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NUMERICAL SIMULATION OF DYNAMIC LARGE DISPLACEMENTS OF HALE AIRCRAFT WINGS

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Abstract. We revisit a classical structural engineering problem, fist solved by Euler and Bernoulli, that of large transversal displacements of cantilever beams. Their pioneering work, in the 16th century, stablished that curvature is proportional to the applied bending moment. In this context, many posterior authors simplified the resulting differential equation by assuming that, for small displacements, this curvature could be taken as the second partial derivative of the beam's axis transversal displacement with respect to the its longitudinal coordinate. This assumption may be adequate for Civil, Naval and Mechanical Engineering usual purposes, as in these fields of application such displacements are usually small. In recent aerospace applications this assumption is no long acceptable. HALE (High-Altitude Long-Endurance) aircraft wings are known to undergo large flexural displacements, due to their relatively small stiffness. Further, they are usually built of new high technology flexible materials. Thus, it is a design necessity to evaluate its deformed shape along time as it interferes with aeroelastic and aerodynamic concerns. In this paper, we present a simple, low cost, numerical solutions of the exact Euler-Bernoulli differential equation of the "elastica", to be compared to contemporaneous nonlinear large-scale Finite Element models via available either academic or commercial codes. The proposed algorithms basically numerically integrates the exact Euler-Bernolli differential equation using the MATLAB ode 45 code. The goal is always simulation of the dynamic behavior of such aircraft wings under turbulent aerodynamic excitation.

Keywords: HALE aircraft, Large displacements, exact curvature.

1 Introduction

The minimum weight criteria in aircraft and aerospace vehicles design, linked to the use of light materials that can undergo large displacements without exceeding their elastic limit, leaded to interest in analyzing flexible structures subjected to static and dynamic loads (Cesnik et al [3]). In aeronautical engineering we mention the building of high-aspect-ratio wings, whose geometry experiment large deformations during normal operating loads, according to geometrically nonlinear theories. The solution of such problems is a very demanding task.

According to Patil and Hodges [11], these aircraft are being considered for unmanned reconnaissance missions, environmental sensing, long-term surveillance, and communications relay (Fig. 1). They have slender wings whose aspect ratio is of the order of 35. Due to its high flexibility, large geometric nonlinear deflections occur, up to 25% of wing semi-spam.

Figure 1. Helios Prototype Flying Wing. Available from:
https://www.nasa.gov/centers/dryden/multimedia/imagegallery/Helios/ED03-0152-2.html

In general, structural nonlinearities are governed by elastic deformations that affect the whole structure. Diversely, concentrated nonlinearities act locally and are commonly found in control mechanisms or in the connecting parts between wing, pylon, engine, or external stores. The most prevalent nonlinearities in aircraft structures are concentrated structural nonlinearities, usually given by cubic stiffness and the freeplay (Woolston, Runyan and Andrews, [14]; Dowell, Edwards and Strganac, [4]). For HALE wings, with moderate-to-large amplitude deformations, its associated nonlinear behaviors are surely more frequent (Tang and Dowell, [13]; Frulla, Cestino and Marzocca, [8]).

Our work presents the study of an elastic analysis of second order, which considers the geometric nonlinearity of linear elastic material structures. Our models of a high-aspect-ratio wing are slender cantilever beams. In future work we will evaluate its the structural coupling between the torsion and edgewise bending moments generated by the vertical displacement (Afonso et al, [1])

2 Background

The first published work regarding deformation of flexible members is due to Leonhard Euler (1707-1783), around 1744, in the appendix of his De Curvis Elastics apud Oldfather, Ellis and Brown, [10]. According to Euler, when a member is subjected to bending, we cannot neglect the slope of the deflection curve in the expression of the curvature, unless the deflections are small (Fertis, [6]). The development of this theory started in the 18th century, by Jacob Bernoulli, his younger brother Johann Bernoulli and Euler himself.

The Euler-Bernoulli law states that the bending moment $M(x)$ is proportional to the change in the curvature produced by the action of the load, as shown in Equation (1):

$$\frac{1}{R} = \frac{d\theta}{dx_0} = \frac{M}{EI} \tag{1}$$

where R is the radius of curvature, θ is the slope at any point x_0 (with x_0 measured along the arc length of the member – see Fig. 2), E is the modulus of elasticity and $I(x)$ is the cross-sectional moment of inertia. For small deformations, it is usually assumed that $x= x_0$; hence, $L=L_0$. On the other hand, for large deformation, L is not L_0 (Patil and Hodges, [11]).

