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OUTPUT-ONLY MODAL PARAMETER IDENTIFICATION OF A RECTANGULAR BEAM USING MIMO AND SIMO TESTS

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Abstract. The dynamic behavior of structures can be evaluated through modal parameters, which can be obtained experimentally, and used to solve vibration problems, in sensitivity analyses, in structural modification, for numerical and experimental model correlation, for updating numerical model, for structural health monitoring, etc. The present work aims to identify the modal parameters (natural frequencies, damping ratios and vibration modes) of a rectangular aluminum beam suspended in the air, simulating the “free-free” boundary condition. The rectangular beam was instrumented with 09 accelerometers arranged equidistantly along the length. Two types of tests were performed: (i) the multiple reference impact testing (MRIT), which simulates MIMO (multiple input/multiple output) test; (ii) the SIMO (single input/multiple output) test, which generates responses in the 09 accelerometers due to the impacts in 01 degree of freedom. Modal parameters were identified using only the vibration responses, through two frequency domain techniques: Enhanced Frequency Domain Decomposition (EFDD) and Frequency-Spatial Domain Decomposition (FSDD). Results showed that the average difference in natural frequencies estimation with the EFDD technique, using MIMO and SIMO tests, was 0.25%, a low and acceptable percentage. When comparing EFDD (MIMO test), FSDD (SIMO test) and FEM model results, the average differences in natural frequencies estimation are 0.33% and 1.63%, respectively. According to the results, both EFDD and FSDD methods can be used to identify the modal parameters, with similar results, using either the MIMO or SIMO tests. As the SIMO test demands less time, it becomes a fast alternative for modal identification of structures.

Keywords: Output-Only Modal Analysis, Frequency Domain Identification Methods, MIMO, SIMO Tests.

1 Introduction

Any type of mechanical system is submitted to different types of shipments during its lifetime. These loads may have diverse natures: external or internal, deterministic or random, controlled or not. The specifications of intensity, duration and periodicity of these forces, when interacting with system's properties, define the dynamic behavior of the same.

Winds, hurricanes, waves, maritime currents, the unbalance itself and misalignment of a mechanical system's components are factors capable of influencing the dynamic behavior of a system.

Controlling the characteristics of these excitation forces is a complex and sometimes impossible task. On the other hand, these forces can lead to behaviors that generate undesirable conditions, such as: fatigue, high stresses and vibration levels, noises, resonance, among others. These operating conditions negatively alter the system performance, causing operational problems and damage to its components.

The dynamic behavior of structures and equipment can be evaluated through modal analysis. Modal parameters are used in vibration solutions, to estimate forced vibration amplitude, in sensitivity analyses, for structural modification, and others.

The present work aims to identify the modal parameters (natural frequencies, damping ratios and vibration modes) of a rectangular aluminum beam suspended in air, simulating the free-free contour condition. The beam was instrumented with 09 (nine) accelerometers way along the length; the excitation was through an impact hammer.

Only vibration responses (output-only) were used to obtain modal parameters, using two techniques in frequency domain: Enhanced Frequency Domain Decomposition (EFDD) and Frequency-Spatial Domain Decomposition (FSDD).

Two types of tests were performed: (i) the multiple reference impact testing (MRIT), which simulates MIMO (multiple input/multiple output) test; (ii) the SIMO (single input/multiple output) test, which generates responses in the 09 accelerometers due to the impacts in 01 degree of freedom.

These methods allow one to find beam modal properties experimentally. In this way, it is possible to correlate, at the end of the article, experimental and numerical data. A comparison between the two techniques used, as a form of experiment validation, will still be made.

2 Output-Only Modal Analysis

The great limitation of the Experimental Modal Analysis (EMA) is the necessity of knowing the excitation forces magnitudes acting on the tested system, so that it becomes possible to estimate its modal parameters. The force measurement in large structures can be very difficult to be accomplished, if not impossible. To excite a complex structure implies in a considerable economical cost, which can cause local damages, even in the cases of laboratory tests, where it is hard to reproduce the "real life" excitations in a reliable way.

For these reasons, the Output-Only Modal Analysis was developed in order to bypass the existing limitation of the traditional method and, in this way, to offer the possibility of modal parameters assessment only from dynamic response information. It can be mentioned that this analysis allows to estimate these parameters in cases of unknown excitation forces (in terms of its magnitude and frequency), applied to linear and time-invariant systems.

This type of study is advantageous because, for being applied in operating systems, it can be used without damaging the operating conditions. In other words, it avoids the obstruction of bridges and viaducts' traffic or possible interruptions in the use of equipment and in offshore structures, for example. Making it a suitable method for monitoring cases of structural integrity and vibration control.

2.1 Enhanced Frequency Domain Decomposition (EFDD)

This modal identification technique is an enhanced version of the Frequency Domain Decomposition (FDD) method proposed by Brincker et al. [1]. Its main advantage, related to the FDD method, is the possibility of a more precise assessment of the natural frequencies and the identification of damping rate.

