

SPACECRAFT ATTITUDE CONTROL SYSTEM DESIGN CONSIDERING FUEL SLOSH DYNAMICS AND PARAMETERS ESTIMATION

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Abstract. The design of the satellite Attitude Control System (ACS) becomes more complex when the satellite structure has great number of components like, flexible solar panels and antennas, mechanical manipulators and tanks with fuel. Besides, the ACS performance and robustness depend on the effects of dynamics interaction between these components being considered in the satellite controller design. When the satellite is performing a translational and/or rotational maneuver the fuel slosh motion can change the center of mass position damaging the ACS accuracy. Therefore, controller performance and robustness depend not only on a good control technique but also on the knowledge of the system interactions characteristics. In this paper one apply the Linear Quadratic Regulator (LQR) technique to designs the ACS for a rigid satellite with a partially filled fuel tank taking into account the slosh dynamics using mechanical analogies type pendulum. During the LQR controller design some physical parameters of the system are estimated using the Kalman filter technique. The focus of the estimation is the length of the pendulum which represents the sloshing dynamics. The investigation is performed considering planar maneuver of the satellite caused by fixed thruster.

Keywords: satellite attitude control, kalman filter, slosh and flexible dynamics

1 Introduction

The problem of interaction between fluid and structure is important when one needs to study the dynamic behavior of offshore and marine structures, road and railroad containers partially filled with a fluid, spinning spacecraft with liquid fuel and ship motion and oil tankers. As for space missions the interaction between fluid and structure is important for a rigid or flexible structure interacting with a fluid under the sloshing effects. An interesting approach to analyze a rigid container mounted on flexible springs interacting with a perfect fluid including sloshing effects has been done by Lui and Lou [1]. Space structures, like rockets, geosynchronous satellites and the space station usually contains liquid in tanks that can represent more than 40% of the initial mass of the system, creating the need for more detailed dynamics studies and for the ACS design. When the fuel tanks are only partially filled and suffer a transversal acceleration, large quantities of fuel moves uncontrollably inside the tanks and generate the sloshing effects. The dynamics of the motion of the fuel interacts with the rigid body dynamics producing attitude instability, in order to minimize these effects new control methods should be employed to assure stability, and to achieve good attitude control effects. The dynamics of Rigid-Flexible satellite with fuel tanks when subject to large angle manoeuvre is only captured by complex non-linear mathematical model Ibrahim [2]. Besides, the remaining vibration can introduce a tracking

error resulting in a minimum attitude acquisition time. Souza [3] has shown the influence of the nonlinearities introduced by the panel's flexibility. It was shown that system parameters variation can degrade the control system performance, indicating the necessity to improve the ACS robustness. An experimental controller robustness and performance investigation was done by Conti and Souza, [4]. The estimation of the platform inertia parameters was introduced as part of the platform ACS design. The problem of designing satellite nonlinear controller has been done by Gonzales and Souza [5] using the SDRE method which is able to deal with high nonlinear plants. Due to the complexity of modeling a fluid dynamic system it is common to use mechanical systems analogies that describe this dynamic. But depending on the complexity of the rigid-flexible satellite control system design one need to obtain some physical parameters by experimental analysis. In this paper it was used the LQR technique Souza [6] and the Kalman filter procedure to estimate such parameters.

