# Damage analysis in metal beams by changes in their natural frequencies.

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**Abstract.** Nowadays, the concern with structural damages in the civil construction is in growth, slender structures, of great dimensions, and old or more economic generate constant concern for the engineers responsible for its maintenance and stability. In this scenario, considering the growing demand for structural damage monitoring, this paper presents the results of laboratory tests performed to indicate the feasibility of a simple signal processing method to identify structural damages. This method consists of experimental tests using a set of accelerometers that were coupled in two metal beams to obtain acceleration responses at various levels of structural integrity (cuts in the cross section of the profile corresponding to 80%, 60% and 40% of the original). The recorded accelerations went through a simple processing of data for identification of natural frequencies. The numerical modeling of the beams was also performed in the ANSYS software based on the finite element method, from which the natural numerical frequencies were extracted. Finally, the experimental natural frequencies were compared with the numerical ones for the validation of the results. It was possible to identify damages in the two beams since changes in the natural frequency values were identified at all levels of section integrity. A good approximation between the experimental and theoretical values was observed, and in this way, it was concluded that the tests were able to detect structural damage correctly.

**Key-words:** Modal Operational Analysis, Dynamic Structures, Element finite Method.

## 1. Introduction

Buildings have become complex and bold, due to various factors such as the development of building material technology, innovative building systems, constantly decreasing of energy resources in the construction phase and emergence of software's capable of performing more accurate analyzes [1].

In the last decades, the need to use numerical tools for calculating civil structures has been growing, due to the concern with the accuracy of the analyzes, to help the understanding and identification of static and dynamic structural behaviors. The analyzes involve the evaluation of state of service and the last state of service, the latter with increasing importance, especially after the 70's, with the rise of tall buildings built with light materials [2].

Still on the numerical analysis, Brincker, Zhang and Andersen [3] point out the importance of structural analysis analyzing dynamic behaviors, also considering external forces acting on the structure in relation to time. Soriano [4] describes that this way it is possible to identify modal parameters (mode shapes, natural frequencies and damping rates), thus allowing a new feature for calculation and monitoring of structures that evaluate large constructions that undergo dynamic loading requests.

Many damage detection methods, such as ultrasound and radiography, are able to detect and identify damage in practice. However, these methods need initial data, such as the proximity of the damage. Fortunately, there are other alternative methods such as vibration reading (modal analysis) that can locate and identify damage more effectively [5].

Obtaining dynamic data in the field, the modal analysis has been able to identify modal parameters of constructions, by reading vibrations due to external dynamic loads that the structures are normally exposed. The data are processed by signal processing techniques, from which their modal parameters are extracted [6].

The usual modal analysis is performed considering the known dynamic excitation in the structure, normally operated through eccentric mass oscillators and impact hammers, this method requires intervention of the structure [7]. In operational modal analysis, it is not necessary to know the excitation efforts. Therefore, the structure does not need to be paralyzed, because only the operational responses are measured and the modal parameters are obtained through probabilistic models, Au, Ni, Zhang [5].

Modal analysis focused on damage identification is based on the assumption that damaged structures present changes in their physical properties, and consequently in their dynamic responses, thus processed dynamic information generates data on the integrity of the actual structure, Soriano [4]

Many methods for damage detection have emerged in recent decades. Damage can be identified from 4 categories: Level 1, the method gives indication that damage may be present in the structure (detection). Level 2, the method gives information of probable damage location, besides detection (localization). Level 3, the method gives damage size information (rating). Level 4, the method gives information of current safety of the structure due to a certain state of damage (consequence) [8].

In this work, tests were performed with the objective of experimentally evaluating and quantifying the alteration of natural frequencies of two metal beams, through free and forced excitations, due to quantified section loss of 20%, 40% and 60% of the original section of the profile. Vibrations will be captured through a high sensitivity accelerometer system, an analog to digital data converter and a computer. Natural frequencies were obtained by processing the recorded vibrations. A numeric modal based on finite element method, was also elaborated, with the same experimental conditions, aiming to compare the experimental and numerical results.

Changes in natural frequency, of metal beams, was observed due to loss of stiffness. The dynamic data collection test, without knowing the acting forces (operational modal analysis focused on frequency identification) that were performed in this work, can be an indicative of the feasibility of level 1 damage identification in structures. This method has benefits, damage detections become simple, easy to perform, low cost and there is no necessity of paralyzing the structure or take any other measures that make it impossible to use the structure under analysis.

## 2. Objectives

Check if it is possible to identify changes in natural frequencies of metal beams when they are subjected to reductions in their cross section.

Plan and perform free excitation (environmental excitation) and forced excitation (random impact) tests on two metal beams, taking into account cross-sectional reductions of 20%, 40% and 60% of original cross-section of the profile.

Perform numerical simulations of the beams and compare results obtained from the experimental tests.

## 3. Search Restrictions and Limitations

In the experimental tests performed in this work, no temperature variations were considered. Temperature differences can lead to slight variations in natural frequencies.

Due to the inability to perfectly simulate the supports and constraints in the numerical model of the beams, there are variations between experimental and numerical results.

Identifying minor damage through natural frequency changes can be a complex process and inconclusive results due to the large number of variables to be analyzed.

## 4. Description of the beans

The beams used have cross section C, with 5cm flange width, 10cm web width. Flange and web thickness equal to 0.24cm. The lengths of the spans are 3,765m for beam B1 and 1,895m for B2. The small curvature between flange and web was estimated and measured experimentally, and has a radius of 1cm as shown in figure 1.

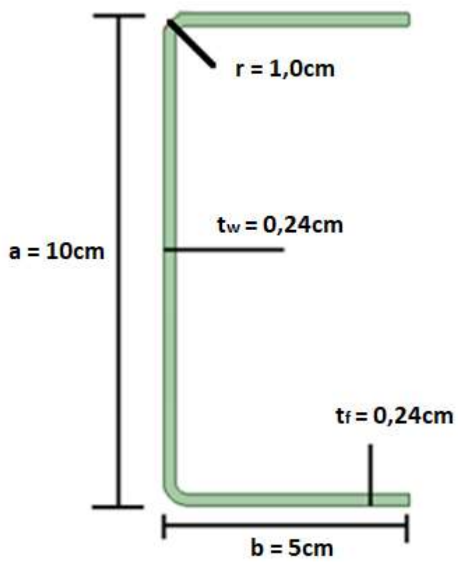


Figure 1 - Profile cross section.

The relevant physical properties for the work are specific weight, longitudinal modulus of elasticity, Poisson's ratio. For the tested profile are given by:

$p_{steel}$ = 7850 kg/m³.
E = 137,671GPa.

The Young's modulus was obtained from a tensile test of a rectangular beam sample taken after the modal tests. The dimensions of this object are 4cm wide, 0.23cm thick and 11cm high. The tensile test was performed in an EMIC press, in the civil engineering laboratory of CEFET-MG.

## 5. Vibration tests

The tests consist of experimental vibration measurements of two, (B1 and B2) beams of same profile and different lengths described in item 4. The experimental measurement was made by positioning the beams with the C-profile table supported by two supports (Figures 2 and 3), which restrict movements in the X, Y and Z directions.

Figure 2 – B1 supported by two supports.

Figure 3 – two supports.

To capture the displacements at 3 different points, a set of three accelerometers (figure 4) with high sensitivity (1.5V / g) and a 20-bit SYSCOM brand acquisition system (figure 4) were used over a period of 255 seconds and acquisition rate of 400Hz. The accelerometers were positioned along the beam at 1/3 L, 1/2 L and 2/3 L of span length (P1, P2 and P3, figure 5), measured from the support.


Figure 4 – three accelerometers and SYSCOM acquisition system.


Figure 5 – accelerometers measurement points.

This procedure was performed for different levels of cross section reduction, which consist of cuts in the profile. There were two cuts in the profile on both sides of the tables (in section located at half the length of the beam) with an increase of 2 cm cut in each step (figures 6 and 7), and the sections were reduced to 80%, 60% and 40% of the liquid section, with an average cut thickness of 2mm.

The experiment was performed considering two forms of excitations, free excitations and forced excitation. The first is to measure accelerations that environment itself induces in the beam, while in forced excitation a rubber hammer was used for applying random blows during test time. Totaling 3 measurements (1/3 L, 1/2 L and 2/3 L) for four levels of integrity (100%, 80%, 60% and 40%) for two excitation types (free and forced) and for each beam (beams 1 and 2). Thus, a total of 48 modal tests were performed.