The essence of the EFDD technique, as described by Jacobsen and Brincker [2], rely on a decomposition of the power spectrum density (PSD) of the measured response. The estimated PSD matrix, $\mathbf{G}_{yy}(i\omega)$, formed by the estimated PSD's for all sensors, which for a complex Hermitian and positive definite matrix, has the following form:

$$\mathbf{G}_{yy}(i\omega) = \mathbf{H}(i\omega)\mathbf{G}_{xx}(i\omega)\mathbf{H}^{\mathrm{H}}(i\omega). \tag{2.1}$$

Where, $\mathbf{G}_{yy}(i\omega)$ and $\mathbf{G}_{xx}(i\omega)$ are the output and input (stochastic) PSD matrices of size $\mathrm{N} \times \mathrm{N}$; N is the number of measurements and $\mathbf{H}(i\omega)$ is the frequency response function (FRF) matrix, which ca be expressed in partial form as (Zhang et al. [3]):

$$\mathbf{H}(i\omega) = \sum_{m=1}^{M} \left(\frac{\mathbf{R}_m}{i\omega - \lambda_m} + \frac{\mathbf{R}_m^*}{i\omega - \lambda_m^*} \right) \tag{2.2}$$

Where M is the number of modes; λ_m is the mth pole, and are related to the damping factor (σ_m), damped modal frequency (ω_{dm}) and modal damping ratio (ζ_m) as given in Eq. (2.3):

$$\lambda_m = -\sigma_m + i\omega_{dm}\,;\ \omega_m = \sqrt{\sigma_m^2 + \omega_{dm}^2};\ \zeta_m = \frac{\sigma_m}{\sqrt{\sigma_m^2 + \omega_{dm}^2}} \tag{2.3}$$

$\mathbf{R}_m$ and $\mathbf{R}_m^*$ are the corresponding conjugate residue matrices pairs and $\mathbf{R}_m$ is given as:

$$\mathbf{R}_m = \boldsymbol{\varphi}_m \boldsymbol{\gamma}_m^{\mathrm{T}} \tag{2.4}$$

Where, $\boldsymbol{\varphi}_m$ and $\boldsymbol{\gamma}_m$ are the mth mode shape and modal participation factor vectors, respectively, and N_{ref} is the number of reference sensors:

$$\boldsymbol{\varphi}_m = [\phi_{1m}, \phi_{2m}, \dots, \phi_{Nm}]^{\mathrm{T}} \tag{2.5a}$$

$$\boldsymbol{\gamma}_m = \left[\gamma_{1m}, \gamma_{2m}, \dots, \gamma_{N_{ref}m}\right]^{\mathrm{T}} \tag{2.5b}$$

When all output measurements are taken as references, $\mathbf{H}(i\omega)$ is a square matrix and:

$$\boldsymbol{\gamma}_m = \boldsymbol{\varphi}_m \tag{2.6}$$

If the excitation spectrum is flat (or input is white noise), then $\mathbf{G}_{xx}(i\omega) = \mathbf{G}_{xx}$ is a constant matrix. Substituting Eq. (2.2) into Eq. (2.1) leads to:

$$\mathbf{G}_{yy}(i\omega) = \sum_{m=1}^{M}\sum_{s=1}^{M} \left(\frac{\mathbf{R}_m}{i\omega - \lambda_m} + \frac{\mathbf{R}_m^*}{i\omega - \lambda_m^*} \right) \mathbf{G}_{xx} \left[\frac{\mathbf{R}_s}{i\omega - \lambda_s} + \frac{\mathbf{R}_s^*}{i\omega - \lambda_s^*} \right]^H \tag{2.7}$$

Where the superscript "H" denotes complex conjugate and transpose. After some mathematical manipulations, the output PSD matrix can be reduced to pole/residue form as:

$$\mathbf{G}_{yy}(i\omega) = \sum_{m=1}^{M} \left(\frac{\mathbf{A}_m}{i\omega - \lambda_m} + \frac{\mathbf{A}_m^H}{-i\omega - \lambda_m^*} + \frac{\mathbf{A}_m^*}{i\omega - \lambda_m^*} + \frac{\mathbf{A}_m^T}{-i\omega - \lambda_m} \right) \tag{2.8}$$

Where $\mathbf{A}_m$ is the corresponding residue matrix. When taking all measurements as references, $\mathbf{A}_m$ is an $\mathrm{N} \times \mathrm{N}$ Hermitian matrix and can be derived as:

$$\mathbf{A}_m = \mathbf{R}_m \mathbf{G}_{xx} \sum_{s=1}^{M} \left(\frac{\mathbf{R}_s^H}{-\lambda_m - \lambda_s^*} + \frac{\mathbf{R}_s^T}{-\lambda_m - \lambda_s} \right) \tag{2.9}$$

