freedom can be categorized as master and slave, then Eq. (1) can be rearranged as follows:

$$\begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} X_m \\ X_s \end{Bmatrix} = \begin{Bmatrix} F_m \\ F_s \end{Bmatrix} \tag{2}$$

In Eq. (2), the subscripts m and s indicate master and slave respectively. Expanding the matrix multiplication on the left side of Eq. (2), we have:

$$K_{mm}X_m + K_{ms}X_s = F_m \tag{3}$$

$$K_{sm}X_m + K_{ss}X_s = F_s \tag{4}$$

In Eqs. (3) and (4), it is observed that the displacements of the slave degrees of freedom have two parts: the first, due to the interaction with the master degrees of freedom (coupled displacement), and the second, due to the external forces acting on them (relative displacements). Thus, manipulating Eqs. (3) and (4), we have:

$$K_r X_m = F_r \tag{5}$$

Equation (5) is the static equilibrium equation corresponding to the master degrees of freedom, where K_r and F_r are known as the reduced model stiffness matrix and equivalent force vector, respectively, and are defined as:

$$K_r = K_{mm} - K_{ms}K_{ss}^{-1}K_{sm} \tag{6}$$

$$F_r = F_m - K_{ms}K_{ss}^{-1}F_s \tag{7}$$

To determine a relationship between the master and slave degrees of freedom, Guyan's method assumes that $F_s = 0$, hence from Eq. (4) we have:

$$X_s = RX_m \ \text{, where,}\ \ R = -K_{ss}^{-1}K_{sm} \tag{8}$$

The matrix R is known as Guyan Condensation Matrix. Note that this matrix is load independent because the external forces in the slave degrees of freedom were ignored. Thus, the displacement vector of Eq. (2) can be expressed as follows:

$$X = \begin{Bmatrix} X_m \\ X_s \end{Bmatrix} = TX_m \ \ \text{, where:}\ \ T = \begin{bmatrix} I \\ R \end{bmatrix} \tag{9}$$

In Eq. (9) the matrix T is known as the Coordinate Transformation Matrix. It is worth remembering that Guyan's reduction method was developed for static problems, but it can be used in dynamic analysis. Therefore, the motion equation of the complete model without damping is considered:

$$M\ddot{X}(t) + KX(t) = F(t) \tag{10}$$

Where M and K are the mass and stiffness matrices of the full model, $\ddot{X}$ and X are the acceleration and mass vectors of all degrees of freedom and F is the vector of external forces acting on the different degrees of freedom of the model.

Similarly, to static analysis, Eq. (10) can be expressed in terms of master and slave degrees of freedom:

$$\begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{X}_m(t) \\ \ddot{X}_s(t) \end{Bmatrix} + \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} X_m(t) \\ X_s(t) \end{Bmatrix} = \begin{Bmatrix} F_m(t) \\ F_s(t) \end{Bmatrix} \tag{11}$$

From Eq. (11) one can obtain:

$$M_{sm}\ddot{X}_m(t) + M_{ss}\ddot{X}_s(t) + K_{sm}X_m(t) + K_{ss}X_s(t) = 0 \tag{12}$$

In Eq. (12), it was assumed that $F_s(t) = 0$, similar to the static problem. Also, assuming that $\ddot{X}(t) = 0$ e $X(t) = 0$, the relationship between primary and secondary degrees of freedom is equal to Eq. (8). Thus, since the transformation matrix *T* is independent of time, deriving twice from Eq. (9), we have:

$$\ddot{X}(t) = T\ddot{X}_m(t) \tag{13}$$

Substituting Eqs. (12) and (9) in Eq. (11) and pre-multiplying by matrix *T* transpose, results:

$$M_r\ddot{X}_m(t) + K_rX_m(t) = F_r(t) \tag{14}$$

Equation (14) is the equation of motion of the reduced model, where M_r and K_r are the reduced mass and stiffness matrices, respectively, and F_r is the equivalent force vector, and are calculated as follows.:

$$K_r = T^TKT \quad , \; M_r = T^TMT \; , \quad F_r(t) = T^TF(t) \tag{15}$$

It should be noted that vibration responses in slave degrees of freedom are difficult to predict if the force vector acting on them is not zero. Therefore, in the reduction process, it is recommended to maintain as master all degrees of freedom whose vibration response is of interest.

2.2 System equivalent reduction expansion process (SEREP) method

The SEREP method was developed for the dynamic condensation of models, Maia *et al* [4] and Qu [1] using the modal approach. Therefore, the solution of Eq. (10) can be as follows:

$$X(t) = \Phi q(t) \tag{16}$$

Where, Φ e q(t) are the modal matrix and modal coordinates, respectively, of the complete model. However, obtaining the modal matrix is practically impossible in large models. In this sense, the modal truncation technique is often used. Thus, if p modes of the complete model are used in modal superposition, Eq. (16) can be rewritten as:

$$X(t) = \Phi_p q_p(t) \tag{17}$$

Similarly to Guyan's method, Eq. (17) can be rearranged as follows:

$$X(t) = \begin{Bmatrix} X_m(t) \\ X_s(t) \end{Bmatrix} = \begin{Bmatrix} \Phi_{mp} \\ \Phi_{sp} \end{Bmatrix} q_p(t) \tag{18}$$

From Eq. (18) one can obtain:

$$X_m(t) = \Phi_{mp}q_p(t) \tag{19}$$

$$X_s(t) = \Phi_{sp}q_p(t) \tag{20}$$

Equation (19) is a description of the responses for the master degrees of freedom in terms of the modal matrix of the master themselves. It can also be noted that Φ_{mp} is generally not a square matrix and depends directly on the degrees of freedom and modes considered. Thus, SEREP considers that the number of master degrees of freedom is greater than the number of modes considered ($m > p$).

