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# SEARCHING FOR "DISTANCE-STABLE" ORBITS FOR A SPACECRAFT TO OBSERVE THE TRIPLE ASTEROID 2001SN$_{263}$

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**Abstract.** Missions to asteroids are very popular in current space research, for many reasons. In the scientific side, it is believed that some asteroids have information about the original cloud of particles that formed the Solar System in the past. There are also commercial reasons, with a large interest in exploring minerals from asteroids, and even planetary defense related studies, searching for alternatives to avoid a collision of an asteroid with the Earth. The objective of the present paper is to search for stable orbits to locate a spacecraft that has the goal of observing the triple system of asteroids 2001SN$_{263}$. This is a very interesting system and a very good candidate to receive a spacecraft. Trajectories near the primary body, in the middle of the orbits of both smaller bodies of the system and outside the orbit of the external satellite body are investigated. A new definition of stability for the orbit is made, with a very practical goal, which is to keep the spacecraft-primary body distance inside a given interval. We called this type of stability "Distance-Stability". Preferred orbits are found in the three regions studied, and a physical explanation is made, based in the integral of the accelerations received by the spacecraft.

**Keywords:** Astrodynamics, Space Trajectories, Asteroids, Stable Orbits, Perturbed Orbits.

## 1 Introduction

Missions to asteroids are very popular nowadays and several missions flew or are schedule to fly [1-7] to those bodies. Asteroids are expected to keep information related to the formation of the Solar System. There is also a long list of papers studying more theoretical aspects of smaller bodies, like the study of their gravitational field, trajectories, etc [8-15].

There are many asteroids having a good potential to be visited by a spacecraft. One of the most interesting ones is the triple system 2001SN$_{263}$ [6]. The three bodies of the system have radius about 1.30 km, 0.39 km and 0.29 km. The largest of the satellite bodies is called Beta and it orbits the central asteroid in an orbit with semi-major axis 16.63 km and eccentricity 0.015. The second satellite body orbits the central body in an orbit with semi-major axis 3.80 km and eccentricity 0.016 [15-18].

The goal of the present paper is to search for stable initial circular orbits around the main body of the system, where a practical definition of stability is made, based in the evolution of the distance spacecraft-main body. Several previous publications showed that natural orbits around Beta and Gamma have very short duration, so requiring intensive station keeping maneuvers [14, 16, 18]. A new concept of stability is defined here, with a very practical form, based in the oscillations of the distance spacecraft-main body. We call these orbits “distance-stable orbits”, in the sense that they keep the oscillations of the distance spacecraft-asteroid inside a fixed limit. The reason is that it is desired to find natural orbits that avoid large fluctuations of this distance, which is interesting from the point of view of treating the data collected. Other definitions of stability were used in some recent researches to explore orbits in this same system, based in the duration of the orbits before a collision of the particle with one of the bodies or the escape of the particle from the system [14, 16, 19, 20]. The literature also has another practical definition of stability, made by Hu and Scheeres [21], based in the maximum eccentricity reached by the orbit of the spacecraft, which is called "shape stability". It is related to the present definition, although not the same, since the spacecraft-asteroid distance also depends on the instantaneous semi-major axis of the orbit, not only from the eccentricity.

The present study considers orbits that are initially circular, since the goal is to keep the oscillations of the spacecraft-main body as constant as possible. Then, those orbits are numerically integrated for a maximum period of 180 days, always measuring the values of the spacecraft-main asteroid distance. It is verified when this distance reaches values below or above a defined limit, and this time is defined as the lifetime of the natural orbit. After this point a maneuver would be required to place the orbit back to the limits previously defined. This limit is different for each region under study. It is 2.0 km for the exterior region, with semi-major axis above 18 km; 1.0 km for the intermediate region, between the orbits of the two smaller asteroids, with semi-major axis varying from 5.0 to 14.0 km; and 0.2 km for the region inside the orbit of Gamma, from 1.5 to 3.0 km. The results presented here complements some previous studies of orbits in this system made in Prado [22], where general perturbation maps (using the nomenclature defined in [23]) were made using an integral approach, but not looking for specific orbits, like done in the present research.

