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APPLICATION OF SMOOTH ORTHOGONAL DECOMPOSITION ON DAMAGE IDENTIFICATION

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Abstract. This is an ongoing research in which a Structural Health Monitoring (SHM) methodology, namely, the Smooth Orthogonal Decomposition (SOD), is employed on damage identification on beams. In a first step, displacement data from the structure is subjected to a Spectrum Analysis Amplitude and the most energetic vibration frequencies are acquired. Subsequently, SOD is applied to the same displacement data in order to obtain the linear mode shapes and natural frequencies of the structure. Finally, most energetic mode shapes are selected by correlation of frequencies obtained on the two previous steps. The procedure is applied to both a damaged structure and a similar healthy one. Damage spot identification is achieved by mode shapes comparison of healthy and damaged models. The technique is tested using Finite Elements models for beams under two boundary conditions, namely, simply supported beams and cantilevered beams. For each of the two case studies above, numerical models concerning damage in different spots are generated for comparison. The results show that the method is successful aiming at identifying structural damage. Robustness and limitations of this procedure are discussed.

Keywords: Structural Health Monitoring, Smooth Orthogonal Decomposition, Damage Identification, Spectrum Amplitude Analysis, Beam models

1 Introduction

During its lifetime, structures are usually subjected to random excitations. Studying these excitation phenomena for predicting the structure behavior and physical properties is, sometimes, difficult or even impracticable. An alternative approach consists in applying SHM (Structural Health Monitoring) techniques for monitoring the structure displacements.

SHM involves the analysis of data extracted from sensors distributed along the structure in order to obtain its characteristics. In case of vibrational behavior study, accelerometers and strain gages can be employed on mode shapes and mode frequencies acquisition. A variety of SHM methods are presented in [1].

[2] applied both power spectrum and time-frequency analysis for damage identification on beam-like structures, while [3] developed a method employing modal analysis and genetic algorithm to identify cracks in beams using a single accelerometer. In a more specific application of SMH, [4] used MDR (Modal Decomposition and Reconstruction) to study the response of an offshore riser subjected to VIV (vortex-induced vibration), a particular class of flow-induced vibration. Later [5] described the architecture and data treatment of a riser real-time monitoring tool using the same MDR method.

The Hilbert Transform and WWA (weighted waveform analysis) were applied by [6] to estimate fatigue damage on risers with 8 sensors. More recently [7] make use of the POD (Proper Orthogonal Decomposition) and WWA to recover the most energetic mode shapes along a offshore riser. Further details regarding the POD method can be found in [8].

Two damage identification methods are also present in [1]: Flexibility Matrix and Rotational Flexibility Matrix. These procedures employ the mode shapes and mode frequencies to reconstruct the flexibility matrix of the structure along time. Changes in the structure flexibility history can be used to locate damage.

As already mentioned, this work is part of an ongoing research developed at the Offshore Mechanics Laboratory (LMO) that aims at identifying damages on structures. Herein, the focus is on the application of Smooth Orthogonal Decomposition (SOD), a variation of POD method in order to identify damage spots on finite elements beam models. A numerical study is developed using beams with different boundary conditions. Simulations based on the Finite Element Method (FEM) using damaged beams are employed as inputs for an algorithm that makes use of SOD for identifying the damage.

2 Methodology description

In a first step, displacement data acquired from sensors along the real structure or numerical model is subjected to an spectrum analysis amplitude in order to obtain the most energetic frequency components of the signals. Subsequently, SOD (Smooth Orthogonal Decomposition) is applied to the same range of signals and excited mode shapes and mode frequencies are recovered.

Both most energetic frequencies and mode frequencies obtained in the previous steps are compared in order to select most energetic mode frequencies. Afterward, the mode shapes associated with most energetic mode frequencies are chosen to compose the mode shape matrix of the structure. The procedure is employed on healthy and damaged structures and mode shape matrices of both are compared for damage spot identification.

The procedure described above is part of a more complete methodology still under development. A second Structural Health Monitoring methodology, namely, RFM (Rotational Flexibility Matrix) will be applied to the mode shape matrices with the purpose of recovering the flexibility matrices of healthy and damaged structures. This technique will provide the damage severity and a more accurate estimations of the damage spot.

The complete methodology can be visualized in the diagram sketched in figure 1. The steps inside the red box are covered in the present work.

For the sake of completeness of this paper, the SOD (Smooth Orthogonal Decomposition) technique is briefly described bellow.