As $m > p$, this means that we have more equations than unknowns. Therefore, Eq. (19) can be placed in the normal form (compatibility of degrees of freedom), projecting this equation as:

$$Y_p(t) = \Phi_{mp}^T X_m(t) \tag{21}$$

Merging Eq. (19) into Eq. (21) yields:

$$Y_p(t) = \Phi_{mp}^T \Phi_{mp} \tilde{q}_p(t) \tag{22}$$

Where, $\tilde{q}_p(t)$ is an approximate solution of $q_p(t)$, and can be calculated by manipulating Eq. (22):

$$\tilde{q}_p(t) = \left(\Phi_{mp}^T \Phi_{mp}\right)^{-1} Y_p(t) \tag{23}$$

Merging Eq. (21) into Eq. (23):

$$\tilde{q}_p(t) = \Phi_{mp}^{+} X_m(t) \tag{24}$$

Where, Φ_{mp}^{+} is the generalized inverse of Φ_{mp} and is defined as:

$$\Phi_{mp}^{+} = \left(\Phi_{mp}^T \Phi_{mp}\right)^{-1} \Phi_{mp}^T \tag{25}$$

Merging Eq. (24) into Eq. (20):

$$X_s(t) = R X_m(t) \tag{26}$$

Where *R* is the SEREP dynamic condensation matrix and is calculated as:

$$R = \Phi_{sp} \Phi_{mp}^{+} \tag{27}$$

Furthermore, the transformation matrix *T* can be calculated by substituting Eq. (24) in Eq. (18):

$$T = \Phi_p \Phi_{mp}^{+} = \begin{bmatrix} \Phi_{mp} \Phi_{mp}^{+} \\ \Phi_{sp} \Phi_{mp}^{+} \end{bmatrix} \tag{28}$$

Thus, the reduced stiffness and mass matrices can be calculated using Eq. (15).

2.3 Criteria used for model assessment

In model reduction techniques, the complete model is transformed to a reduced model, which contains only the master degrees of freedom, where the transformation matrices, *T*, can be given by Eq. (9), in physical space, or (28), in modal space, and through them it is possible to calculate the reduced mass and stiffness matrices, M_r and K_r, by Eq. (15), through which it is possible to verify the accuracy of the reduction technique used.

Following are the criteria used, derived from the experimental modal analysis, for the verification of the reduced model, by comparing the reduced modal matrix and the complete modal matrix. These criteria will also be used to correlate the reduced model with experimentally obtained data.

2.3.1 Modal assurance criteria (MAC)

It is a simple way to correlate two mode shapes, check its linear dependence, verifying the modal assurance between modes, it is calculated as:

$$MAC(\varphi_A, \varphi_B) = \frac{\left|\varphi_A^T \varphi_B\right|^2}{(\varphi_A^T \varphi_A)(\varphi_B^T \varphi_B)} \tag{29}$$

Where φ_A e φ_B are the mode shapes *A* e *B*, respectively. A *MAC* value close to 1 suggests that the two modes are well correlated, and values close to 0 suggest bad correlated modes. According to Qu [1] the *MAC* correlation is the first step in the correlation process.

2.3.2 Co-ordinate modal assurance criterion (COMAC)

According to Friswell et. al. [3] and Ewins [5], *MAC* is an important tool in mode correlation, but may pose a challenge in correlation of modes that are closely spaced in frequency or when the selected locations for measurement or modeling are insufficient. In this sense, a variant of *MAC*, called a co-ordinate *MAC* or *COMAC*, can be used for error finding. *COMAC* values reflect the discrepancy between the compared modal forms and can be calculated as follows:

$$COMAC(i) = \frac{\left(\sum_{j=1}^{L} (\Phi_A)_{i,j} (\Phi_B)_{i,j}\right)^2}{\left(\sum_{j=1}^{L} (\Phi_A)_{i,j}{}^2\right)\left(\sum_{j=1}^{L} (\Phi_B)_{i,j}{}^2\right)} \tag{30}$$

Where *L* e *i* represent respectively the number of modes being compared and the coordinate being evaluated and Φ_A e Φ_B represent the modal matrices being correlated. *COMAC* values close to 1 indicate that all mode coordinates associated with degree of freedom i are equal, values below 0.9 indicate discrepancy in the evaluated degree of freedom.

2.3.3 Relative difference between modes (DR)

The relative difference assesses the level of variances in amplitudes of each degree of freedom between the modes being compared and is calculated as follows:

$$DR(i,j) = \left|\frac{(\Phi_A)_{i,j}-(\Phi_B)_{i,j}}{(\Phi_A)_{i,j}}\right| \text{ , se } (\Phi_A)_{i,j} = (\Phi_B)_{i,j} \rightarrow DR(i,j) = 0 \quad (31)$$

Where, Φ_A e Φ_B represent the modal matrices being compared and indexes i and j represent the degree of freedom and the mode being evaluated respectively. Values close to 0 indicate that the amplitudes of the degrees of freedom analyzed are alike.