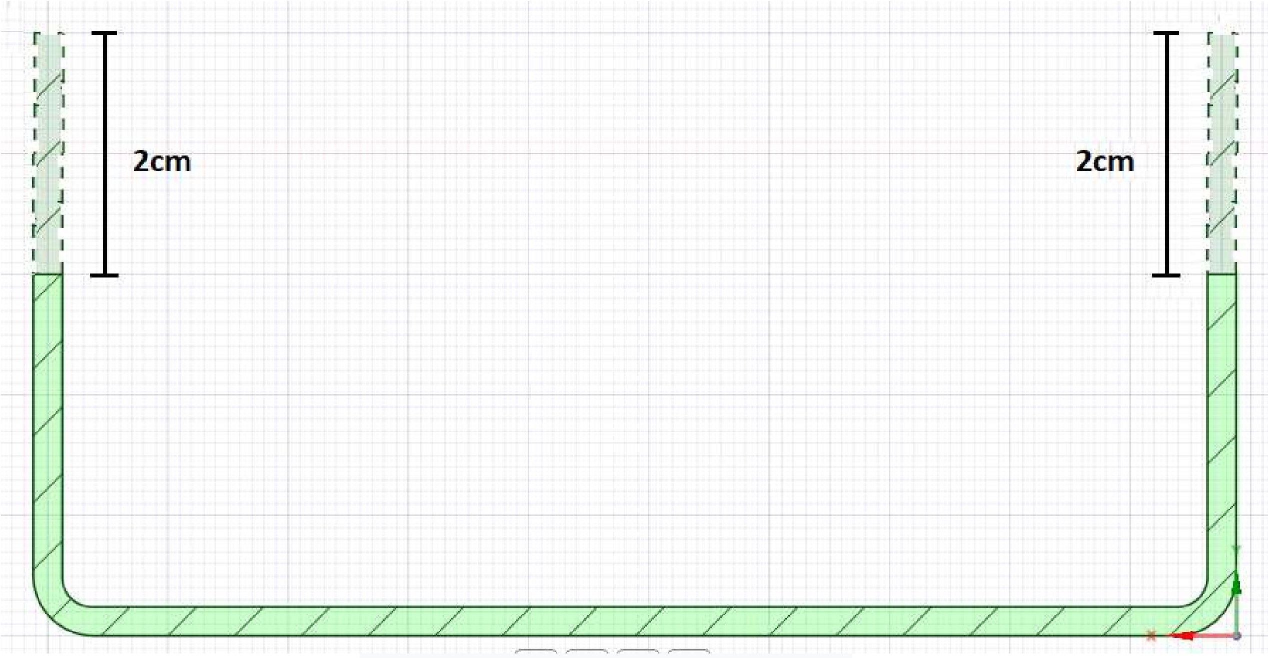


Figure 6 – Cross section integrity of 80%.

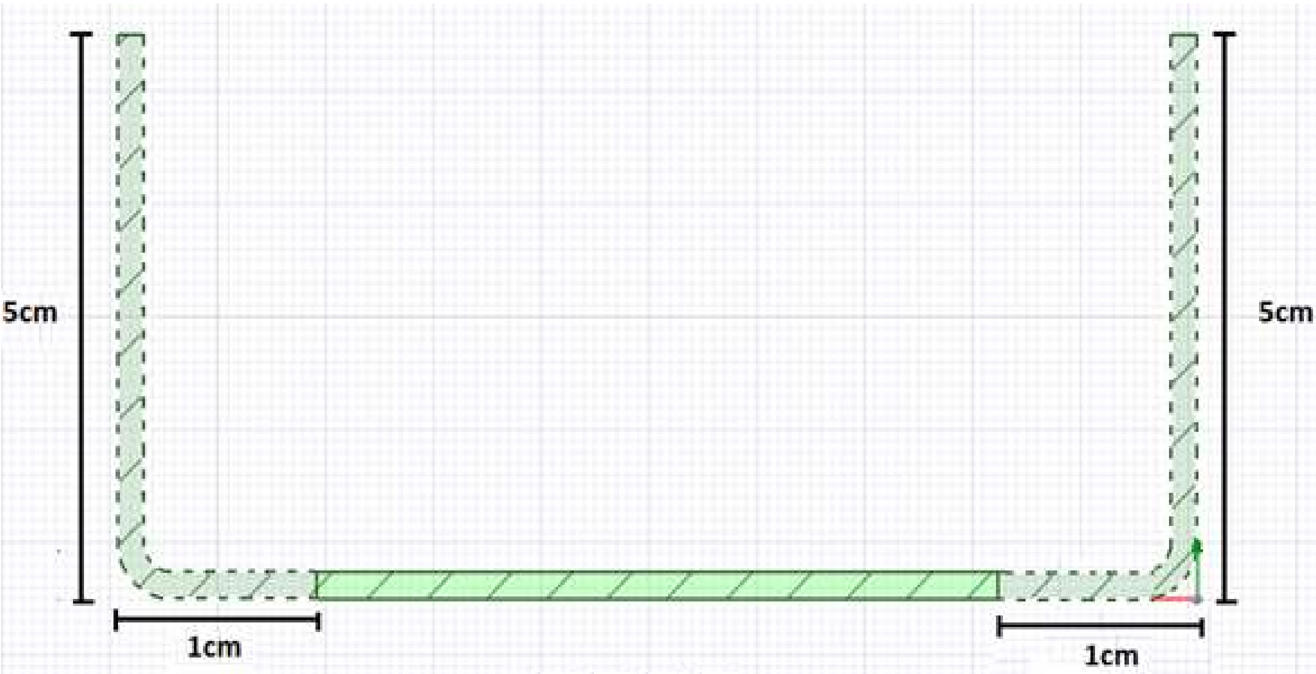


Figure 7 – Cross section integrity of 40%.

## 6. Numeric Model

In order to validate the experimental results and analyze the mode shapes, a numerical test was simulated from a modeling of the beams. The test was performed in a software of finite element method (ANSYS R17.0). For each beam, models were generated considering beam integrity ranging from 100% to 40% (Figure 8). Figure 9 shows the MEF mesh adopted on beam B1, with 40% of the section.

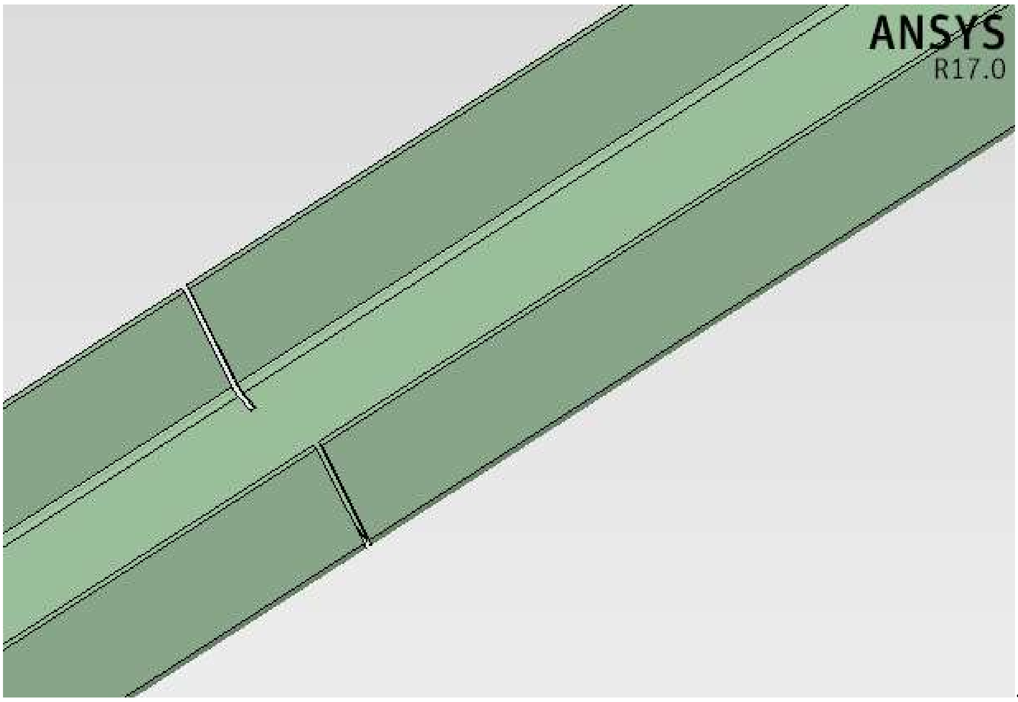


Figure 8 – Beam B1 with 40% cross section integrity.