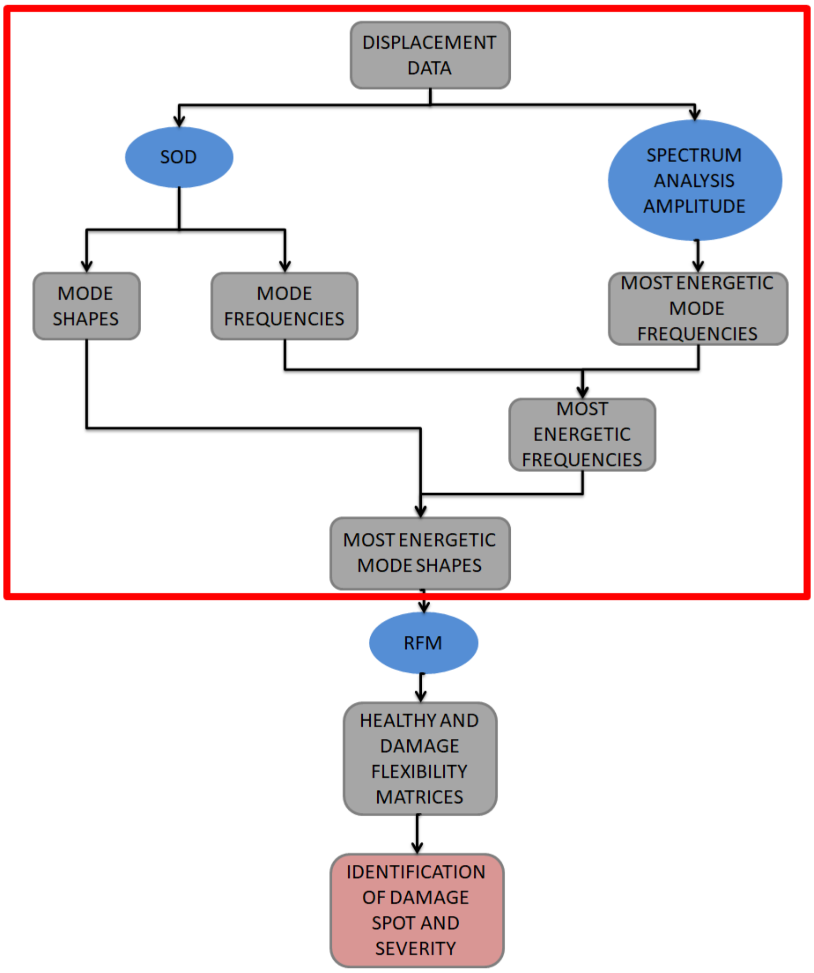


Figure 1. Procedure diagram.

2.1 Smooth Orthogonal Decomposition

The SOD method is from the same family as the POD method, but overcomes some of POD's major deficiencies. Contrary to POD, SOD doesn't require an *a priori* knowledge of mass matrix of the structure; furthermore, SOD provides mode shapes and mode frequencies while POD produces only structure mode shapes.

The basic idea is to look for projections of the signal measurements, obtained from displacement sensors along the structure, that are smooth on time [9].

Let $\mathbf{X}$ being the matrix whose columns represent the sensors spots and the rows the discrete displacement acquisitions in time. One can build the velocity matrix $\mathbf{V} = \mathbf{D}\mathbf{X}$, where $\mathbf{D}$ is the differential operator given by Eq. (1):

$$\mathbf{D} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix} \tag{1}$$

Defining $\Sigma_\mathbf{x}$ as the covariance matrix of $\mathbf{X}$ and $\Sigma_\mathbf{v}$ as the covariance matrix of $\mathbf{V}$, the following generalized eigenvalue problem can be obtained Eq. (2):

$$\mathbf{\Sigma_x \Phi = \Lambda \Sigma_v \Phi} \tag{2}$$

Being $\mathbf{\Lambda}$ the diagonal matrix of eigenvalues, which approximate the square of the modal frequencies and $\mathbf{\Phi}$ the matrix of eigenvectors, which approximate the mode shapes.

Reference [9] has proved that this method works for lightly damped structures subjected to free vibration. Later, [10] showed that this same method was also valid for randomly excited systems. To exemplify, they applied the SOD to a eight-degree-of-freedom linear vibratory system subjected to white noise excitation, mode shapes and mode frequencies were obtained with good accuracy.

3 Results

On the actual stage of the project, SOD and spectrum analysis amplitude were successfully correlated and the most energetic modes for healthy and damaged beam models have been obtained. Despite the RFM technique is not applied to the present following results, damages can already be identified and robustness conclusions can already be addressed.

The methodology is applied for two cases: a simply supported beam and a cantilevered beam. The physical characteristics of the healthy beam are described in table 1.