2.3.4 Model correlation

If the differences in mass and stiffness between the numerical model and real structures are small, and if a set of mode shapes for the projection is chosen well, then it is assumed that a projection of the numerical mode shapes in the subspace of the experimental ones yields a correlation process, Brincker *et al* [6].

The subset of numerical (B_a) modes is defined according to the Local Correspondence principle (LC), in this method the number of mode shapes is ranked after the distance in frequency to the mode shape to be correlated and the mode shape cluster are constructed by including an increasing number of mode shapes, from the ranking, at each cluster iteration. The optimal number of mode shapes to be used in the correlation is calculated by optimizing the *MAC* value of the clusters Brincker *et al* [6].

Therefore, the correlation process is defined as:

$$\hat{a} = B_a\hat{p} \quad (32)$$

Where, $\hat{a}$ is the numerical mode shape matrix fitted with the experimental results, $\hat{p}$ is the transformation matrix calculated according to equation *2.35* and B_a^+ is the pseudo inverse of the subset of numerical modes chosen for the fitting process.

$$\hat{p} = B_a^+ a \quad (33)$$

3 Methodology

Items 2.1 and 2.2 presented the model reduction methods that were used in this work and item 2.3 presented the criteria used to verify the reduced model (when compared with the complete model) and the correlation with experimentally obtained data.

As the objective of this work is to present a methodology that allows to estimate and correlate models of complex structures, developed using the finite element method, we used two approaches applied to the case of a rectangular beam in the free body condition.

The beam under analysis is an aluminum beam with 2145 mm length, 6.17 mm height and 25.42 mm wide. A multiple reference impact testing modal analysis was performed, the experimental apparatus for the modal analysis consisted of 9 equally sparse accelerometers attached to the described aluminum beam which was hung at the ends of the wires to simulate the rigid body condition in horizontal plane movement as can be seen on Fig. 1. The experimental modal characteristics was calculated using Experimental Modal Analysis (EMA).

3.1 First approach – Guyan reduction

The full model for the beam, developed in a commercial finite element software was used as the starting point for the GUYAN reduction method (which is implemented in the software used), a reduction of up to 9 degrees of freedom (translation) was performed in order to make it compatible with the measured degrees of freedom. Figure 2 presents a flowchart of the GUYAN 's method reduction process.

3.2 Second approach – Guyan-Serep mixed reduction

As mentioned in items 1 and 2.1, GUYAN 's method (because it is in physical space) relies heavily on the selection of master degrees of freedom, and complex structure models can have millions of degrees of freedom, therefore, too many master degrees of freedom would be necessary to perform the reduction satisfactorily and in most cases making it impossible to match the model with the experimental degrees of freedom.

Figure 1. Experimental Apparatus.

On the other hand, exporting the mass and stiffness matrices (from commercial software) of the complete model for complex structures is not always recommended, due to their size and the high computational effort necessary for their processing.

In this context, in order to streamline the reduction process, the second approach has two steps: the first stage partially reduces the model using GUYAN 's method (in physical space) to a number of manageable, computationally low-cost, master degrees of freedom. The second stage is to export GUYAN's reduced mass and stiffness matrices for use as input data to SEREP and to perform the final reduction (in modal space), thus matching the number of degrees of freedom of the reduced model with the experimental ones. Figures 2, 3 and 4 shows the flowchart of the GUYAN, SEREP and the GUYAN-SEREP mixed method process, respectively.

3.3 Model correlation

The reduced mode shape matrix was fitted with the experimental data, this process was undertaken aiming the smoothing of the mode shapes amplitudes with the measured values of the real structure.

The smoothing process adjusts the numerical model to the real structure properties, this allows a subsequent step of expansion to the full set of degrees of freedom originally developed at the full numerical mode. This expansion allows the prediction of the behavior at non measured degrees of freedom. The fitted numerical data is correlated with the experimental data and the results are compared with the non-fitted data in order to verify the effectiveness of the fitting process.

It is noteworthy that the criterion used for the verification of the reduced model with the complete model was the *MAC*. For the correlation between the results of the reduced model and the experimental ones, all the criteria presented in item 2.3 (Fig. 5) were used.