The results showed that some particular locations in space have orbits which are much more "distance-stable" then others. Even neighbor orbits have large differences in terms of the "distance-stability" of the orbits. The reason is the combined effects of all the perturbations acting in the spacecraft. To explain this point in more details, an integral scalar index is used to quantify the level of perturbation received by a satellite. An "Integral Index" is a new concept that appeared in the literature [22-31], and can be applied in the present research. The "integral index" used here is the one proposed by Lara [30]. The integral is calculated for each component of the forces acting in the spacecraft. This type of integral allows the compensations of positive and negative values during the orbital period of the spacecraft. This fact makes this definition more adequate for mapping orbits based in a long term timescale. The evaluations of those integrals were made during the whole trajectory, and the results divided by the duration of the trajectory, so expressing the average of the effects accumulated over the time. The result of this integration process represents the contribution of the total force acting in the trajectory of the spacecraft. This same idea was used in [31] to quantify the effects of the Sun and the Moon in spacecraft trajectories. Some preliminary results of this type were presented in [19], but more realistic results obtained with the inclusion of the solar radiation pressure, as well as a physical explanation of the orbits found.

## 2 Mathematical Model

The asteroid system is assumed to be formed by three bodies. The largest one is called Alpha and is located in the center of the reference frame. Its mass is assumed to be $m_0$. The two other asteroids are smaller bodies and they disturb the orbit of the spacecraft. They will be called Beta, the largest of the two smaller bodies; and Gamma, the smaller body of the system. They are assumed to be in elliptical orbits in different orbital planes, having a mutual inclination of 14 degrees. Figure 1 shows the system in detail. In this system of asteroids there is a spacecraft with negligible mass travelling around the main body, disturbed by the gravity fields of the two smaller asteroids and the solar radiation pressure. The solar radiation pressure is a major perturbing force in this system, so it cannot be neglected. It is also considered the flattening of the central body. The reference plane used to study the motion of the spacecraft is the orbital plane of the body Beta. The gravity of the bodies are obtained directly by the Newton's law of universal gravitation, which is proportional to the product of the two masses involved divided by the square of the distance between the two bodies. The system of reference considered in the present study is not inertial, but centered in the largest asteroid of the system. This is done because we are interested in the behavior of the orbits with respect to this body. Other forces, like the gravity of the planets, are acting in all the bodies of the system, and the differential effects are very small, since the system of asteroids have bodies very close to each other. More details will be shown later in the present paper. The orbit of the spacecraft is defined by its keplerian elements: a (semi-major axis), e (eccentricity), i (inclination), ω (argument of perihelion), Ω (longitude of the ascending node) and mean motion n.

To evaluate the level of perturbation of the orbits, the integral of the accelerations received by the spacecraft are evaluated, according to Eq. 1, where ($a_x$, $a_y$, $a_z$) are the components of the accelerations acting in the spacecraft and T is the period of integration of the orbit. A Keplerian orbit would give zero for this integral, so the numbers obtained represent the level of perturbation received by the spacecraft from the perturbing forces as well as from the main body, due to the fact that the orbit is no longer Keplerian. The physical and orbital data of the bodies that compose the system $2001SN_{263}$ are shown in Table 1 [16].

$$Pi3 = (\frac{1}{T})\left(\left(\int_0^T a_x dt\right)^2 + \left(\int_0^T a_y dt\right)^2 + \left(\int_0^T a_z dt\right)^2\right)^{1/2} \quad (1)$$

Table 1: Physical and orbital components of the system $2001SN_{263}$[16]

| **Body** | ***a*** | ***e*** | ***I (deg)*** | **Radius (km)** | **Mass (kg)** |
|---|---|---|---|---|---|
| Alpha | 1.99 UA | 0.48 | 6.7 | 1.3 | $917.47 \times 10^{10}$ |
| Beta | 16.633 Km | 0.015 | 0.0 | 0.39 | $24.04 \times 10^{10}$ |
| Gamma | 3.804 Km | 0.016 | 13.87 | 0.29 | $9.77 \times 10^{10}$ |