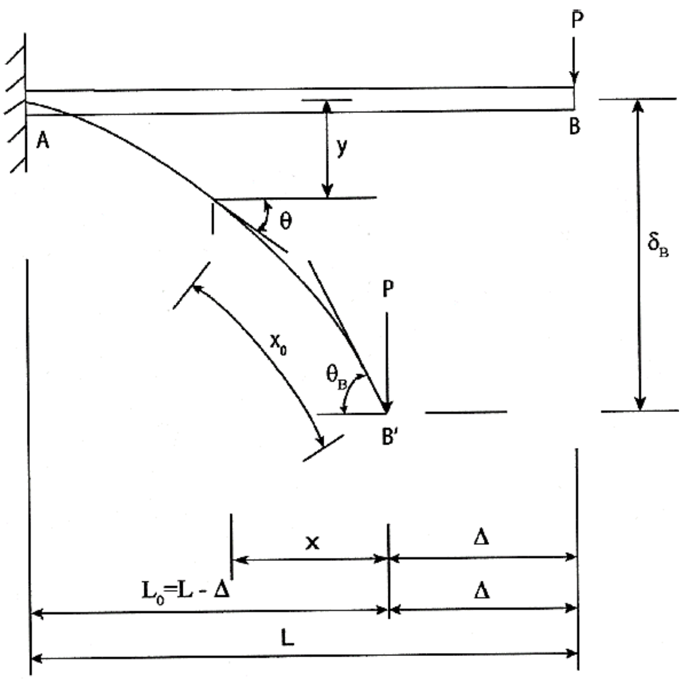

Figure 2. Large deformation of cantilever beam with uniform cross section. Available from Fertis (2006)

Equation (1) can be rewritten as Eq. (2), a second-order nonlinear differential equation showing that the deflection of a member is a nonlinear function of the bending moment. Its exact solution is very challenging to obtain, as the principle of superposition can no longer be applied.

$$\frac{1}{R} = \frac{y''}{[1 + (y')^2]^{\frac{3}{2}}} = -\frac{M(x)}{EI} \qquad (2)$$

According to Fertis [6], because of the difficulties involved in solving Eq. (2), most investigators turned their efforts to the use of the finite element method to that. However, other difficulties were developed, regarding the representation of rigid body motions of oriented bodies subjected to large deformations.

There is little analytical research about the inelastic behavior of flexible structures. Substantial work has According to Fertis [6], because of the difficulties involved in solving Eq. (2), most investigators been done by D.G. Fertis [5] and C.T. Lee [7] on the inelastic analysis of flexible bars, by using simplified nonlinear equivalent systems, for general inelastic behavior of both prismatic and non-prismatic members. The inelastic deformation of a uniform cantilever beam of rectangular cross-section with concentrated load at the free end was studied by G. Prathap and T.K. Varadan [12], considering the inelastic analysis of second-order for a Ramberg–Osgood material. Fig. 3 show the comparison between linear material under linear theory and under nonlinear theory analysis.

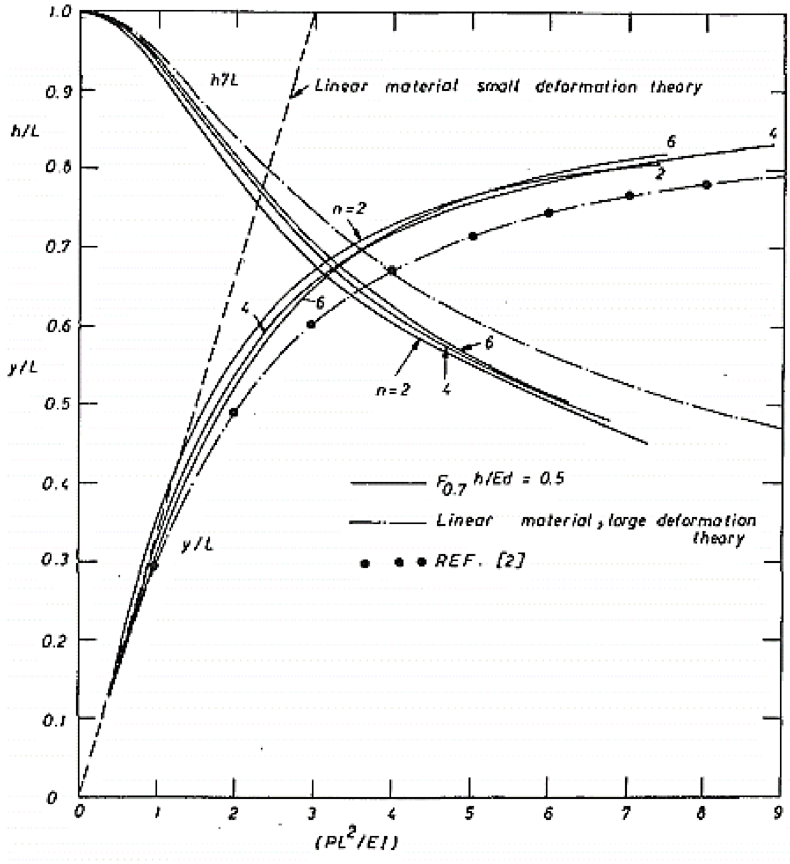

Figure 3. Tip deflections for uniform cantilever beam under point load at free end. Available from Prathap and Varadan [12]

C.C. Lo and S.D. Gupta [9] also worked on the same problem using the logarithmic strain definition for the regions where the material was stressed beyond its elastic limit (Fertis, [6]), obtaining the horizontal coordinate of beam tip within 1% error compared with the Euler-Bernoulli theory studied by Bishop and Drucker [2].