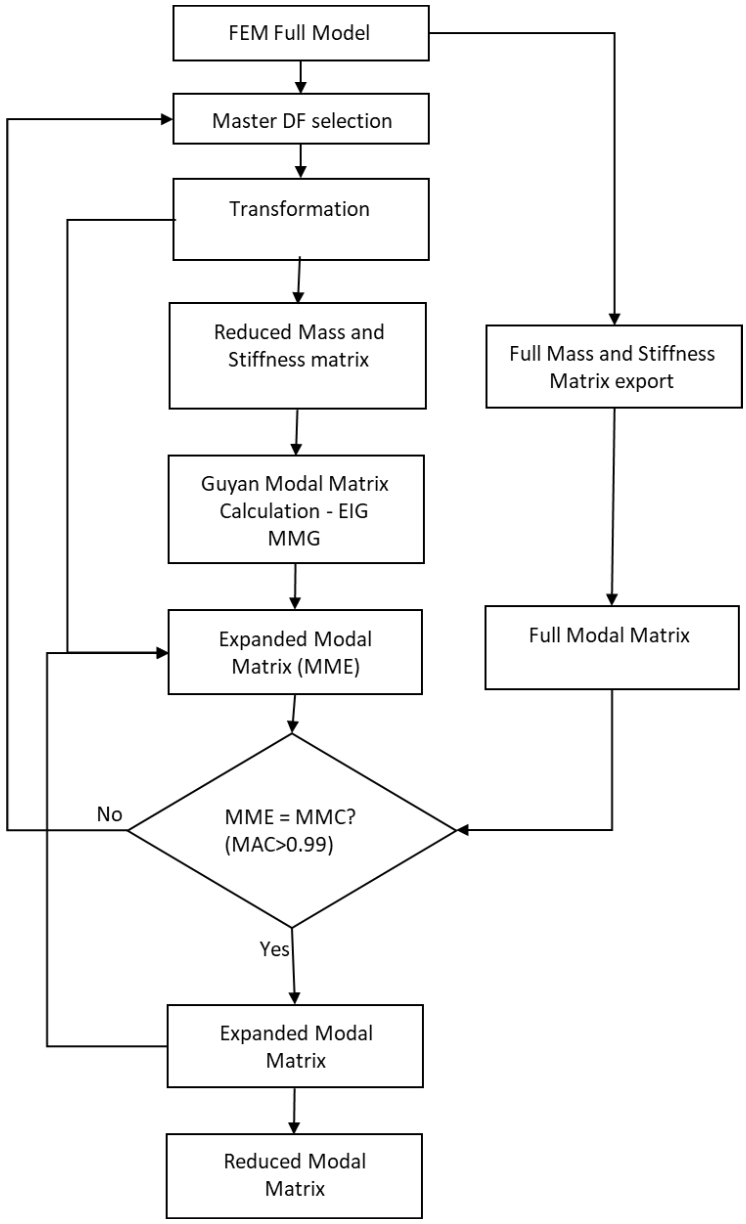


Figure 2. Processing with the GUYAN reduction method.

4 Results and discussion

The results presented in this paper are in the following order: First the reduction process is validated by means of comparing its *MAC* and *DR* with the original FE model, this validation process is done for the two reduction processes, GUYAN and then GUYAN-SEREP. After that the numerical-experimental comparison is shown as a comparison of the reduced model (GUYAN-SEREP) and the EMA experimental results. Finally, the results for the model correlation is shown by giving the improvement of the criteria used to compare the numerical results with the experimental.

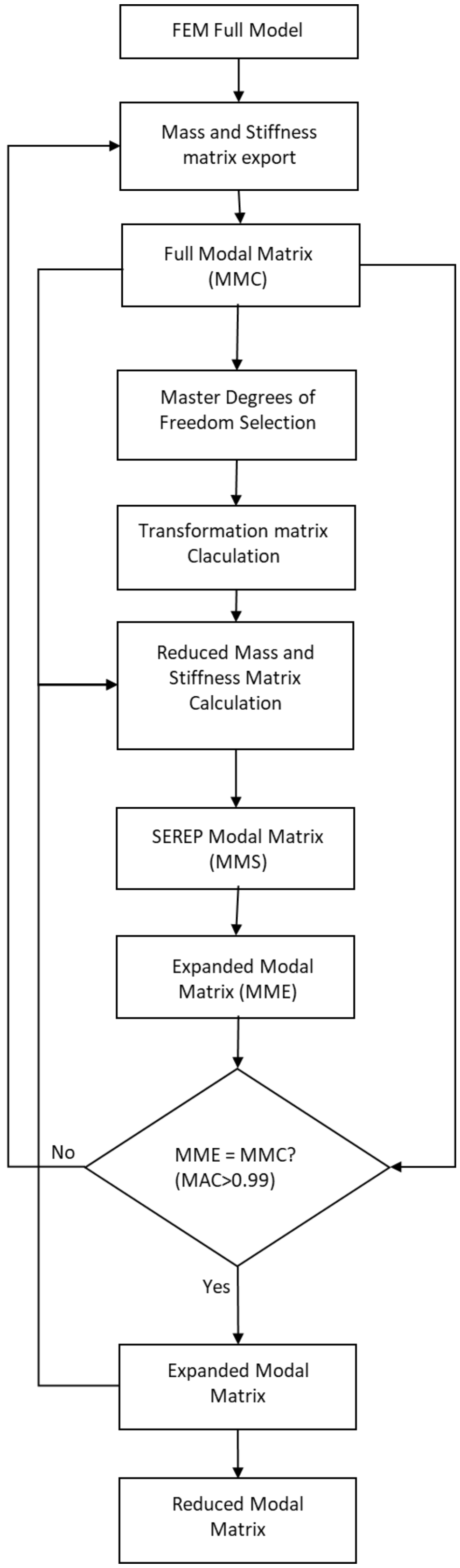


Figure 3. Processing with the SEREP reduction method.

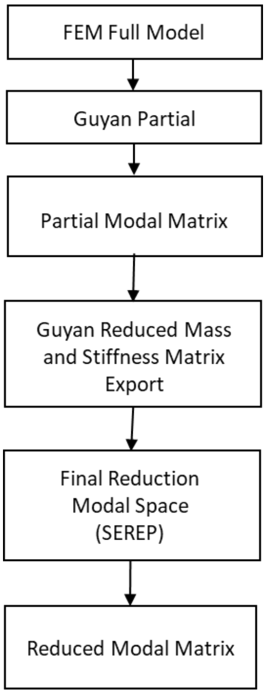


Figure 4. Processing with the GUYAN-SEREP mixed method.