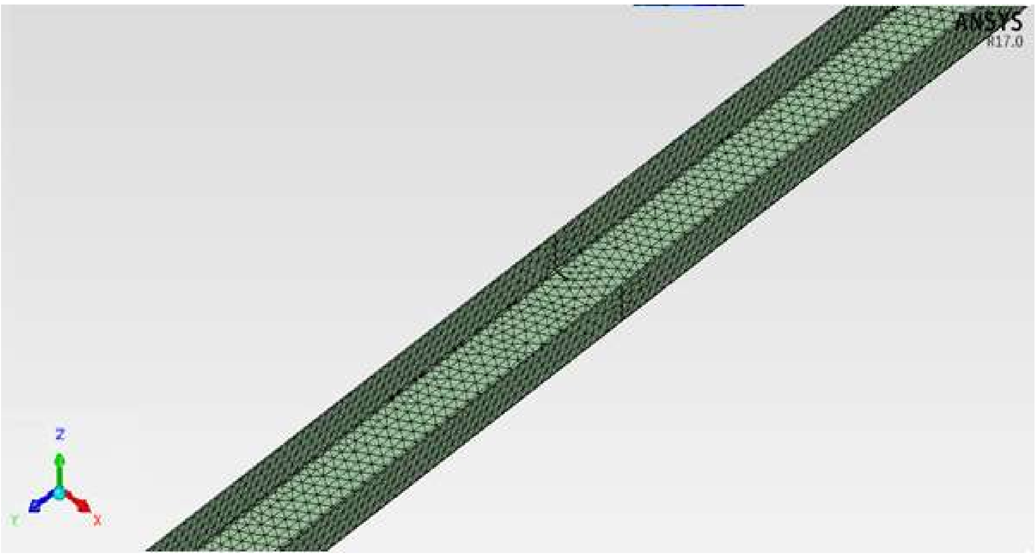


Figure 9 – mesh of Beam B1 with 40% cross section integrity.

## 7. Experimental Results

For each experimental trial, an acquisition rate of 400Hz and data collection time of 255 seconds were considered, totaling 102000 records for each of the 3 points (1/3 L, 1/2 L and 2/3 L). Acceleration results were processed using a private software developed in the Fortran language. The processing followed the steps: First, the time acceleration records were partitioned into 100 parts. Then these parts were summed. This procedure aims to eliminate the random portion of the response, based on the concept of random signals, for which the sum of several random signals is null. In fact, the structure responses during the modal test are made up of two plots: The first is the impulsive response due to excitation, which is the random portion of the response. The second installment corresponds to free vibration, which is not random. Thus, after the procedure of summing 100 parts, only the portion relative to free vibrations will be present in the signal. Then the Fourier transform is performed to obtain the frequency spectrum of free vibrations. The peaks present in this spectrum, theoretically correspond to the natural frequencies of the structure, but noise and harmonic oscillations can generate peaks that must be identified and disregarded. In this work, this procedure was performed comparing the experimental free vibration spectrum to the numerical values. When the experimental values do not have a corresponding value in the numerical spectrum, the peak is discarded. Only the peak related to the first natural frequency can be identified. This was due to the large presence of noise and the short duration of the test. This difficulty is common to operational modal analysis and can be circumvented by robust stochastic signal identification procedures [9]. Such procedure is beyond the objective of the work, which is to use simple data analysis procedures.

Figures 10 and 11 show the frequency spectrum of accelerations recorded for B1 (3.765m beam) and B2 (1.895m beam) considering free excitation at 100%, 80%, 60% and 40% integrity levels. It is noteworthy that for beam 1, integrity level in 40% was not collected, since the structure presented deflection above the service limits.

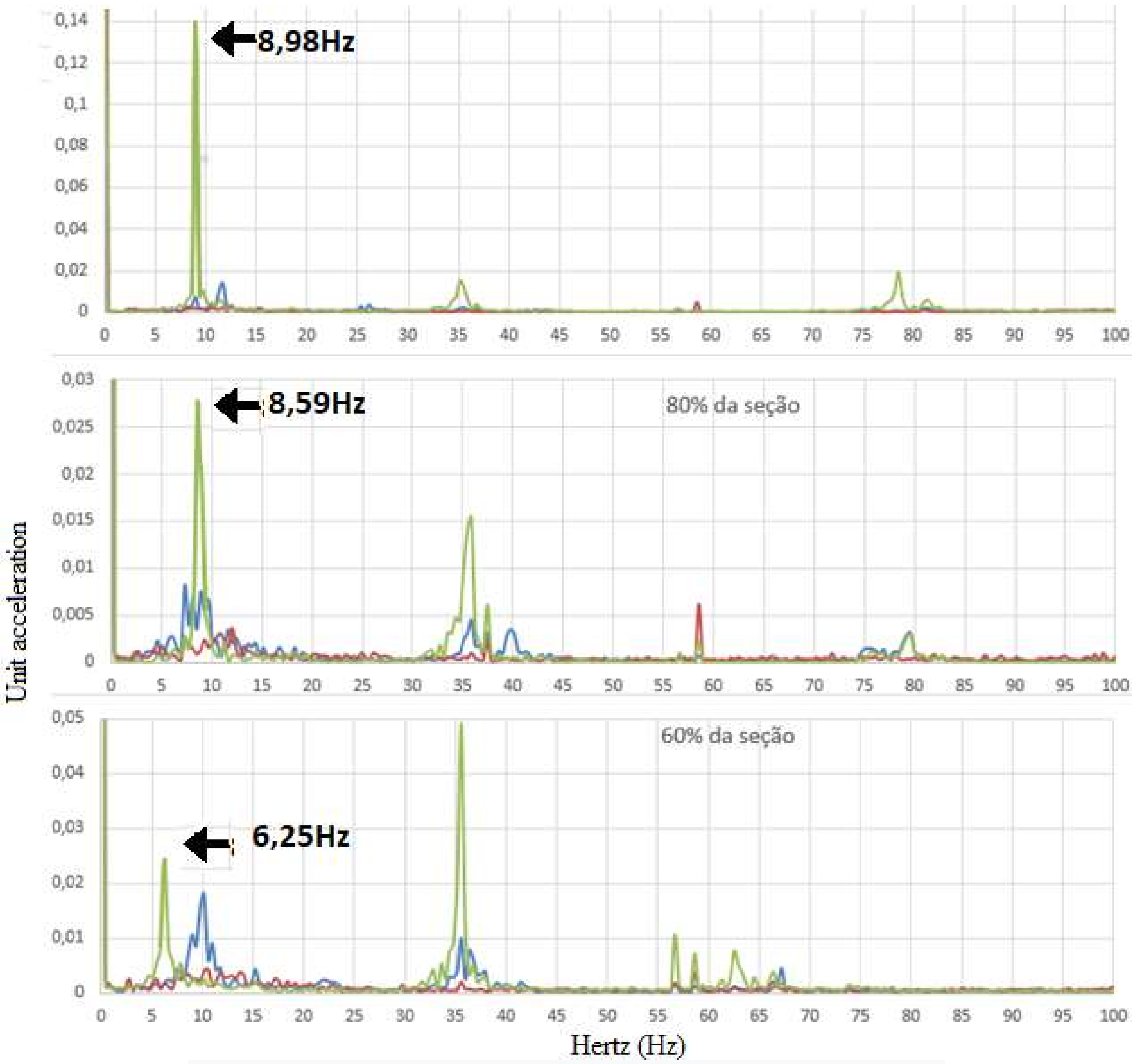


Figure 10 – Unit Acceleration x Frequency (Hz), B1, free excitation.