Taking Eq. (2.4) into account, the output PSD can be modally decomposed as:

$$\mathbf{G}_{yy}(i\omega) = \sum_{m=1}^{M} \left(\frac{\boldsymbol{\varphi}_m \boldsymbol{\chi}_m^{\mathrm{T}}}{i\omega - \lambda_m} + \frac{\boldsymbol{\varphi}_m^* \boldsymbol{\chi}_m^{\mathrm{H}}}{i\omega - \lambda_m^*} + \frac{\boldsymbol{\chi}_m \boldsymbol{\varphi}_m^{\mathrm{T}}}{-i\omega - \lambda_m} + \frac{\boldsymbol{\chi}_m^* \boldsymbol{\varphi}_m^{\mathrm{H}}}{-i\omega - \lambda_m^*} \right) \tag{2.10}$$

Where, $\boldsymbol{\varphi}_m$ and $\boldsymbol{\chi}_m$ are the mth mode shape and operational reference vectors, respectively. The PSD derived above has a 4-quadrant symmetry and is called full PSD.

The operational reference vector is a function of the modal parameters and the PSD of the unknown random input forces. The modal participation factors and, consequently, the modal scale factors cannot be determined from an OMA test.

Full-PSD is normally estimated via the periodogram approach and a tradeoff between stochastic uncertainties and bias errors introduced by leakage. Therefore, additional noise reduction would be preferable, and it is achieved by estimating only the positive poles of the output PSD:

$$\mathbf{G}_{\boldsymbol{yy}}(i\omega) = \sum_{m=1}^{M} \left(\frac{\boldsymbol{\varphi}_m \boldsymbol{\chi}_m^{\mathrm{T}}}{i\omega - \lambda_m} + \frac{\boldsymbol{\varphi}_m^* \boldsymbol{\chi}_m^{\mathrm{H}}}{i\omega - \lambda_m^*} \right) \tag{2.11}$$

The PSD with only the positive poles is called positive PSD, or half PSD (h-PSD), which is like the modally decomposed FRF in format.

The key step in the EFDD technique is to apply a singular value decomposition (SVD) to the h-PSD estimated at a discrete frequency, $\omega = \omega_k$ (Jacobsen et al. [4]):

$$\widehat{\mathbf{G}}_{\boldsymbol{yy}}(i\omega_k) = \mathbf{U}(\omega_k)\mathbf{S}(\omega_k)\mathbf{U}(\omega_k)^{\mathrm{H}} \tag{2.12}$$

Where the diagonal matrix $\mathbf{S}(\omega_k)$ is the singular value matrix and $\mathbf{U}_{\boldsymbol{k}} = [\mathbf{u}_{k1}, \mathbf{u}_{k2}, \dots, \mathbf{u}_{kM}]$, is the unitary matrix, consisting of unitary vectors $\mathbf{u}_k$, which are interpreted as the modal shape of the system at each natural frequency. In this case, the right side of Eq. (2.12) implies that all output measurements were used as references.

It is observed that singular values are a function of the excitation frequency. When the frequency approaches a modal frequency ω_m, the PSD matrix approximates a rank one matrix as:

$$\widehat{\mathrm{G}}_{yy_{\omega \to \omega_m}}(i\omega_k) = \mathrm{s}_1(\omega_m)\mathbf{u}_1(\omega_m)\mathbf{u}_1^{\mathrm{H}}(\omega_m) \tag{2.13}$$

The first singular value reaches a maximum, $\mathrm{s}_1(\omega_m) \to max$. The corresponding singular vector $\mathbf{u}_1(\omega_m)$ is an estimate of the mth mode shape $\widehat{\boldsymbol{\varphi}}_m = \mathbf{u}_1(\omega_m)$ with unitary normalization.

In the vicinity of the natural frequency it is possible to obtain singular vectors having a high MAC value which enable the establishment of a Single-Degree-Of-Freedom (SDOF) spectral density function, for a specific mode, which is transformed to the time domain yielding an auto-correlation function of the SDOF system. From this auto-correlation function, the natural frequency is obtained by determining the number of zero-crossing as a function of time using a simple least-squares fit. The damping ratio is obtained from the logarithmic decrement of the auto-correlation function again using a simple least-squares fit (Jacobsen et al. [4]).

MAC is basically a linear and quadratic regression correlation coefficient that measures the consistency between two vectors. MAC values range from 0 to 1, where 0 indicates inconsistency or orthogonality between vectors and 1 indicates perfect consistency (differing only by a scale factor).