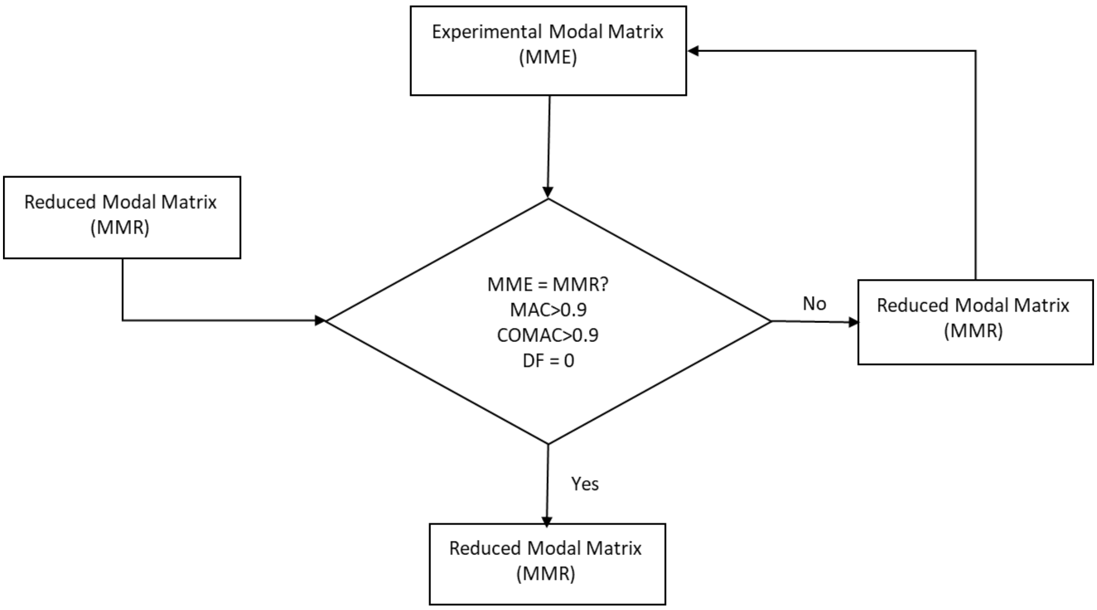


Figure 5. Numerical-Experimental correlation process.

4.1 Verification of reduction process

4.1.1 First approach

The full model was reduced, using the GUYAN method, down to nine master nodes (located exactly where the accelerometers were placed). It exhibited good correlation with the full model. On Fig. 6 can be seen the first seven mode shapes where it was plotted the numerical reduced, experimental and the analytical mode shapes. On top of each figure the correlation *MAC* number between the reduced and the full model is indicated for each.

Fig. 7 presents the complete *MAC* results for the correlation of the whole modal matrix on the left bar plot and on the right the diagonal numbers are indicated in more detailed.

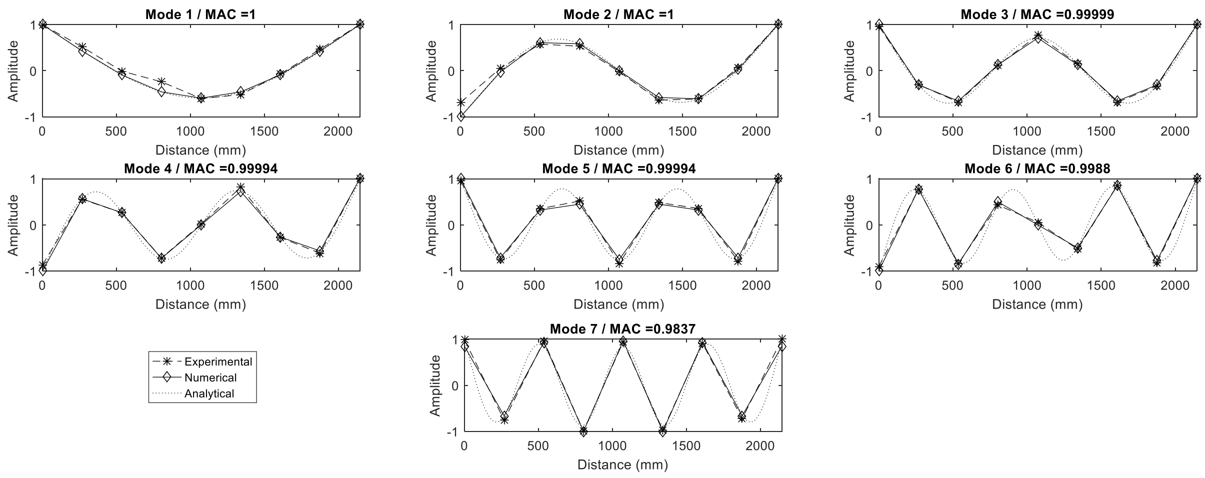


Figure 6. Modal plots for GUYAN reduced model.