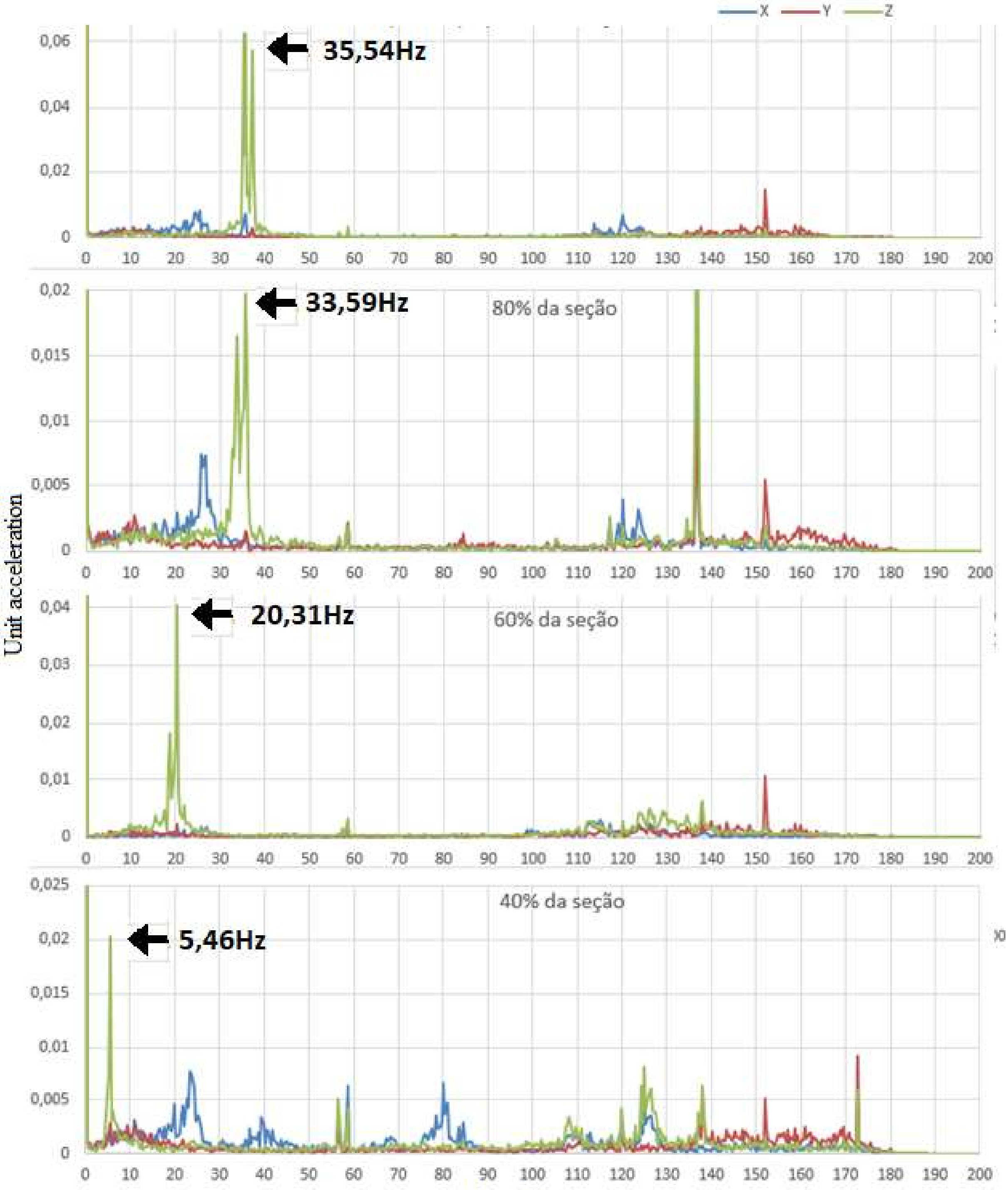


Figure 11 – Unit Acceleration x Frequency (Hz), B2, free excitation.

Figures 10 and 11 show that the first natural frequencies found for beam B1 correspond to 8.98Hz, 8.59Hz, 6.25Hz, for the 100%, 80% and 60% integrity sections. For beam B2 natural frequencies of 35.54Hz, 33.59Hz, 20.31Hz and 5.46Hz were identified, for section integrity levels equal to 100%, 80%, 60% and 40%. Note that frequencies decrease with decreasing integrity, as expected.

Figures 12 and 13 show a frequency spectrum of accelerations recorded for B1 and B2, considering forced excitation at integrity levels 100%, 80%, 60% and 40%.

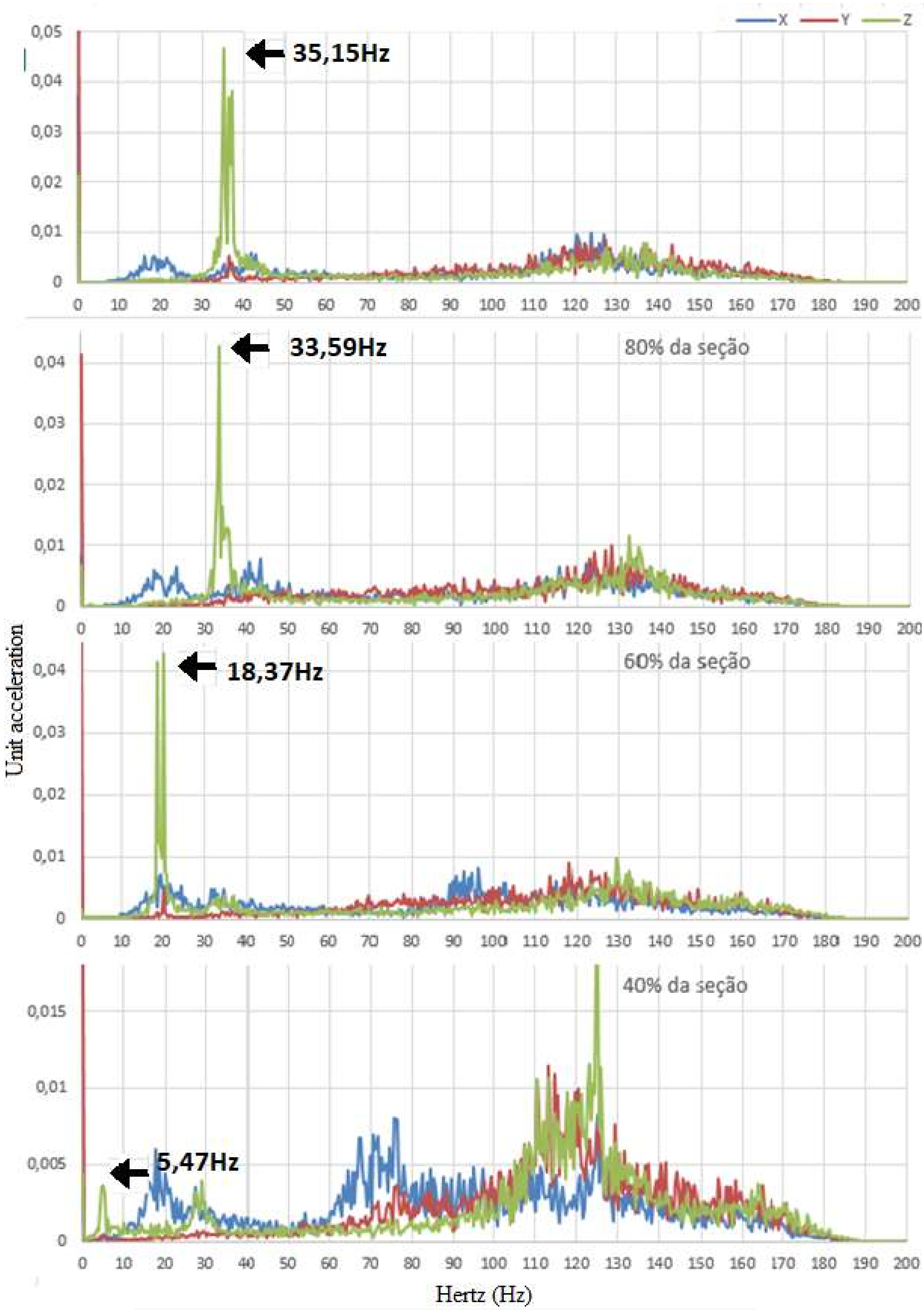


Figure 12 - Unit Acceleration x Frequency (Hz), B2, forced excitation.