Considering the estimation, by two different methods, of the mth vibration modes, $\{\phi_m^1\}$ e $\{\phi_m^2\}$, the MAC between these modes is given by:

$$\mathrm{MAC}(\{\phi_m^1\}, \{\phi_m^2\}) = \frac{|\{\phi_m^1\}^H \{\phi_m^2\}|^2}{(\{\phi_m^1\}^H \{\phi_m^1\})(\{\phi_m^2\}^H \{\phi_m^2\})} \tag{2.14}$$

When MAC is calculated between vibration modes estimated by only one method, Eq. (2.15) is changed to:

$$\mathrm{MAC}(\{\phi_m\}, \{\phi_n\}) = \frac{|\{\phi_m\}^H \{\phi_n\}|^2}{(\{\phi_m\}^H \{\phi_m\})(\{\phi_n\}^H \{\phi_n\})} \tag{2.15}$$

In this paper, MAC was used to verify the correlation between the vibration modes estimated by EFDD-MIMO, EFDD-SIMO and SFDD-SIMO.

2.2 Frequency-Spatial Domain Decomposition (FSDD)

The Frequency-Spatial Domain Decomposition (FSDD) technique is an alternative approach utilizing curve-fitting of a Single-Degree-Of-Freedom (SDOF) spectral density function directly in the frequency domain. The main benefit is a more accurate estimation of the natural frequencies and damping ratios, both in case of pure stochastic excitation and in the presence of deterministic excitation (Zhang et al. [3]) and the mode shapes are found as in the original EFDD technique as described above.

The key step in the FSDD technique is to conduct a spectrum decomposition of the h-PSD estimated at a discrete frequency, $\omega = \omega_k$ (Zhang et al. [3]):

$$\widehat{\mathbf{G}}_{\boldsymbol{yy}}(i\omega_k) = \mathbf{U}(\omega_k)\mathbf{S}(\omega_k)\mathbf{V}(\omega_k)^{\mathrm{H}} \tag{2.16}$$

Where the diagonal matrix $\mathbf{S}(\omega_k)$ is consisting of real scalar values, s_{kj}, and $j = 1, 2, \dots, \mathrm{M}$; $\mathbf{U}_{\boldsymbol{k}} = [\mathbf{u}_{k1}, \mathbf{u}_{k2}, \dots, \mathbf{u}_{kM}]$ and $\mathbf{V}_{\boldsymbol{k}} = [\mathbf{v}_{k1}, \mathbf{v}_{k2}, \dots, \mathbf{v}_{kM}]$ are the right and left unitary matrices, consisting of unitary vectors $\mathbf{u}_k$ and $\mathbf{v}_k$, respectively. When taking all the output measurements as references, $\mathbf{U}_{\boldsymbol{k}} = \mathbf{V}_{\boldsymbol{k}}$.

The spectrum decomposition can be implemented in singular value decomposition (SVD) and in eigen-value decomposition (EVD), if the measurements were taken as references. When implemented via SVD, $\mathbf{S}(\omega_k)$ is the diagonal matrix with singular values sorted in descending order and, in the vicinity of the natural frequency, the singular vector $\mathbf{u}_k$ is considered as a mode shape.

When implementing via EVD, $\mathbf{U}_{\boldsymbol{k}} = \mathbf{V}_{\boldsymbol{k}}$ becomes an eigenvector matrix and $\mathbf{S}(\omega_k)$ is a diagonal matrix with eigen-values, which is normally sorted in ascending order. The spectrum decomposition procedure is known also as principal decomposition analysis.

In the EFDD method, since only truncated data, i.e. the data near the peak of the singular value plot are utilized for the inverse FTT to calculate approximate correlation functions of the corresponding S-DOF system, EFDD may cause bias error in damping estimation. Moreover, when dealing with closely spaced modes, beat phenomena would be encountered, this can lead to inaccurate estimation of damping ratio by the logarithmic decrement technique (Zhang et al. [3]).To eliminate these shortcomings, FSDD utilizes the property of a unitary singular matrix to derive an enhanced output PSD, $\widehat{e\mathrm{G}}_{yy}(i\omega_k)$, via modal filtering, i.e. pre- and post-multiplying a singular vector corresponding to the mth damped modal frequency,

$$e\widehat{\mathrm{G}}_{yy_{\omega\to\omega_{dm}}}(i\omega) = \mathbf{u}_1^{\mathrm{H}}(\omega_m)\widehat{\mathbf{G}}_{\boldsymbol{yy}}(i\omega)\mathbf{u}_1(\omega_m) \approx \left(\frac{2\mathrm{c}_m}{i\omega - \lambda_m}\right) \tag{2.17}$$

Where c_m is a real scalar and λ_m is the mth pole defined in Eq. (2.3).

It is seen that the output PSD is enhanced in the vicinity of the mth modal frequency and behaves like an SDOF system. In other words, the singular vector corresponding to a modal frequency acts as a modal filter. In most cases, the enhanced PSD in a specific frequency range (narrow band) can be approximated as an SDOF system, and therefore an SDOF curve fitter utilizing the spectral lines in the vicinity of the modal frequency can be adopted to estimate the relevant modal frequency and damping ratio.