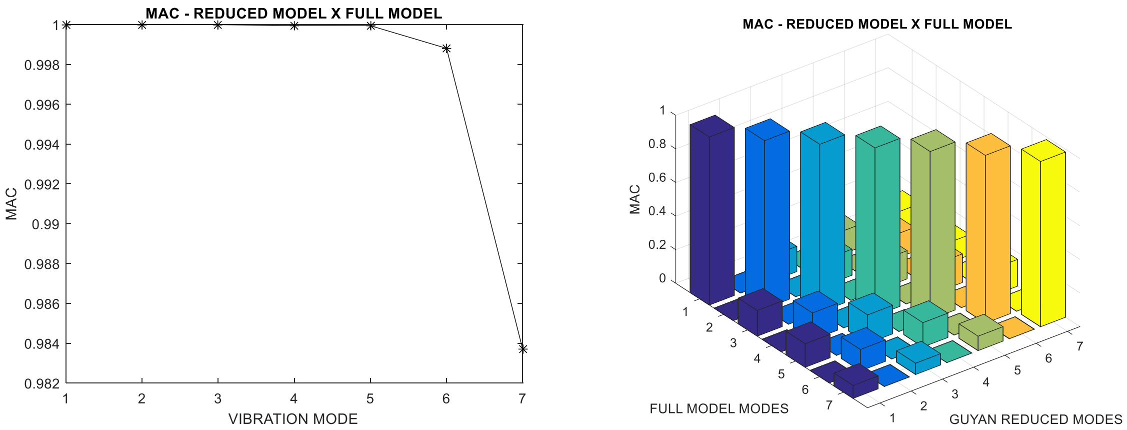


Figure 7 *MAC* for GUYAN reduced and full models.

Figure 8 presents the relative differences between full and reduced models on each master node for all nodes respectably.

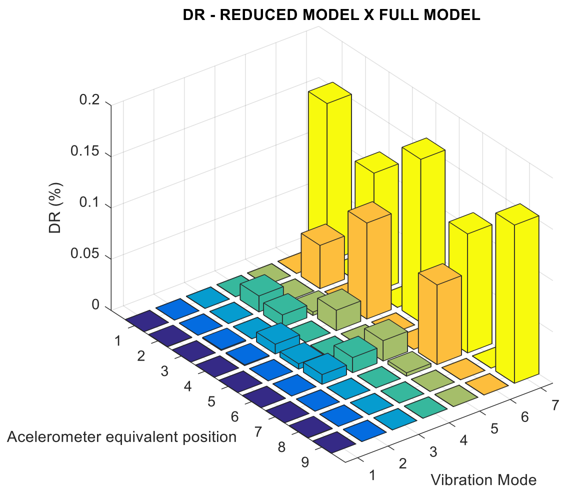


Figure 8. *DR* for GUYAN reduced and full models.

4.1.2 Second approach

The same reduction applied above was undertaken, now using the GUYAN-SEREP mixed method and it also presented good correlation with the full model, when the correlation criteria were used. On Fig. 9 can be seen the first seven mode shapes where it was plotted the numerical reduced, experimental and the analytical mode shapes. On top of each figure the correlation *MAC* number between the reduced and the full model is indicated for each.

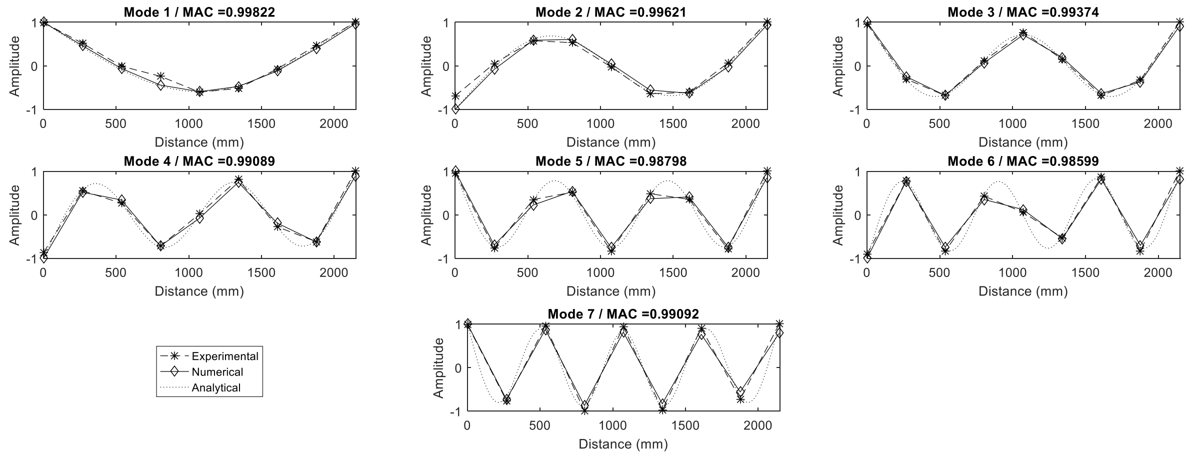


Figure 9. Modal plots for GUYAN-SEREP reduced model.

Figure 10 presents the complete *MAC* results for the correlation of the complete modal matrix, on the left as a bar plot and on the right the *MAC* matrix diagonal numbers are plotted individually for a more detailed view.