2 Sloshing Dynamics

The phenomenon of sloshing is given by the movement of a free surface of a liquid that partially fills a compartment. The movement caused by the liquid is an oscillating movement, which depends on shape of the tank, the acceleration of gravity, or axial acceleration of the tank. As representative of the behavior of the total weight of the system it is accepted that when the mass of the liquid oscillates the mass center of the rigid body also oscillates, thereby disturbing the rigid-flexible part of the vehicle under consideration. It is natural to consider the oscillating movement as a wave generated by the movement of the liquid as a stationary wave which has oscillation modes. Each mode of oscillation has a special feature of this phenomenon under study, and one observes, in a quantitative sense, how much mass is displaced. Among all the modes that cause the greatest disruption in the system the first and second modes are the more important. These modes have lower frequency of oscillation but they are able to shift the center of mass (CM) of the liquid. The other oscillation modes act as a less aggressive and may not even vary the position of its CM due to the symmetry of the wave which on average causes no displacement. Due to its complexity, the sloshing dynamics is usually represented by mechanical equivalent systems that describes a similar and reproduce faithfully the actions and reactions due to forces and torques acting on the system. The main advantage of replacing the fluid model with an equivalent oscillating model Ibrahim [2] is simplifying the analysis of motion, in the rigid body dynamics, compared to the fluid dynamics equations. The figures 1 and 2 represent the type mass-spring and pendulum analogous mechanical for the slosh dynamics, respectively.

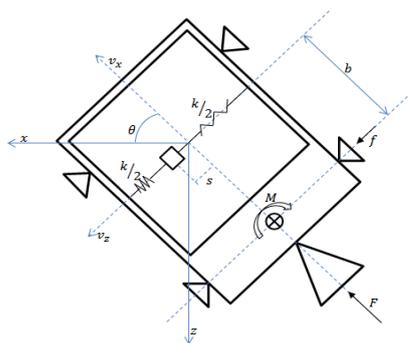


Figure 1. Mechanical analogous mass-spring type

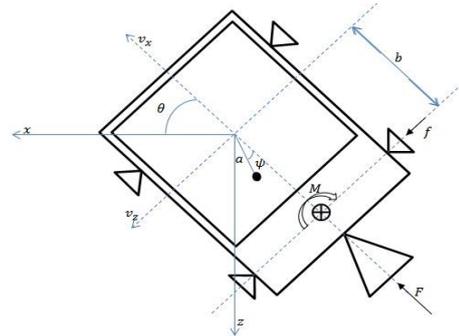


Figure 2. Mechanical analogous pendulum type.

3 Satellite Mathematical Model

For this study it was used the model with a analogous pendulum type, like shows in Figure 2.

Due to the complexity of the fluid moving inside the tank, one assumes the following: 1) small displacements, 2) a rigid tank, 3) no viscous, incompressible and homogeneous liquid, 4) the dynamic of the sloshing is approximated by a pendulum type and 5) the spacecraft is rigid moving in a fixed

plane and include the first lowest frequency slosh mode. The satellite's mass and moment of inertia are m and I ; and the fuel's mass and inertia moment are m_f and I_f , respectively. One assumes that the transverse force f generates a pitch moment M and that the thrust F acts on the spacecraft longitudinal axis. The velocity of the center of the fuel tank v_x, v_z , and θ is the attitude angle of the spacecraft. Fig. 2 shows the linear velocity V and the angular velocity ω of the spacecraft and the pendulum angle ψ , which is assumed $\dot{\psi} = 0$. As a result, the Lagrange equations [7] are:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial V} \right) + \omega^\times \frac{\partial L}{\partial V} &= \tau_t \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \omega} \right) + \omega^\times \frac{\partial L}{\partial \omega} + V^\times \frac{\partial L}{\partial V} &= \tau_r \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\psi}} \right) - \frac{\partial L}{\partial \psi} + \frac{\partial R}{\partial \dot{\psi}} &= 0 \end{aligned} \quad (1)$$

where L is the Lagrangian, R is the Rayleigh dissipation function, τ_r and τ_t are the intern and external torque. One assumes $R, \tau_r, \tau_t, \omega, V$ as [7]:

$$R = \frac{1}{2} \varepsilon \dot{\psi}^2; V = \begin{bmatrix} v_x \\ 0 \\ v_z \end{bmatrix}; \omega = \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}; \tau_t = \begin{bmatrix} F \\ 0 \\ f \end{bmatrix}; \tau_r = \begin{bmatrix} 0 \\ M + fb \\ 0 \end{bmatrix} \quad (2)$$