The derivation of the major formulation for the FSDD method assumes of a flat excitation spectrum, or a white noise excitation. However, since the FSDD method is conducted in a narrow band with spectrum lines in the vicinity of a modal frequency, the only assumption required is the broadband excitation, i.e. the excitation spectrum is flat in the vicinity of a modal frequency.

3 Experimental Tests

To perform the experiments 09 accelerometers, 01 impact hammer with steel tip, 01 data acquisition system and 01 notebook were used. Ten (10) impacts were given in each degree of freedom, totaling 90 impacts on the rectangular beam. This data will be used for an operational analysis (outputs only). Table 1 shows the beam's dimensions and Fig. 1 shows the experimental apparatus.

Table 1. Dimensions of the aluminum beam

Dimensions	
Thickness	6.17 mm
Width	25.42 mm

Length	2145 mm

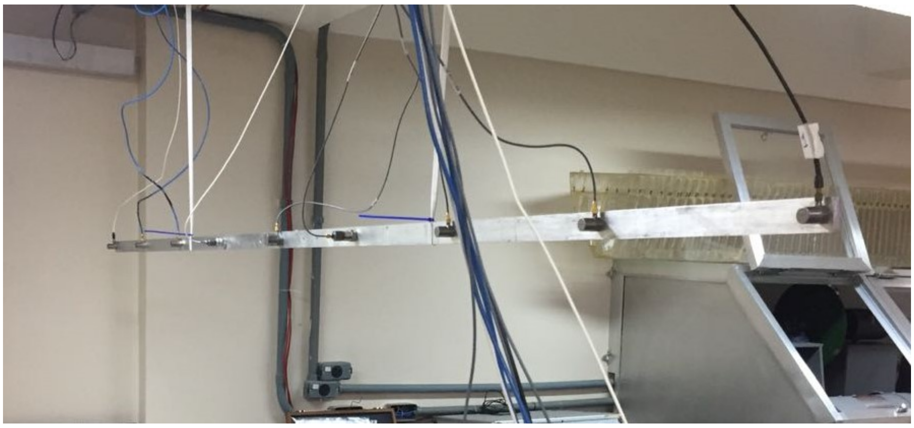

Figure 1. Rectangular aluminum beam and accelerometer positioning in the experiment (accelerometer no. 1 and DOF 1 is in the right side)

Accelerometers were positioned so that the distances between two consecutive sensors were equivalent. As the total length of the beam is 2145 mm, the distances between each sensors is 268 mm. Table 2 shows the weights and accelerometer sensitivities.

Table 2. DOF number and accelerometers characteristics

Degree of Freedom (DOF)	Sensibility	Weight
1	100 mV/g	9 g
2	100 mV/g	9 g
3	100 mV/g	9 g
4	9.72 mV/g	17 g
5	100 mV/g	27 g
6	9.65 mV/g	17 g
7	100 mV/g	9 g
8	100 mV/g	9 g
9	100 mV/g	9 g

Vibration signals were acquired with an acquisition rate of 5128 Hz and preprocessed as shown in Table 3.

Table 3. Signal preprocessing step

Detrending	Yes
Frequency Range in the Analysis	0 – 200 Hz
Spectral Density Estimation (Resolution)	1024 lines

4 Results and Discussion

4.1 Natural frequency and damping ratio estimation using EFDD and MIMO tests

Table 4 shows the estimates of natural frequency and damping ratio when using EFDD method and data from MIMO (MRIT) test.

Table 4. Modal parameters estimated using EFDD-MIMO

Vibration Modes	Natural Frequency [Hz]	Damping Ratio [%]
1	6.291	3.631
2	17.665	1.249
3	34.425	0.749
4	56.92	0.555
5	84.815	0.598
6	119.255	0.397
7	153.814	0.326

4.2 Natural frequency and damping ratio estimation using EFDD and SIMO tests

The same analysis was performed with EFDD method but, in this case, considering impacts only in the first degree of freedom (DOF 1), which is the position of the accelerometer no. 1. Table 5 shows the results obtained.

Table 5. Modal parameters estimated using EFDD-SIMO

Vibration Modes	Natural Frequency [Hz]	Damping Ratio [%]
1	6.312	2.137
2	17.591	1.381
3	34.404	0.44
4	56.935	0.856
5	84.898	0.697
6	119.41	0.352
7	153.941	0.314

4.3 Natural frequency and damping ratio estimation using FSDD and SIMO tests

The same analysis was performed with FSDD method, considering impacts only in the first degree of freedom (DOF 1). Table 6 shows the results obtained.

Table 6. Modal parameters estimated using FSDD-SIMO

Vibration Modes	Natural Frequency [Hz]	Damping Ratio [%]
1	6.344	2.298
2	17.612	1.188
3	34.405	0.418
4	56.986	0.721

5	84.925	0.627
6	119.408	0.331
7	154.001	0.297

4.4 Natural frequencies comparison

To compare results obtained through different identification methods and test setups, the EFDD-MIMO was used as reference, since it uses MIMO tests which allow excitation at multiple DOFs. To compare EFDD and FSDD, the EFDD method was used as reference. Table 7 shows the differences in natural frequencies estimation among methods.