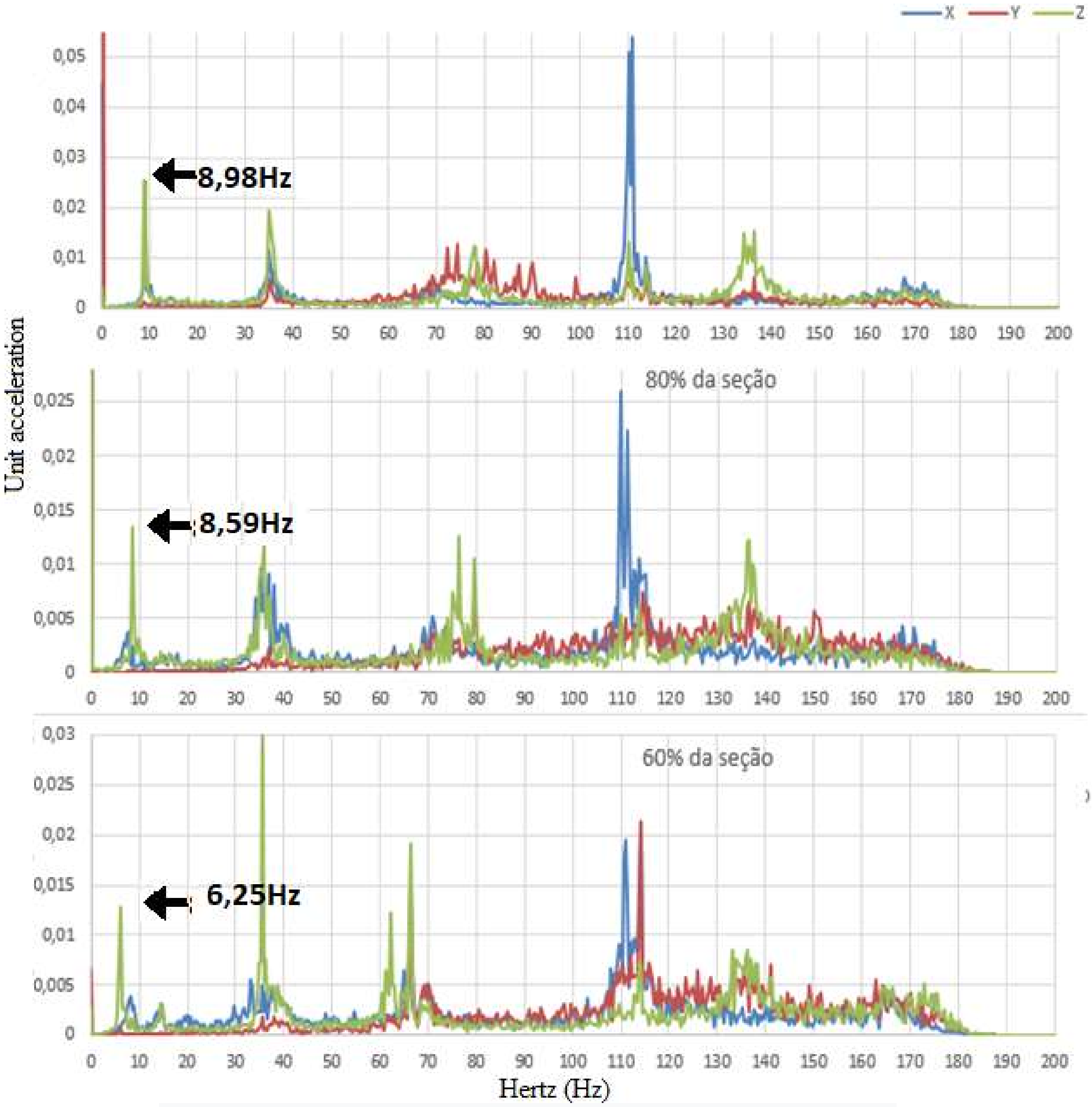


Figure 13 - Unit Acceleration x Frequency (Hz), B1, forced excitation.

For the considered frequency spectrum, of forced excitation (figures 12, 13), the first natural frequencies found for beam B1 correspond to 8.98Hz, 8.59Hz, 6.25Hz, considering the cross sections 100%, 80% and 60% of integrity. For beam B2, natural frequencies of 35.15Hz, 33.59Hz, 18.375Hz and 5.47Hz were identified, for section integrity levels of 100%, 80%, 60% and 40%. It was observed that a decrease in section integrity influenced the drop of natural frequencies.

## 8. Numeric Results

Tables 1 and 2 show the natural frequency values for the first 10 mode shapes of the two beams B1 and B2, obtained from numerical modeling via ANSYS R17.0 software. Figures 14 show the first 8 mode shapes for undamaged beam. Figures 15 to 18 show the 1st vibration mode for beam B2, with the cross section ranging from 100% to 40% of their initial integrity.

Table1 – Natural Frequencies of B1

| | **Beam 1,9m 100% section** | **Beam 1,9m 80% section** | **Beam 1,9m 60% section** | **Beam 1,9m 40% section** |
|---|---|---|---|---|
| MODE | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) |
| 1 | 9,48 | 9,20 | 6,32 | 1,67 |
| 2 | 14,03 | 13,90 | 12,92 | 12,13 |
| 3 | 30,67 | 30,67 | 30,67 | 28,34 |
| 4 | 34,05 | 33,91 | 31,68 | 30,46 |
| 5 | 36,91 | 36,89 | 36,80 | 36,32 |
| 6 | 70,61 | 69,30 | 61,50 | 57,04 |
| 7 | 72,21 | 70,93 | 64,95 | 62,26 |
| 8 | 97,58 | 97,58 | 97,54 | 97,51 |
| 9 | 120,06 | 120,05 | 120,03 | 116,98 |
| 10 | 127,21 | 127,14 | 127,25 | 122,69 |

Table2 – Natural Frequencies of B2

| | **Beam 1,9m 100% section** | **Beam 1,9m 80% section** | **Beam 1,9m 60% section** | **Beam 1,9m 40% section** |
|---|---|---|---|---|
| MODE | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) |
| 1 | 39,07 | 36,79 | 20,21 | 4,79 |
| 2 | 43,71 | 42,05 | 31,76 | 25,73 |
| 3 | 124,16 | 124,16 | 120,03 | 106,52 |
| 4 | 131,59 | 130,85 | 124,10 | 117,42 |
| 5 | 131,65 | 131,58 | 131,31 | 122,55 |
| 6 | 256,29 | 250,76 | 226,20 | 217,91 |
| 7 | 261,35 | 252,61 | 226,96 | 219,31 |
| 8 | 301,04 | 301,64 | 301,49 | 301,64 |
| 9 | 310,87 | 310,67 | 308,85 | 305,21 |
| 10 | 311,63 | 311,58 | 311,48 | 308,49 |

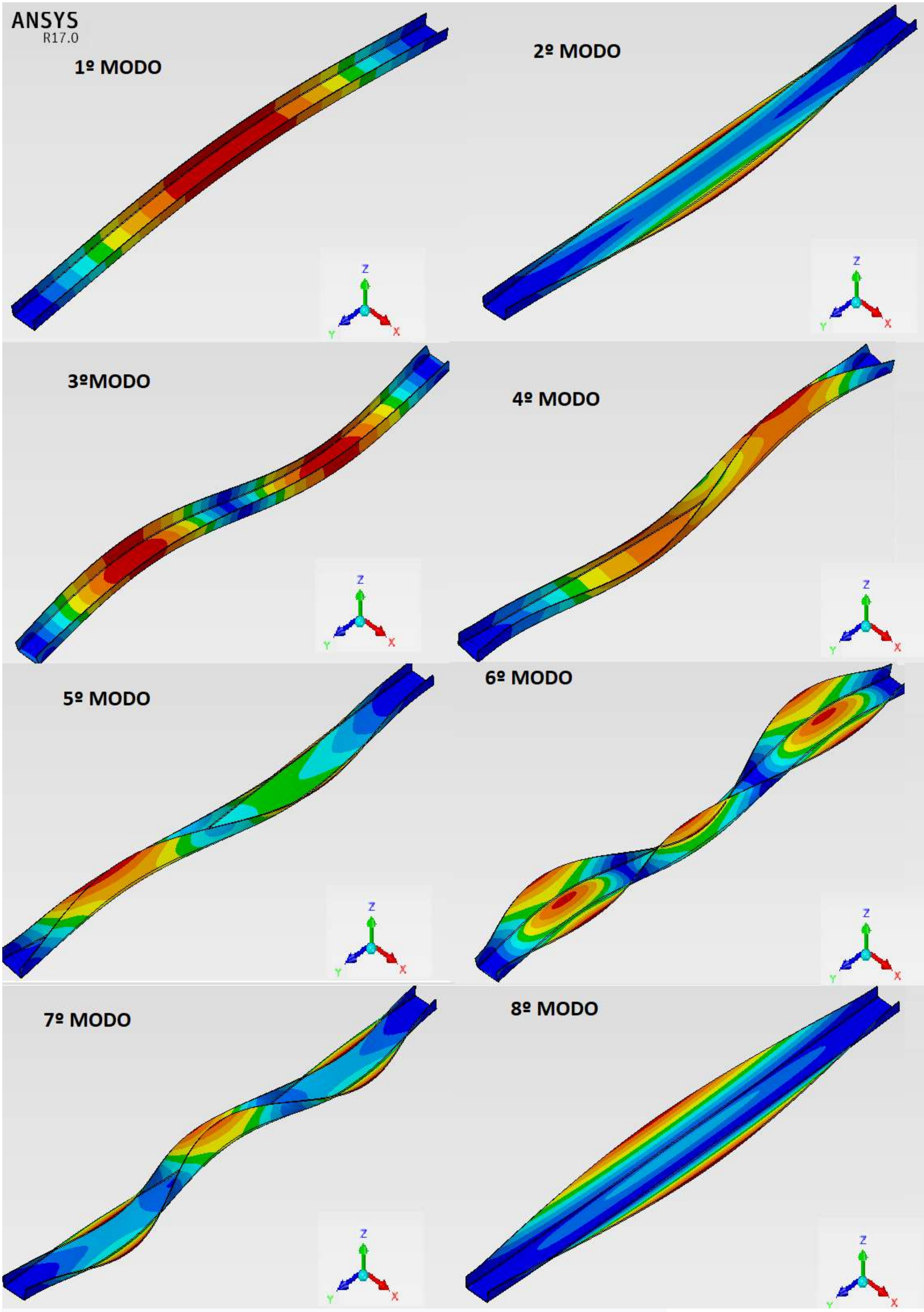


Figure 14 – B2 eight firsts mode shapes.