Table 1. Beam characteristics.

Length	201	cm
Height	5	cm
Width	5	cm
Density	7860	$\mathrm{kg/m^3}$
Young modulus	200	GPa
Poison's ratio	0.25	-

The structure is modeled by Finite Element Method using MATLAB®. The model is composed of 21 Timoshenko beam elements where 20 elements are 10 cm long and one element, in which the damage is inserted, is 1 cm long and 2.5 cm high.

3.1 Simply supported beam

The investigated simply-supported beam model is illustrated in figure 2 and the first 3 natural frequencies of the healthy structure are show in table 2. The damage was inserted in element 5 (x = 40 cm).

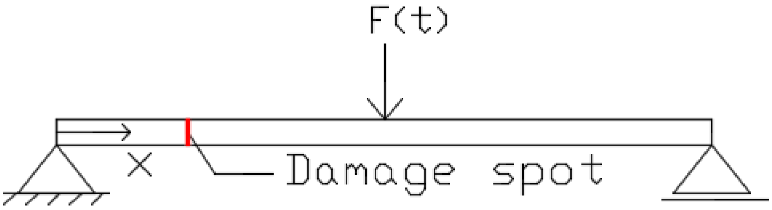


Figure 2. Simply supported beam model. Damage in element 5 indicated in red.

Table 2. First 3 natural frequencies of healthy simply supported beam.

First natural frequency	178	Hz
Second natural frequency	711	Hz
Third natural frequency	1601	Hz

Both healthy and damaged beams were subjected to a periodical excitation applied to node 21 with 1 kN amplitude and 1617 Hz (frequency of the third mode of healthy simply-supported beam).

Displacement data from nodes 2 to 20 were acquired with a time sample of 0.001 second. Standard spectral analysis based on the Discrete Fourier was applied to the healthy and damage structures displacement data as shown in figure 3.

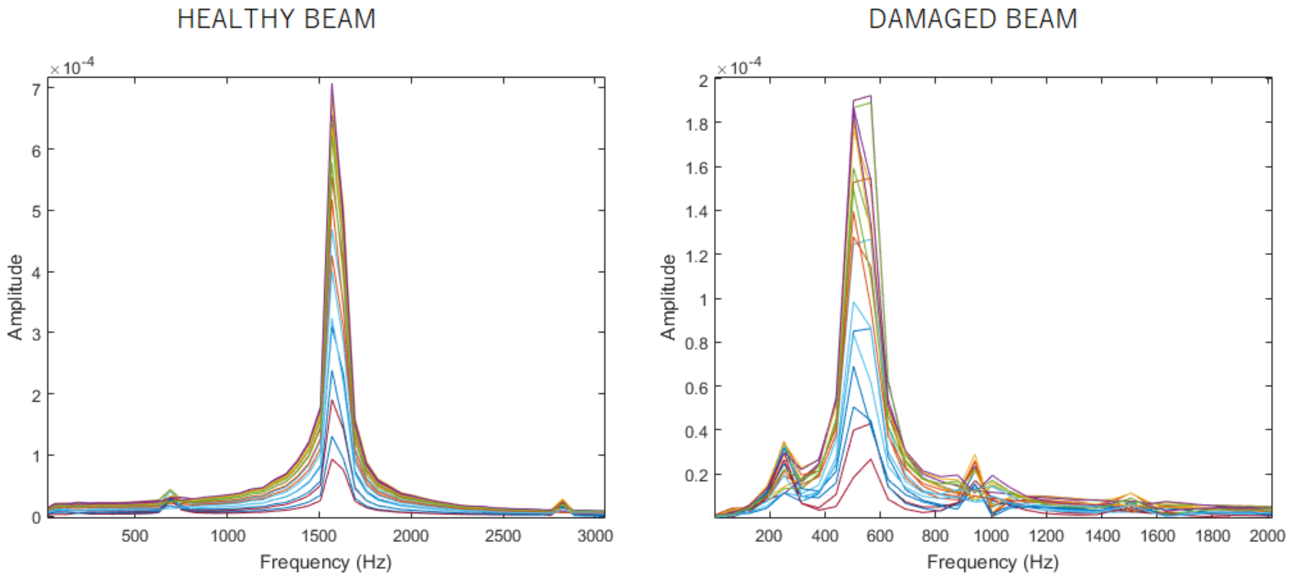


Figure 3. Amplitude spectra for healthy (left) and damaged (right) simply supported beam.