Table 7. Differences in natural frequencies estimation among methods and tests

Vibration Modes	EFDD-MIMO vs EFDD-SIMO	EFDD-MIMO vs FSDD – SIMO	EFDD-SIMO vs FSDD – SIMO
1	-0.33%	-0.84	-0.507
2	0.42%	0.30	-0.119
3	-0.06%	0.06	-0.003
4	-0.03%	-0.12	-0.090
5	-0.10%	-0.13	-0.032
6	-0.13%	-0.13	0.002
7	-0.08%	-0.12	-0.039

It is possible to observe that all differences are less than 1%. So, we can conclude that all identification methods are good for identifying natural frequencies, even when SIMO tests were used.

4.5 Damping ratios comparison

Damping ratios estimation were compared using the same references as above. Table 8 shows the differences in the estimation among methods.

It can be seen that the differences are larger than those of natural frequencies, but remained below 1.5%. It is known that damping ratios estimation are much more susceptible to measurement noise than the natural frequencies (Rainieri and Fabbrocino [5]).

Table 8. Differences in damping ratios estimation among methods and tests

Vibration Modes	EFDD-MIMO vs EFDD-SIMO	EFDD-MIMO vs FSDD – SIMO	EFDD-SIMO vs FSDD – SIMO
1	1.49%	1.33%	-0.16%
2	-0.13%	0.06%	0.19%
3	0.31%	0.33%	0.02%
4	-0.30%	-0.17%	0.14%
5	-0.10%	-0.03%	0.07%
6	0.05%	0.07%	0.02%
7	0.01%	0.03%	0.02%

4.6 Comparison Between EFDD-MIMO and Beam's FE model

Another comparison was made using a Finite Element Model (FEM) of the rectangular beam. Table 9 shows the natural frequencies calculated for a simple beam in free-free boundary condition. It also shows the difference between FEM and EFDD-MIMO results.

Table 9. Comparison between natural frequencies obtained using EFDD-MIMO and FE model

Vibration Modes	EFDD-MIMO	FEM	Difference (%)
1	6.291	6.770	-7.61
2	17.665	18.070	-2.29
3	34.425	34.980	-1.61
4	56.92	57.810	-1.56
5	84.815	85.550	0.87
6	119.255	121.000	-1.46
7	153.814	155.030	-0.79

It can be observed that all natural frequencies, except for the 5th mode shape, were overestimated, with the largest difference being 7.61%, for the 1st mode shape, as shown in Tables 9. These differences can be reduced if a numerical model calibration is performed. Nevertheless, it can be concluded that the FE model represents correctly the beam´s dynamical behavior in the frequency range tested.

4.7 Mode shapes comparison

One possible method to quantify the correlation between two different vibration modes is by using the MAC indicator, which is basically a linear and quadratic regression correlation coefficient that measures the consistency between two vectors. MAC values vary in the range of 0 to 1, indicating inconsistency between the vectors (MAC = 0) and perfect consistency (MAC = 1). As can be seen in the chart below, Fig. 2, the vibration modes estimated with EFDD-MIMO and FSDD-SIMO are well correlated.

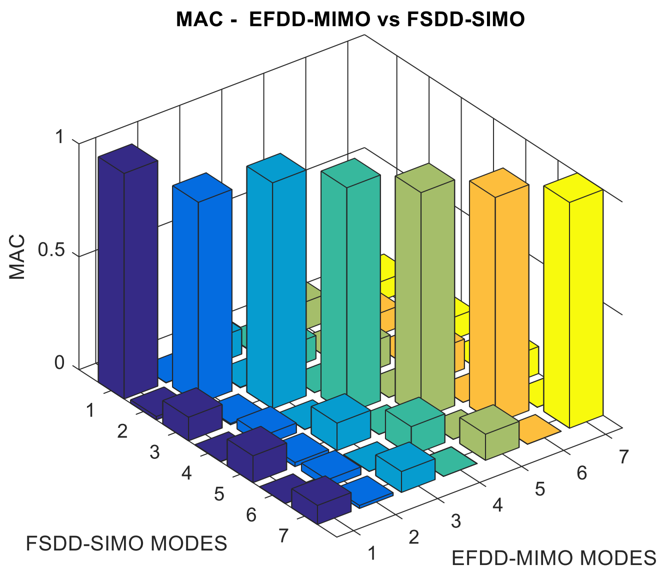

Figure 2. Mode shapes comparison for FSDD-SIMO and EFDD-MIMO

As can be seen in the chart below, Fig. 3, the vibration modes estimated with FSDD-SIMO and EFDD-SIMO are also well correlated.