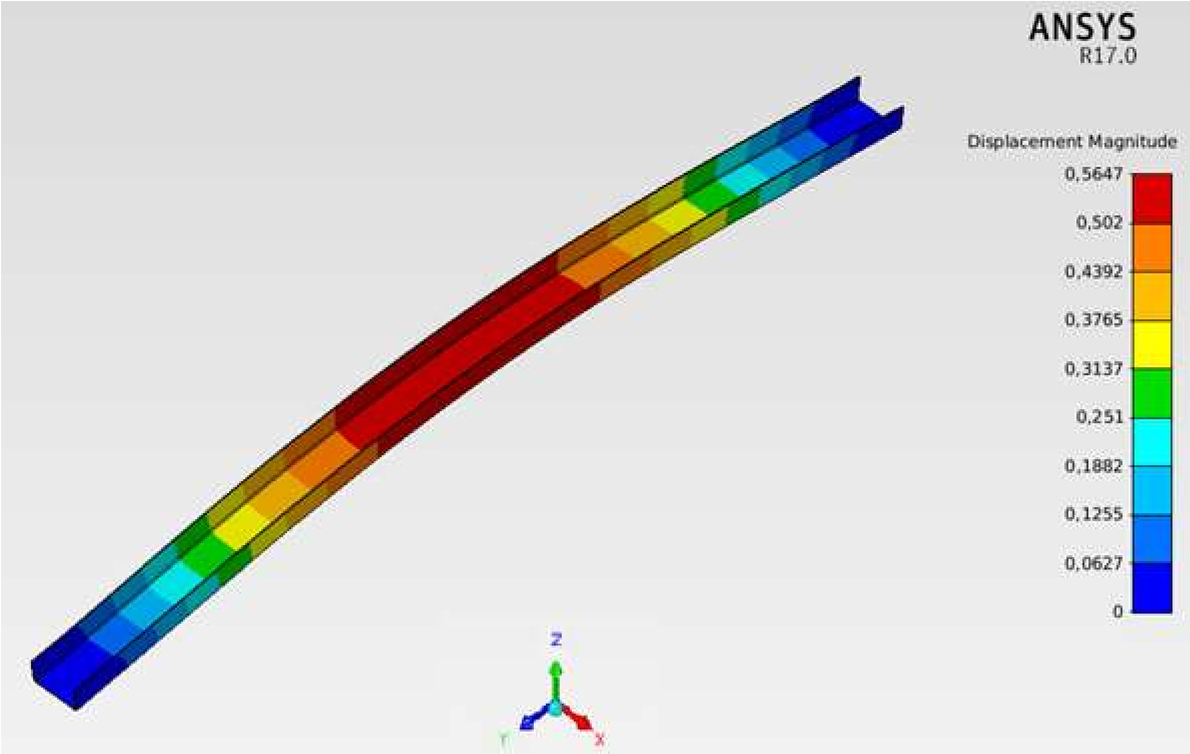


Figure 15 – first mode shape, 100% section, B2

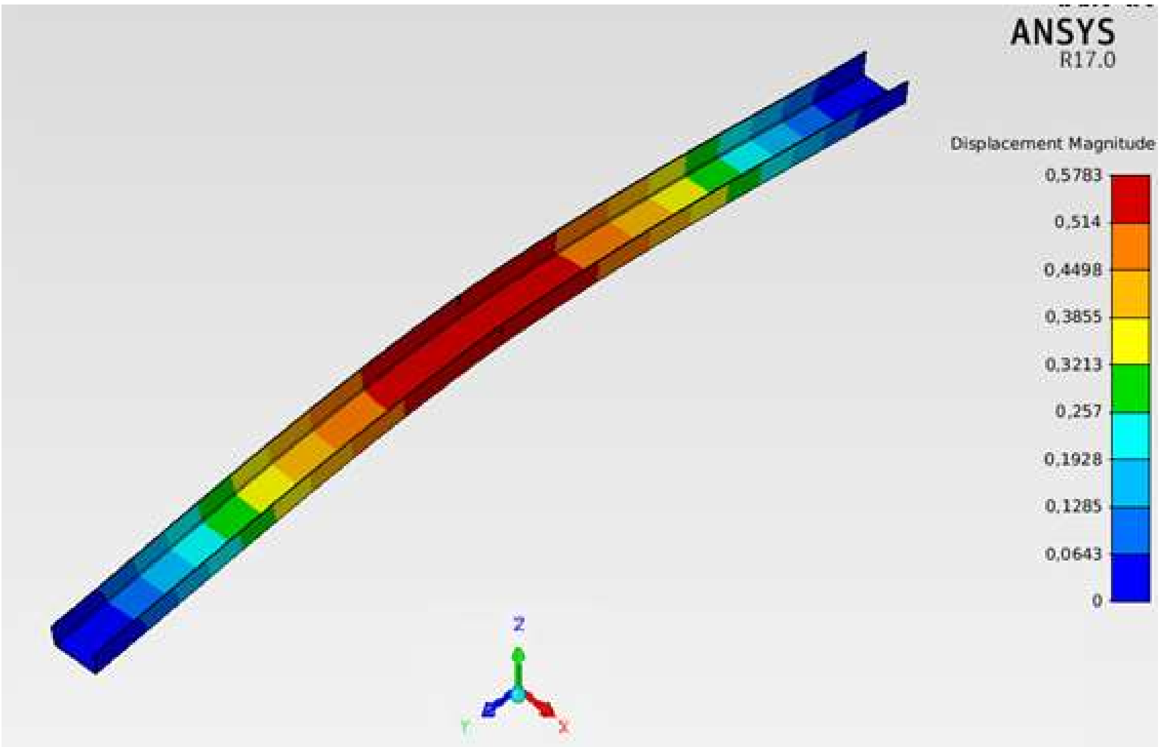


Figure 16– first mode shape, 80% section, B2

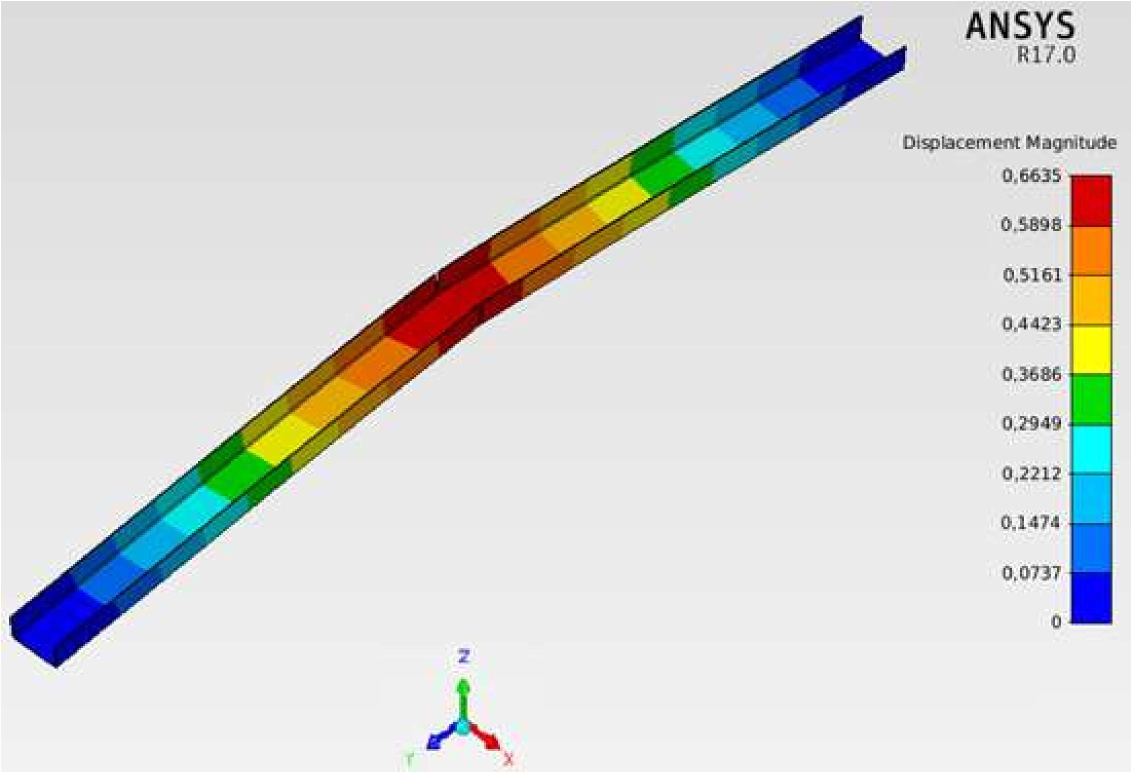


Figure 17 - first mode shape, 60% section, B2

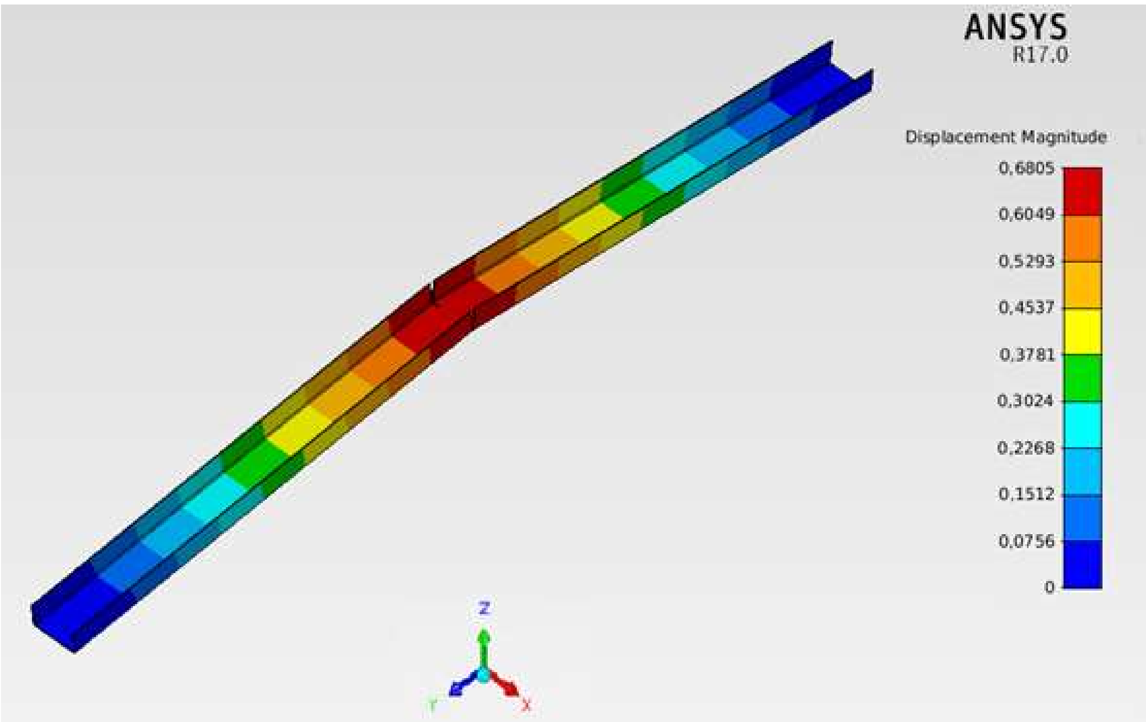


Figure 18 - first mode shape, 40% section, B2

## 9. Comparison of experimental results

As expected, beam B1 had lower natural frequencies than beam B2, as it has greater span and consequently lower stiffness.

It was possible to identify a reduction of natural frequencies in an increasing way, according to decrease of the section integrity level, in the beams B1 and B2 (table 5 and figures 19 and 20), as expected, showing that the section loss generated a drop of stiffness on the beams, and reduction in natural frequencies.

The natural frequencies found for free excitations and forced excitations are almost identical (Tables 3 and 4), as expected. The small discrepancy between some frequencies is believed to occur due to a strong presence of noise in the data collection phase. Thus, it can be said that the free excitation test is more efficient, as it is significantly more practical to perform.

It can be seen that the percentage changes in natural frequencies due to cut levels were slightly more severe in beam B2 (Table 5), this is believed to occur because the cut in the smaller span structure resulted in proportionally greater damage compared to beam B1.