The SOD method was also applied to the displacement data for both healthy and damage structures. The mode shapes associated with the mode frequencies nearest to the most energetic frequency obtained in the amplitude spectra analysis were selected for normalizing the mass matrix. Figure 4 presents most energetic mode shapes for healthy and damaged models.

One can notice that the modal displacement at the damage spot of the damaged beam has increased in comparison with the modal displacement of the healthy model. This result indicates that the structure stiffness at this position has decreased because of the damage.

3.2 Cantilevered beam

The dealed cantilevered beam model is represented in figure 5 and the first 3 natural frequencies of the healthy cantilever are shown on table 3. Once again, the damage was insert in element 5 (x = 40 cm).

Table 3. First 3 natural frequencies of healthy cantilevered beam.

First natural frequency	63	Hz
Second natural frequency	397	Hz
Third natural frequency	1120	Hz

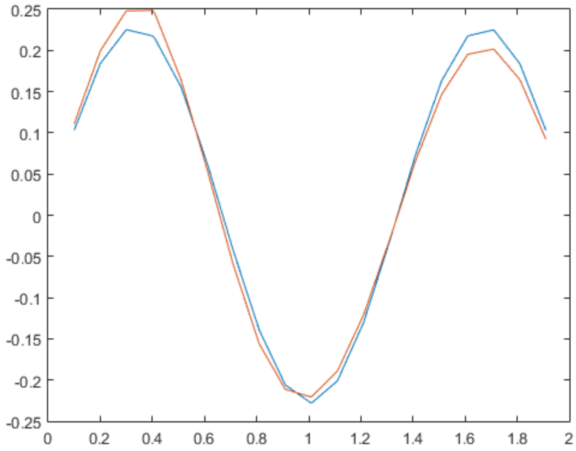
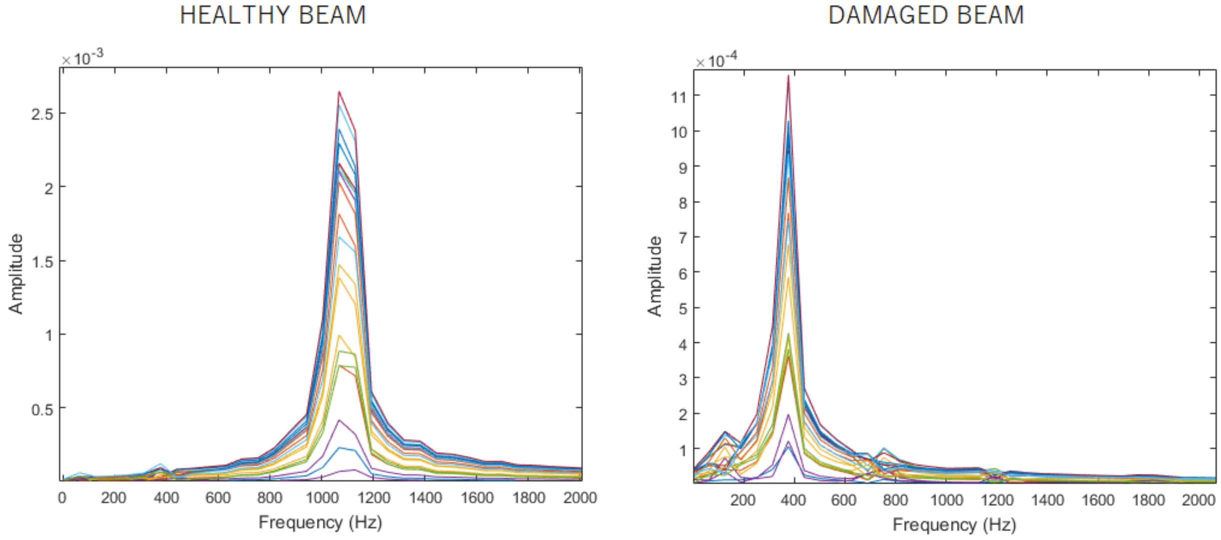


Figure 4. Mode shape of healthy (blue) and damage (red) simply supported beam.

Figure 5. Cantilevered beam model. Damage in element 5 indicated in red.

Both healthy and damaged beams were subjected to a periodical excitation applied to node 21 with 1 kN amplitude and 1617 Hz (frequency of third mode of healthy cantilevered beam). Displacement time-histories from nodes 2 to 20 were acquired with a time sample of 0.001 second. Spectrum analysis amplitude was applied to both the healthy and damaged structures displacement data, as shown on figure 6.


HEALTHY BEAM

DAMAGED BEAM

Figure 6. Amplitude spectra for healthy (left) and damaged (right) cantilevered beam.