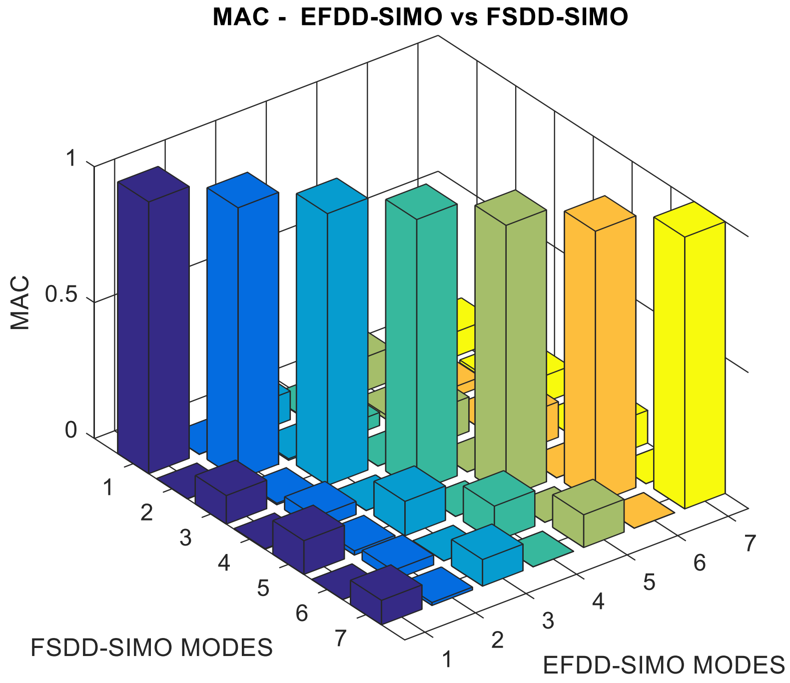

Figure 3. Mode shapes comparison for FSDD-SIMO and EFDD-SIMO

Figure 4 shows the comparison between mode shapes from EFDD-MIMO and beam's FE model. All MAC values are above 0.975, which means that the vibration modes are well correlated.

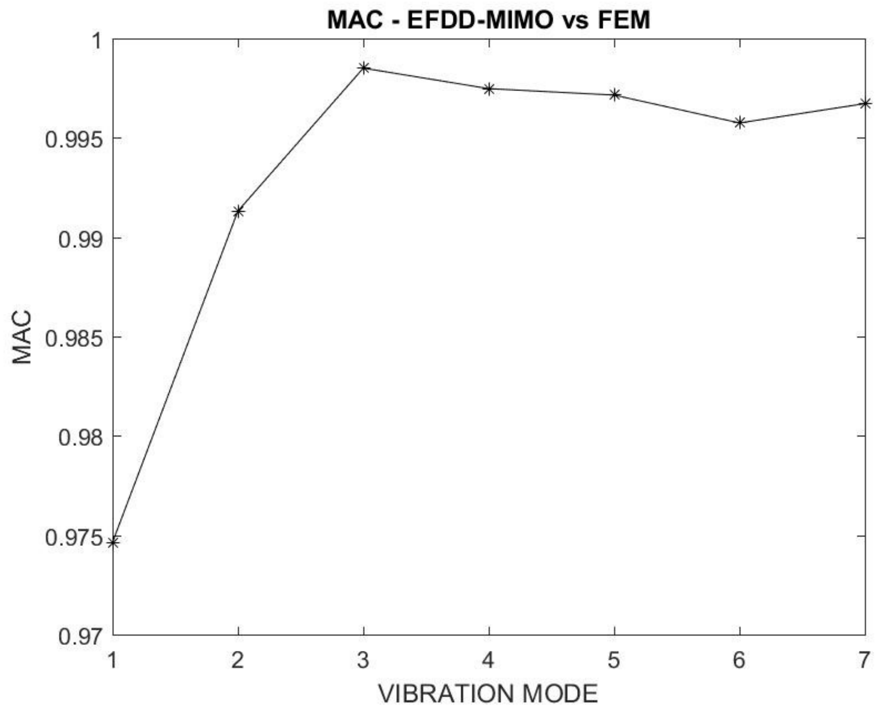

Figure 4. Mode shapes comparison for EFDD-MIMO and FE model

Another way of comparison would be through a visual inspection. Figure 5 compares the experimentally obtained modes with those obtained through the FE model. All modes have been normalized to unity.