Table 3 – Natural Frequencies of first mode shape. B1

| FIRST MODE SHAPE BEAM B1 | | | | |
|---|---|---|---|---|
| | **100% section** | **80% section** | **60% section** | **40% section** |
| TYE OF EXCITATION | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) |
| FREE EXCITTION | 8,98 | 8,59 | 6,25 | -- |
| FORCED EXCITATION | 8,98 | 8,59 | 6,25 | -- |
| **Frequencies percentual variation of Fee Excitation and Forced Excitation** | | | | |
| -- | Δ% 100% section | Δ% 80% section | Δ% 60% section | Δ% 40% section |
| -- | 0,00% | 0,00% | 0,00% | -- |

Table 4 - Natural Frequencies of first mode shape. B2

| FIRST MODE SHAPE BEAM B2 | | | | |
|---|---|---|---|---|
| **100% section** | **100% section** | **100% section** | **60% seção** | **40% seção** |
| TYE OF EXCITATION | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) |
| FREE EXCITTION | 35,54 | 33,59 | 20,31 | 5,46 |
| FORCED EXCITATION | 35,15 | 33,59 | 18,75 | 5,47 |
| **Frequencies percentual variation of Fee Excitation and Forced Excitation** | | | | |
| -- | Δ% 100% section | Δ% 80% section | Δ% 60% section | Δ% 40% section |
| -- | 1,10% | 0,00% | 7,68% | -0,18% |

Table 5 – Percentual Reduction of first mode shape of experimental tests.

| Percentual reduction 1º mode shape | | | |
|---|---|---|---|
| B1 | Δ% 100-80 CUT | Δ% 100-60 CUT | Δ% 100-40 CUT |
| | 4,34% | 30,40% | - |
| B2 | Δ% 100-80 CUT | Δ% 100-60 CUT | Δ% 100-40 CUT |
| | 4,44% | 46,66% | 84,44% |