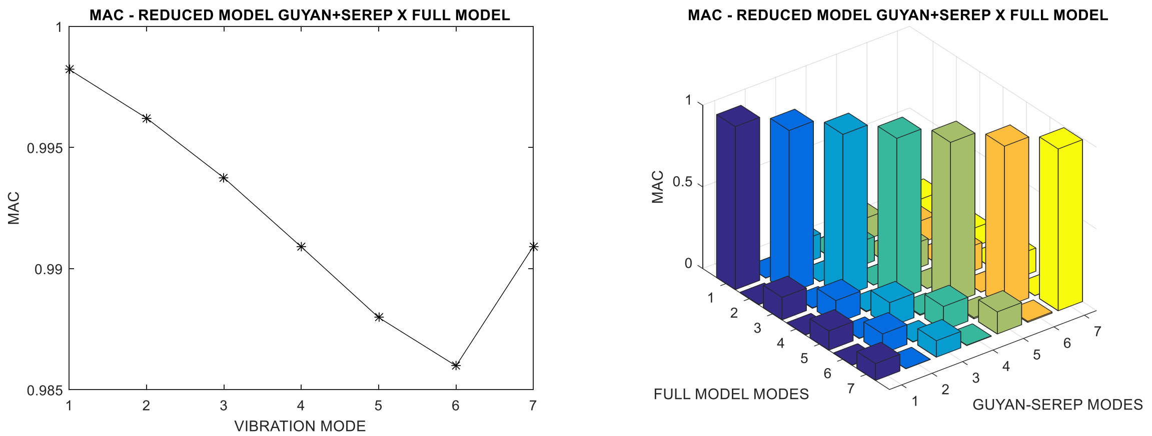


Figure 10. *MAC* for GUYAN-SEREP reduced and full models.

Figure 11 presents the relative differences between full and reduced models on each master node for all nodes respectably.

4.2 Validation and experimental Correlation

4.2.1 Numerical-experimental comparison

As the reduced model is validated relative to the full model, as showed on the results above, then, the correlation criteria between the reduced model and the experimental results was then employed.

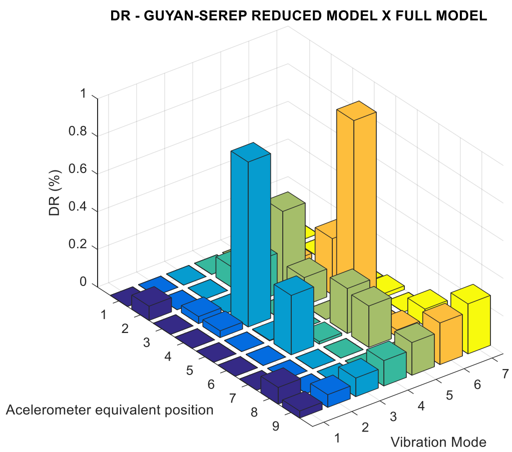


Figure 11. *DR* for Guyan-SEREP reduced and full models.

Figure 10 already includes the experimental data in the mode shapes, so it can be compared to the mode shapes of the reduced method and the analytical results. For the defined criteria, Fig. 12 presents the complete *MAC* results for the correlation of the complete modal matrix, on the left as a bar plot and on the right the *MAC* matrix diagonal numbers are plotted individually for a more detailed view.

Figure 13 on the left (a) presents the relative differences between experimental and reduced models on each master node for all nodes respectably and on the right (b) the *COMAC* values for the same correlation.

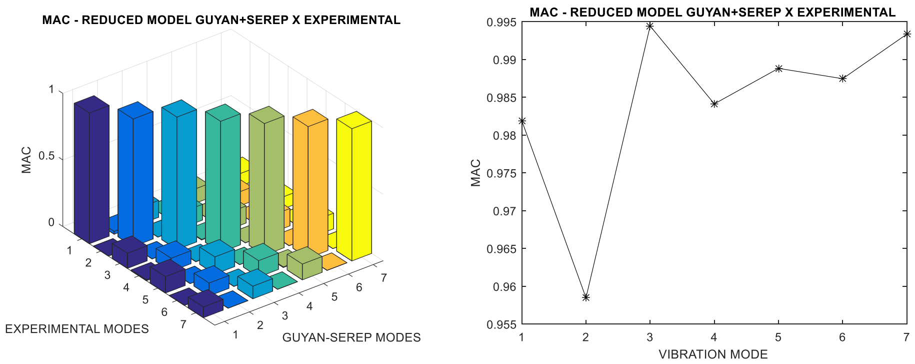
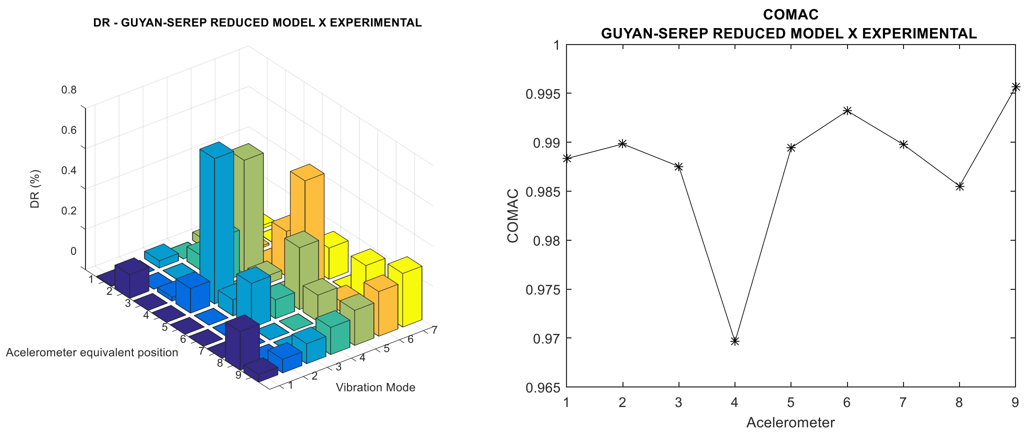


Figure 12. *MAC* for GUYAN-SEREP reduced and experimental models



(a) (b)

Figure 13. *DR* for GUYAN-SEREP reduced and experimental data (a) *COMAC* values (b).