The same methodology applied for the simply supported beam was also employed for the cantilevered beam problem. Figure 7 presents most energetic mode shapes for healthy and damaged cantilevered models.

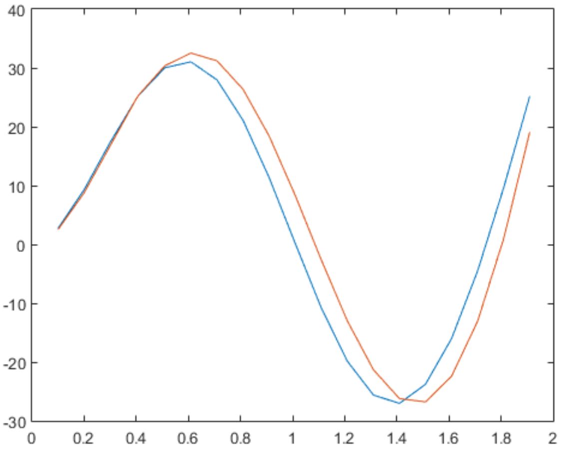


Figure 7. Mode shape of healthy (blue) and damaged (red) cantilevered beam, damage in element 5.

The procedure has repeated for the cantilevered beam model with a damage inserted in element 15 (x = 140 cm) represented on figure 8. The most energetic mode shapes for both the healthy and damaged structures are presented on figure 9 for this new condition.

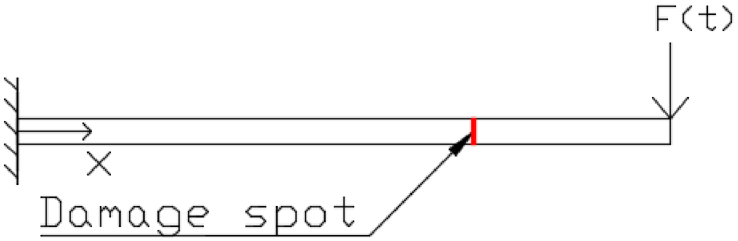


Figure 8. Cantilevered beam model. Damage in element 15 indicated in red.

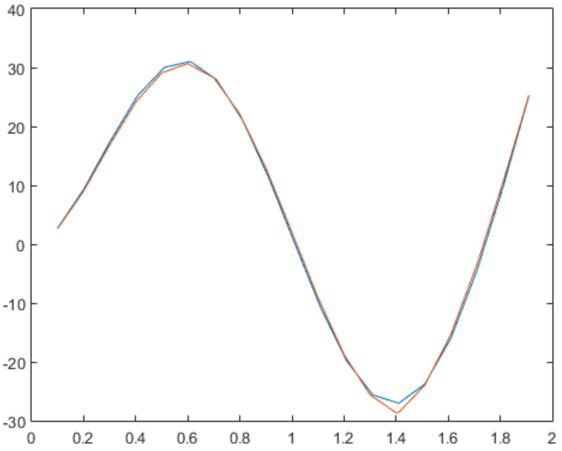


Figure 9. Mode shape of healthy (blue) and damaged (red) cantilevered beam, damage in element 15.

Comparing the results of damage location for cantilevered (element 5 ou 15), one can note that, in the first case the mode shape variation is more prominent. The possible reason for this observation is that for the third mode shape of the cantilevered beam model the element 5 has more strain energy (i.e., large curvature) than the element 15, leading to the conclusion that the method is more efficient when damage occur in a high strain energy spot.

4 Conclusions

Part of methodology for structures real-time monitoring has been presented. Smooth Orthogonal Decomposition has been briefly described as well as the procedure for using these technique on damage identification.

The part of the methodology was applied in Timoshenko beam models with two different boundary conditions: simply-supported and cantilevered. Preliminary results have demonstrated the efficiency of the methodology. Damage spot have been successfully identified in both boundary conditions.

On cantilevered beam model case two damage spot conditions were put into test: damage in node 5, closer to fixed end; and damage in node 15, closer to free end of the beam. First condition presented more strain energy on the third mode shape than the second one. One could note that changes in modal parameters between healthy and damaged structures were more prominent when damage was inserted in spots with more strain energy.

As already mentioned, this paper is part of an ongoing work on methodologies for assessing damage problems in beam models. Ongoing works include application of Rotational Flexibility Matrix technique to the mode shapes of the structure in order to obtain its flexibility matrix. Comparison of healthy and damaged flexibility matrices will allow the identification of damage spot with more accuracy and measure of damage severity. The main goal of this research line, recently introduced in the LMO group, is to develop a series of numerical tools for damage identification in risers.

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