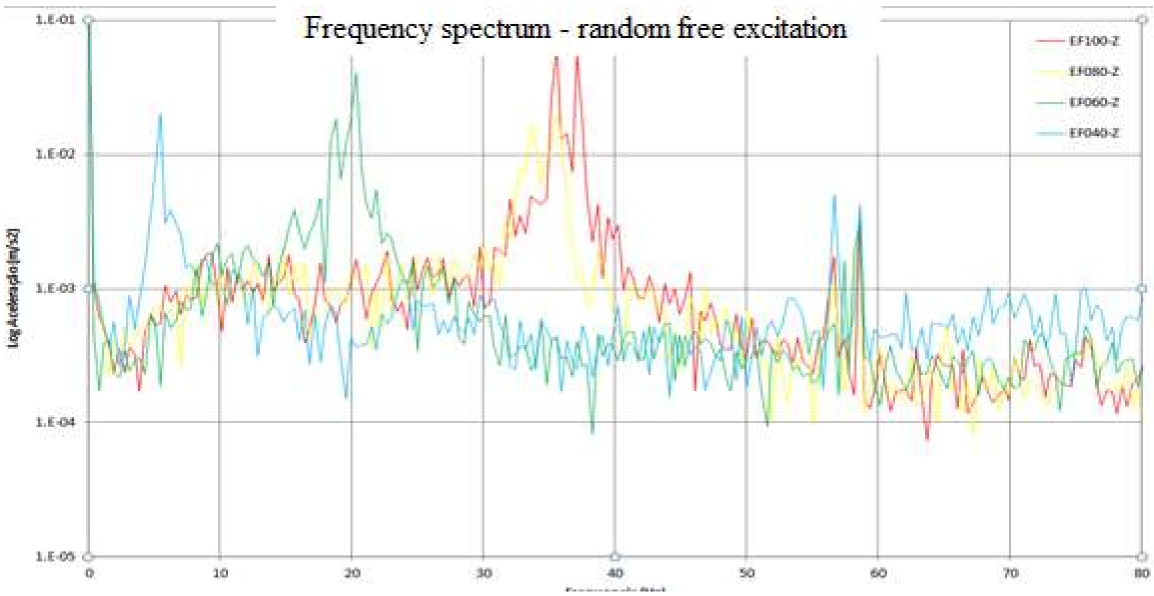


Figure 19 - Acceleration x frequency on free excitation. B2

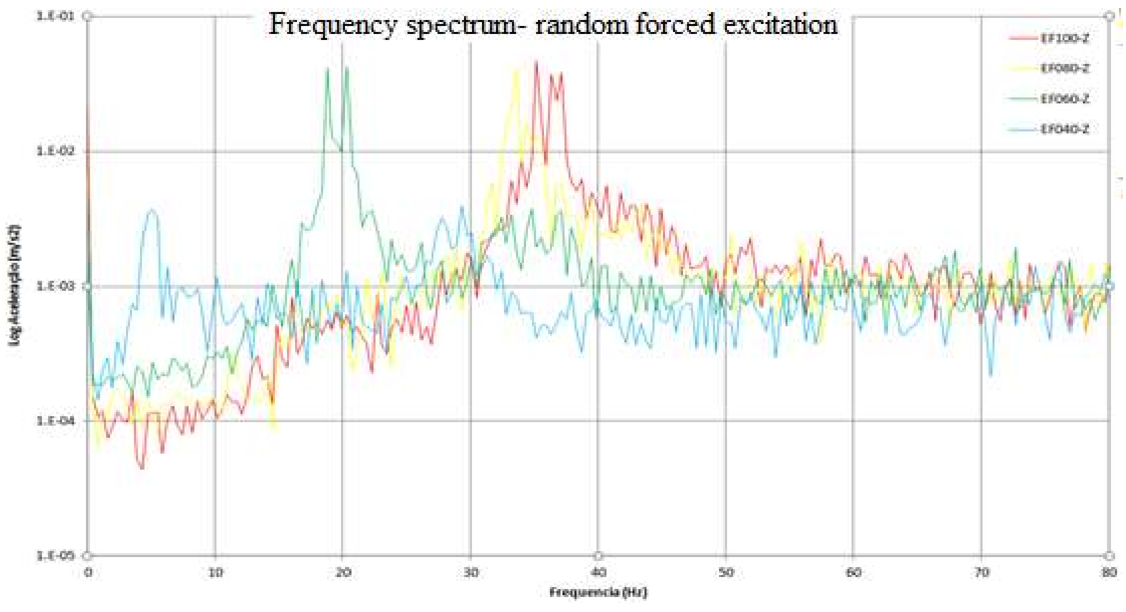


Figure 20 – Acceleration x frequency on forced excitation. B2.

## 10. Comparison of numerical results

Like experimental results, the natural frequencies of beam B1 are lower than those of B2, this occurs because the free span in B1 is larger, which makes the structure stiffness smaller.

Table 6 shows the percentage variation of natural frequencies for each vibration mode, it can be seen that in the first mode shape the natural frequency rapidly decreased, caused by the loss of stiffness, due to the lost in section integrity levels.

Other mode shapes have not changed so significantly (Table 6). Since reduced section integrity implies more loss of stiffness in vertical flexion. From the evaluation of figures 15 to 18 it is observed that the mode shapes have more flexible configurations as the damage increases.

Like the experimental results, it can be seen that the percentage changes in natural frequencies due to cut levels were slightly more significant in beam B2 (Table 6). This occurs because cutting beam B2 implies proportionally greater damage than in beam B1.

Table 6 – Percentual variation in natural frequencies.

| **Beam 3765mm** | | | |
|---|---|---|---|
| MODE | Δ% 100-80 CUT | Δ% 100-60 CUT | Δ% 100-40 CUT |
| 1 | 3% | 33% | 82% |
| 2 | 1% | 8% | 14% |
| 3 | 0% | 0% | 8% |
| 4 | 0% | 7% | 11% |
| 5 | 0% | 0% | 2% |
| 6 | 2% | 13% | 19% |
| 7 | 2% | 10% | 14% |
| 8 | 0% | 0% | 0% |
| 9 | 0% | 0% | 3% |
| 10 | 0% | 0% | 4% |
| **Beam 1882,5mm** | | | |
| MODE | Δ% 100-80 CUT | Δ% 100-60 CUT | Δ% 100-40 CUT |
| 1 | 6% | 48% | 88% |
| 2 | 4% | 27% | 41% |
| 3 | 0% | 3% | 14% |
| 4 | 1% | 6% | 11% |
| 5 | 0% | 0% | 7% |
| 6 | 2% | 12% | 15% |
| 7 | 3% | 13% | 16% |
| 8 | 0% | 0% | 0% |
| 9 | 0% | 1% | 2% |
| 10 | 0% | 0% | 1% |

## 11. Experimental x Numerical Results

Table 7 shows a comparison between the experimental and numerical results for beams B1 and B2. It can be observed that in all cases of section loss there was a decrease in natural frequencies, indicating that loss of stiffness was detected and it was possible to identify damage.

The variation between the natural frequencies of the experimental trial and the numerical model did not exceed 13%, and according to table 8, the percentage decrease of natural frequencies between the two models was very close (no more than 4% difference), indicating proximity between both procedures. It is believed that the difference in natural frequencies identified between the models is due to the inability of the numerical model to perfectly simulate the movement restrictions.

Table 7 – Percentual Variation between experimental and numerical results

| FIRST MODE SHAPE OF THE BEAMS | | | | |
|---|---|---|---|---|
| | B1 100% section | B1 80% section | B1 60% section | B1 40% section |
| ANALISIS TYPE | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) |
| NUMERIC | 9,48 | 9,2 | 6,32 | 1,67 |
| EXPERIMENTAL | 8,98 | 8,59 | 6,25 | -- |
| | B2 100% section | B2 80% section | B2 60% section | B2 40% section |
| ANALISIS TYPE | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) | FREQUENCY (HZ) |
| NUMERIC | 39,07 | 36,79 | 20,21 | 4,79 |
| EXPERIMENTAL | 35,15 | 33,59 | 18,75 | 5,47 |

| PERCENTUAL VARIATION BETWEEN NUMERIC AND EXPERIMENTAL FREQUENCIES | | | |
|---|---|---|---|
| B1 100% section | B1 80% section | B1 60% section | B1 40% section |
| 5,57% | 7,10% | 1,12% | -- |
| B2 100% section | B2 80% section | B2 60% section | B2 40% section |
| 11,15% | 9,53% | 7,79% | -12,43% |

Table 8 –Natural frequencies reduction of experimental and numerical results

| **Percentual Variation first mode B1** | | | |
|---|---|---|---|
| ANALISIS TYPE | Δ% 100-80 CUT | Δ% 100-60 CUT | Δ% 100-40 CUT |
| NUMERIC | 2,95% | 33,33% | 82,40% |
| EXPERIMENTAL | 4,34% | 30,40% | - |
| **Percentual Variation first mode B2** | | | |
| ANALISIS TYPE | Δ% 100-80 CUT | Δ% 100-60 CUT | Δ% 100-40 CUT |
| NUMERIC | 5,84% | 48,27% | 87,74% |
| EXPERIMENTAL | 4,44% | 46,66% | 84,44% |
| **Difference of percentual variation between numeric and experimental frequencies** | | | |
| BEAM1 | 1,39% | -2,93% | -- |
| BEAM2 | 1,40% | 1,62% | 3,30% |

## 12. Conclusion

The results found based on frequency spectrum analysis showed reductions in values (up to 84% frequency reduction for 60% section cut) almost identical for both types of excitations. The numerical model also presented a reduction of its natural frequencies, noting that it presented values very close to the experimental one (variations of maximum 13%). The decrease in natural frequencies for the different cut levels were very close in both methods (experimental and numerical) which is indicative of validity of the experiment.

The operational modal analysis was effective to identify level1 damage, according to Rytter [8] scale, for the two metal beams analyzed. It was possible to verify a decrease of natural frequency clearly even with the profile integrity in 80%. Errors between the numerical and experimental model did not exceed 13%, which is acceptable, as it is difficult to perfectly simulate the actual structure conditions. The free excitation assay was found to be more efficient, as it is significantly more practical to perform and obtained almost identical records.

The use of Operational Modal Analysis in Brazil for damage identification is in its infancy, there are still several factors to be studied for better efficiency of this method, such as the verification of possibility of identifying damage in large structures. However, this work shows that it is possible, from simple frequency spectrum analysis, to detect changes in structural stiffness, quickly and without paralyzing the structure.

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