3 Mathematical modeling

The new idea presented in this paper is to integrate numerically the second order nonlinear ordinary differential equation (2). We take advantage of the fact that a wing is basically a clamped cantilever beam with restrained displacements and rotations at its root at the aircraft fuselage. Thus, we transform this boundary problem into an initial value one. Further, we transform this second order EDO into a pair of nonlinear first order ordinary differential equations, in order to use powerful integration tools such as the ODE 45 routine available in the MATLAB environment for this sort of initial value problem.

First, the following variables transformation is performed.

$$x_1 = y \tag{3}$$

$$x_2 = y' \tag{4}$$

rendering the set of two first order differential equations

$$x'_{!} = x_2 \tag{5}$$

$$x'_2 = \frac{M(x)}{EI}[1 + (x_2)^2]^{\frac{3}{2}} \tag{6}$$

4 Results of some examples

Next, we present results of some examples applying the proposed algorithm for a few hypothetic aircraft wings. Two are simple prismatic cantilever beam models under arbitrarily chosen loading patterns. The last one is a more realistic HALE wing with variable section, under reasonably realistic aerodynamic loading.

4.1 First example: prismatic beam with constant bending moment

Our first example, displayed in Fig. 4, is a horizontal cantilever prismatic beam made of isotropic uniform linear elastic material subject to constant *M* bending moment. Its length is *L*, the elasticity modulus is *E* and moment of inertia of the cross section is *I*.

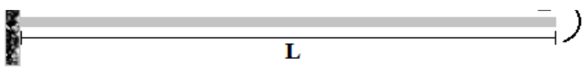

Figure 4. First example: prismatic cantilever subjected to constant bending moment

The well-known exact solution is a circle segment of *L* length, constant radius *R*, as shown in Eqs. (1) and (2), with internal angle θ given by

$$\theta = \frac{ML}{EI} \tag{7}$$

leading to coordinates of the displaced tip of the beam to be

$$X = R.sin\theta \qquad Y = R(1 - cos\theta) \tag{8}$$

Adopted numerical values are: *L*=1m, *E*=1 N/m², *I*=1m⁴. The implementation using ode45 function of MatLab resulted deformed beam tip coordinates X = 0.8563m and Y = 0.4837m, quite close to exact solutions of Equations (8), which are X = 0.8415m and Y = 0.4597m.

4.2 Second example: prismatic beam with uniform transverse loading

Next, we consider the horizontal cantilever prismatic beam of Fig. 5 made of isotropic uniform linear elastic material subject to a uniform distributed transverse load *p* = 1 N/m, emulating aerodynamic forces. Again, the beam length is *L* = 1 m, the elasticity modulus is *E* = 1 N/m², and moment of inertia of the cross section is *I* = 1 m⁴.

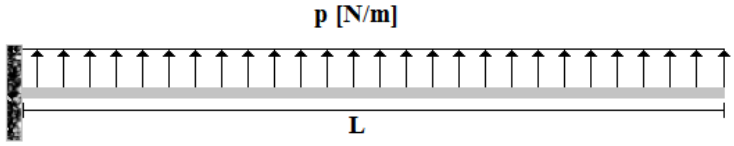

Figure 4. First example: prismatic cantilever subjected to constant bending moment

The implementation using ode45 function of MatLab resulted deformed beam tip coordinates X = 1 m and Y = 0.1264 m. These are to be compared to a Finite Element NASTRAN model simulated with 100 elements of type "beam", whose deformed beam tip coordinates are X=1.0054m and Y=0.1252m, leading to a difference of -0,54% and -0,96% for horizontal and vertical coordinates, respectively.

4.3 Third example: variable section wing under aerodynamic loading

Finally, a more realistic example was analyzed. We consider a 3 m half-span HALE aircraft wing. The chosen airfoil is NACA0012, a usual symmetrical profile, whose chord is made to vary from 600 mm at the root to 300 mm at the tip of the wing. The resulting variation of the moment of inertia along the wing, in m^4, is:

$$I(x) = 2.0x10^{-5}(.6 - .1x)^4 \quad (9)$$

It is adopted a fictitious composite material with elastic modulus E = 4 GPa. Based on Prandtl's aerodynamic theory, we adopted a variable transverse net lift load resulting the following bending moment equation, in Nm:

$$M(x) = 5(.6425x^4 + .5083x^3 + 3x^2 - 101.115x + 210.2325) \quad (10)$$

The resulting displacements at the tip of the wing where -7.3 cm in the horizontal direction and 44 cm in the vertical direction. Those are very large displacements, compatible with those expected of a HALE aircraft.

5 Conclusions

We presented a simple numerical solution by direct Runge-Kutta integration of the exact Euler-Bernoulli differential equation of the Elastica, implemented using ode45 function of MatLab. Two cantilever prismatic beam models, isotropic uniform linear elastic material, were analyzed, for constant bending moment and for uniform distributed transverse load. Another more realistic variable inertia wing subjected to Prandtl's lift was also analyzed. Numerical results were very good with relatively small computational cost.

Acknowledgements

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