The position vector of the CM of the body with respect to the position of the tank is $\vec{r} = (x-b)\hat{i} + z\hat{k}$, and the velocity are $v_x = \dot{x} + z\dot{\theta}$ and $v_z = \dot{z} - x\dot{\theta}$, $\dot{\vec{r}} = v_x\hat{i} + (v_z + b\dot{\theta})\hat{k}$. The fuel mass position is given by $\vec{r} = (x - a\cos(\psi))\hat{i} + (z + a\sin(\psi))\hat{k}$ and the velocity is $\dot{\vec{r}} = (v_x + a\sin(\psi)(\dot{\theta} + \dot{\psi}))\hat{i} + (v_z + a\cos(\psi)(\dot{\theta} + \dot{\psi}))\hat{k}$. Then the Lagrangian for this system is given by

$$L = \frac{1}{2} m \dot{\vec{r}}^2 + \frac{1}{2} m_f \dot{\vec{r}}_f^2 + \frac{1}{2} I_f (\dot{\theta} + \dot{\psi})^2 + \frac{1}{2} I \dot{\theta}^2 \quad (3)$$

After derivations using the previously equations the spacecraft equations of motion are

$$(m+m_f)(\dot{v}_x + v_z\dot{\theta}) + mb\dot{\theta} + m_f a(\ddot{\psi} + \ddot{\theta})\sin(\psi) + m_f a(\dot{\theta} + \dot{\psi})^2 \cos(\psi) = F \quad (4)$$

$$(m+m_f)(\dot{v}_z - v_x\dot{\theta}) + m_f a(\ddot{\theta} + \ddot{\psi})\cos(\psi) - m_f a(\dot{\theta} + \dot{\psi})^2 \sin(\psi) - mb\ddot{\theta} = f \quad (5)$$

$$(I_f + mb^2)\ddot{\theta} + mb(\dot{v}_z - v_x\dot{\theta}) - \varepsilon\dot{\psi} = M + bf \quad (6)$$

$$(m_f a^2 + I_f)(\ddot{\theta} + \ddot{\psi}) + m_f a((\dot{v}_x + v_z\dot{\theta})\sin(\psi) + (\dot{v}_z - v_x\dot{\theta})\cos(\psi)) + \varepsilon\dot{\psi} = 0 \quad (7)$$

Assuming the relations $a_x = \dot{v}_x + v_z\dot{\theta}$, $a_z = \dot{v}_z - v_x\dot{\theta}$ and substituting into Eq.(4) and Eq.(5), one has

$$a_x = \frac{F - mb\dot{\theta} - m_f a(\ddot{\psi} + \ddot{\theta})\sin(\psi) - m_f a(\dot{\theta} + \dot{\psi})^2 \cos(\psi)}{m + m_f} \quad (8)$$

$$a_z = \frac{f - m_f a(\ddot{\theta} + \ddot{\psi})\cos(\psi) + m_f a(\dot{\theta} + \dot{\psi})^2 \sin(\psi) - mb\ddot{\theta}}{m + m_f} \quad (9)$$

The linearized equations of motions are given by

$$\ddot{\theta}(I_f + m^*(a^2 - ba)) + \psi(I_f + m^*a^2) + am_f^*F\psi + \varepsilon\dot{\psi} = -am_f^*f \quad (10)$$

$$\ddot{\theta}(I + m^*(b^2 - ba)) - m^*ab\psi - \varepsilon\dot{\psi} = M + b^*f \quad (11)$$

where $b^* = \frac{bm_f}{m + m_f}$, $m^* = \frac{mm_f}{m + m_f}$ and $m_f^* = \frac{m_f}{m + m_f}$