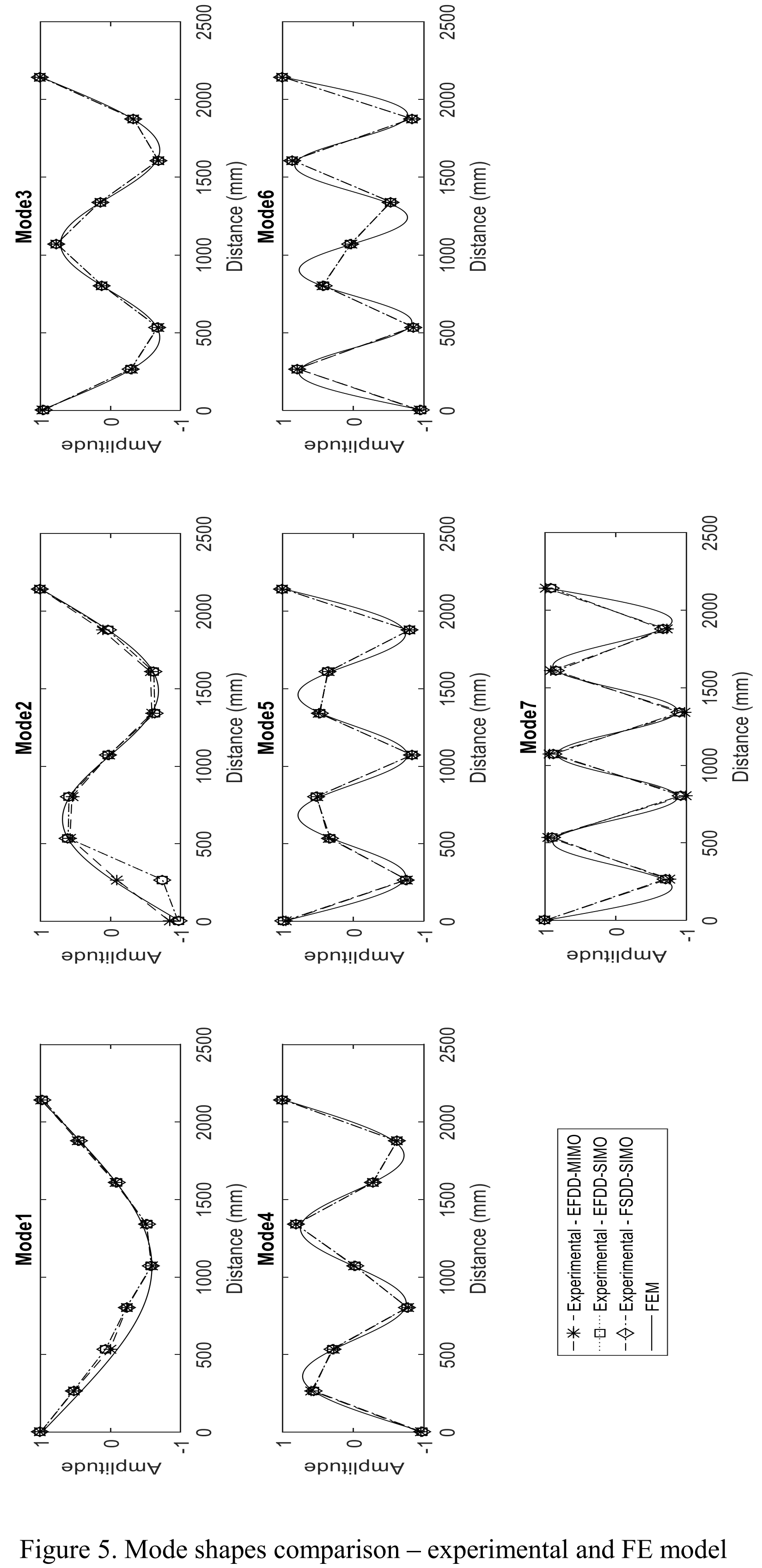

Figure 5. Mode shapes comparison – experimental and FE model

5 Conclusions

Initially, an impact test was performed with the objective of experimentally obtain the modal parameters of a rectangular aluminum beam, considering the hypothesis of free-free boundary condition tests. In this article, 02 (two) output-only modal analysis methods were used: EFDD and FSDD. That is, methods that can use only the vibration responses to estimates the modal parameters. In this way, it is possible to simulate a real condition, in which the inputs (loads) are normally not available.

The EFDD method was performed with both the MIMO and the SIMO approach, while the FSDD method was used only with the SIMO approach.

When comparing the three approaches: EFDD-MIMO, EFDD-SIMO, FSDD-SIMO, very satisfactory results were obtained since all differences were below 1.5%, for natural frequencies and damping ratios. Regarding to mode shapes, MAC criteria and visual comparison of experimentally and numerically identified modes were made and all were well correlated.

Regarding the damping ratios, it can be seen that the differences are larger than those of natural frequencies, but remained below 1.5%. It is known that damping is more susceptible to measurement noises and still the object of many ongoing researches.

In addition, a numerical FE model was also used for comparison purpose with results from EFDD-MIMO. It was observed that all natural frequencies, except for the 5^{th} mode shape, were overestimated with the largest difference being 7.61% (1^{st} mode shape). From the mode shapes point of view, MAC values above 0.975 confirms the good correlation between numerical and experimental modes.

Finally, it is possible to conclude that all methods can be used to identify the modal characteristics of a dynamic system using only the measured responses.

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