4.2.2 Model correlation

On Fig. 14 can be seen the first seven mode shapes where it was plotted the fitted numeric reduced, numeric reduced (non-fitted), experimental and the analytical mode shapes. On top of each figure the correlation *MAC* number between the fitted numeric reduced and the experimental model is indicated for each.

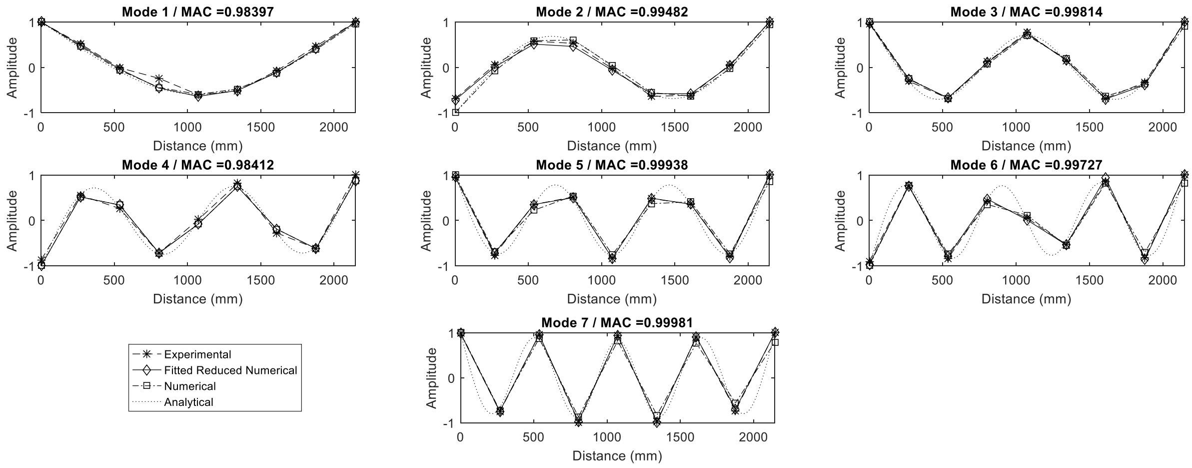


Fig. 14. Modal plots comparing fitted, non-fitted and experimental mode shapes.

Figure 15 shows the *MAC* value, between the reduced numeric and experimental, of each mode shape comparing the result for the fitted and non-fitted.

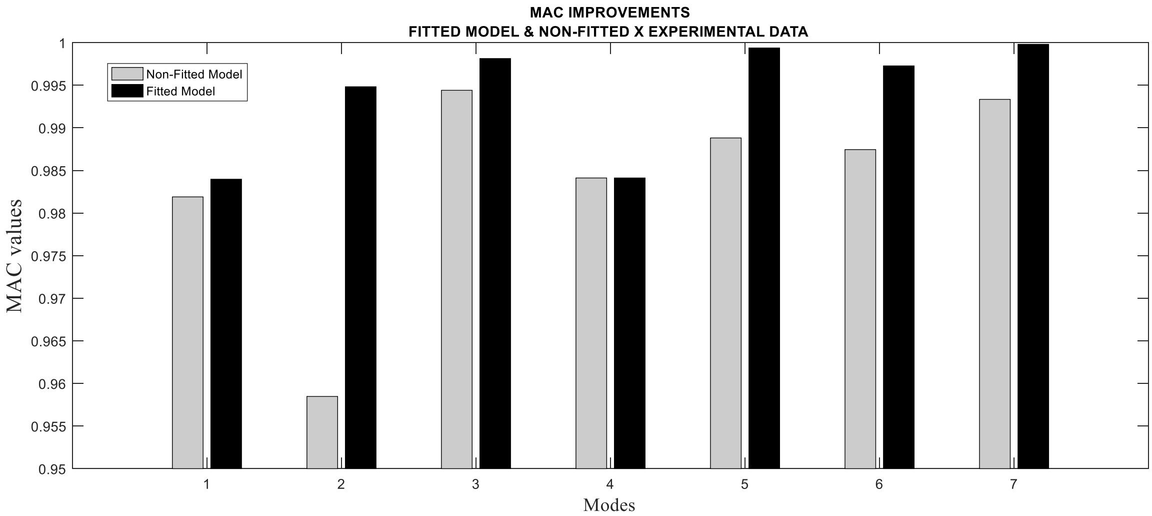


Figure 15. *MAC* improvements within the fitting process.

5 Conclusions e recommendations

The results for the model reduction GUYAN and GUYAN-SEREP mixed approaches proposed in this work showed good correlation with the full model. Therefore, the reduced models can be used in subsequent analysis as good representation of the full model.

The GUYAN-SEREP showed comparable results with the GUYAN reduction process with the advantage of being able to operate better with complex structures as the matrixes exported from the finite element software are smaller in size and therefore computationally cheaper to operate in the second part of the reduction process.

A clear improvement can be seen on the modal assurance criteria correlation when the fitting process is undertaken, meaning that the numeric results when fitted are closer to the experimental ones used on the fitting process. Thus, the fitted results when used to expand the measurements onto non measured degrees of freedom are more prone to yield closer to reality results.

Finally, the presented methodology can be applied as an intermediate step for the prediction of fatigue failure of structural elements and equipment used in offshore units, especially those that need to have high reliability, as well as the identification of dynamic loads.

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