4 Simulations Results

The spacecraft model used in the simulations are given by Eq.(10) and Eq.(11) and LQR Method is used to design the control law and the Kaman filter technique is used to estimate the instantaneous state of the system. The data used are: $m = 600kg$, $m_f = 100kg$, $I = 720kg/m^2$, $I_f = 90kg/m^2$, length of the pendulum rod $a = 0.2m$, $b = 0.3m$, $F = 2300N$, $\varepsilon = 0.19kgm^2/s$. The initial condition: $\theta = 2^\circ$, $\dot{\theta} = 0.57^\circ/s$, $\psi = 30^\circ$ and $\dot{\psi} = 0^\circ$. Figure 3 shows the evolution in time of state variables of the system controlled by LQR in about fifteen seconds.

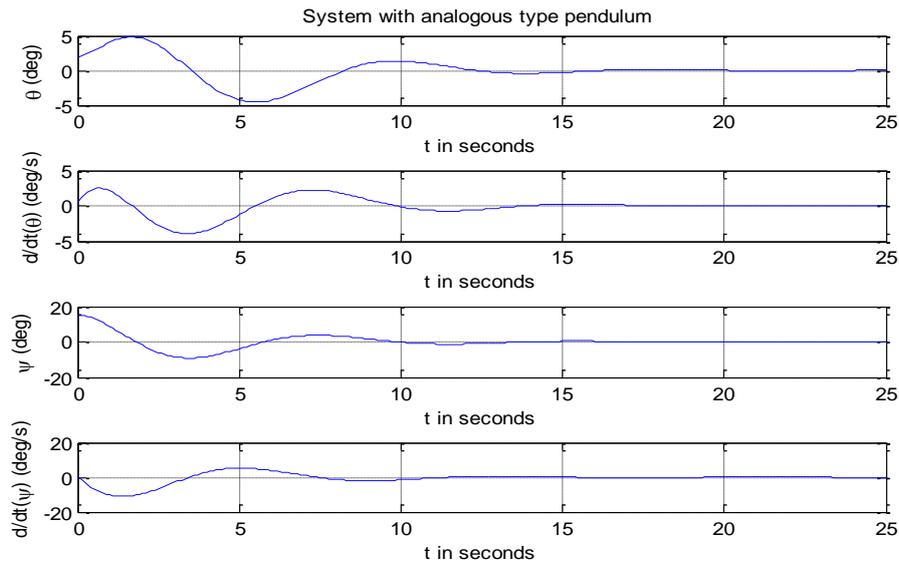


Figure 3. LQR Controller performance without the Kalman filter

In the simulations considering Kalman filter the closed loop system measuring $\theta, \dot{\theta}$, and though one estimates the five state, the rod length a is the focus of the estimation. The importance of estimate de rod length is because this value directly proportional to the vibration of the slosh [7]. Figure 4 shows that the system is still controlled in less than twenty seconds, with the same performance than before, with perfect knowledge of state. The value of the rod length is estimated as $a = 0.18902m$, remembering that the real value of the rod is $0.20m$.

5 Conclusions

The paper describes briefly the concepts of the phenomenon of sloshing, dealing with the dynamics of a liquid that partially fills a reservoir. The pendulum type is used to model the slosh dynamics. Using this model it was

possible to estimate a parameter and designs a LQR controller which keeps all states of the system controlled. When one introduce the Kalman filter is fed by the θ and $\dot{\theta}$ measurements, the control system the performance of the LQR controller is not so disturbed presenting an adequate response and controlling all states in less than 20 seconds and the value of the rod length is estimated as $a = 0.18902m$, which is a very close the real value of the rod $0.20m$.

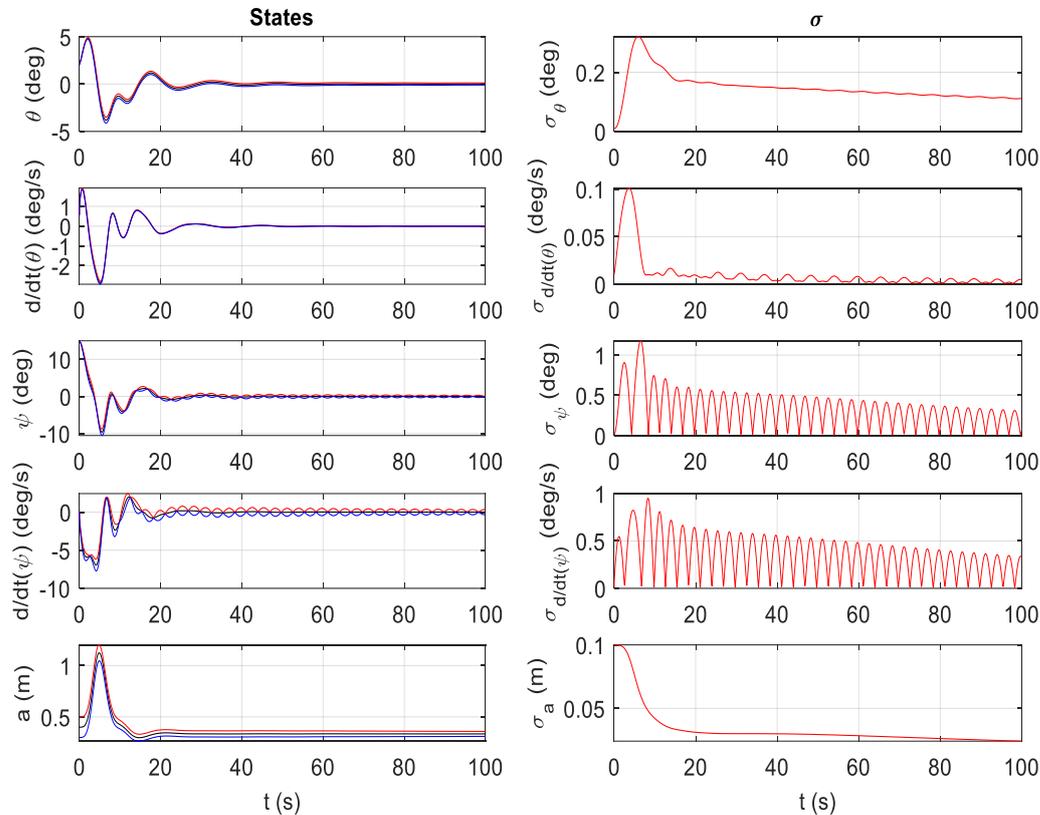


Figure 4. The LQR controller performance with Kalman Filter

References

- [1]Lui A. P. and Lou J. Y. K. Dynamic coupling of a liquid-tank system under transient excitations. *Ocean Engineering*, 17(3):263-277, 1990.
- [2]Ibrahim, R.A., 2005, "Liquid sloshing dynamics, theory and applications", Cambridge university press, New York.
- [3]Souza L. C. G., 1996, "Robust controllers design for flexible space system using a combination of LQG/LTR and PRLQG methods". In: *Dynamics and Control of Structure in Space III*. U.K.: C. L. Kirk and D. J. Inman, 151-166.
- [4]Conti. G.T. and Souza L.C.G., 2008. Satellite attitude control system simulator. *Shock and Vibration*. 15, 3-4, 395-402.
- [5]Souza, L.C.G. & Gonzales,R.G. (2012).Application of the state-dependent Riccati equation and Kalman filter techniques to the design of a satellite control system. *Shock & Vibration*,19(5),939-946.
- [6]Souza, L.C.G. (2006). Design of Satellite Control System Using Optimal Nonlinear Theory. *Mechanics Based Design of Structures and Machines*, 34(4), 351-364.
- [7] Souza, L.C.G.; Souza, A. G. Satellite Attitude Control System Design considering the Fuel Slosh Dynamics. *Shock and Vibration*, v. 2014, p. 1-8, 2014.