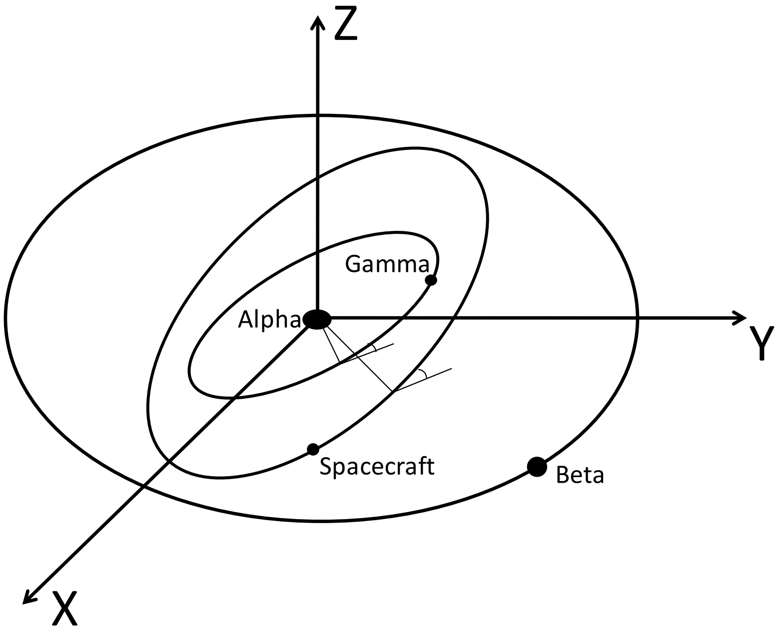



Fig. 1. The $2001SN_{263}$ system showing the triple asteroid and the spacecraft.

## 3. Results

The results of the present paper are concentrated in searching for "distance-stable" trajectories for a spacecraft orbiting the triple asteroid $2001SN_{263}$. Several ranges of semi-major axis are considered. Trajectories outside the orbits of both smaller bodies, from 18 to 50 km from the central body, are interesting for the mission. The idea is to use them to place the spacecraft when it arrives at the system, in order to study in more detail the shape and masses of the three bodies, to obtain important information to make a final decision related to which orbits around each body can be used for the mission. At this point it is important to avoid risks of collisions with one of the bodies or space debris from the system, as well as to avoid escapes due to the gains of energy coming from close approaches between the spacecraft and the satellite bodies of the system. After that, trajectories closer to the central body are also investigated. They are divided in two more regions: the one between the orbits of the satellite bodies and the interior region, below the orbit of Gamma. Those orbits can be interesting to observe more than one body at the same time, without orbital maneuvers. The perturbations considered in the present model are due to the gravity field of the two satellite bodies (Beta and Gamma); the main body Alpha and its flattening at the poles, expressed by the $J_2$ term of its gravity field, and the solar radiation pressure. Reference [22] makes a detailed study of the gravity forces acting in the spacecraft. Instead of considering the forces in specific points, it makes an integral of the effects of each force for one orbital period of the spacecraft. It gives a better comparison, by considering different orbital regions. Several figures show the results in details in this reference, but the summary is shown in Table 2 [22]. Note that the larger perturbations, except the ones coming from the bodies of the system, comes from Mars and the Sun, but they are 1000 times smaller when compared to the perturbation from Gamma, which gives the largest perturbation, and 100 times smaller than the perturbations coming from Beta. It is interesting to note that the perturbations from Gamma are stronger than the one from Beta, although it has a smaller mass. It happens because Gamma is very close to Alpha, and it influences the motion of the central body. Indirectly, it modifies the orbit of the spacecraft around Alpha [22]. The calculations are made assuming a circular orbit with zero inclination and semi-major axis of 10 km, but are not much different for other orbits. The smaller bodies of the system are assumed to be in elliptical and non coplanar orbits. A limit of 180 days was used for the numerical integrations, because this time is about half of the duration of the mission, expected to last around one year, and that will also study Beta and Gamma. This time can also be considered long enough to observe the bodies during the mission.

An orbit was considered "distance-stable" if the distance spacecraft-Alpha remains inside a specified limit. This limit was fixed in 2.0 km for exterior orbits, the ones with semi-major axis ranging from 18.0 to 50.0 km. For the internal orbits, in the range of semi-major axis from 1.5 to 3.0 km, the limit used was 0.2 km. For the intermediate orbits, in the middle of the orbits of the two smaller bodies, with semi-major axis ranging from 5.0 to 14.0 km, the limit used was 1.0 km. The limits are not the same because orbits closer to the bodies need to remain inside a smaller range of variation.

Table 2: Perturbations Due to the other bodies of the Solar System [22].

| Planet | Mean Value (m/s) |
|---|---|
| Alpha | 0.0186 |
| Beta | 0.0024 |
| Sun | $1x10^{-5}$ |
| Mercury | $1.99\times10^{-12}$ |
| Venus | $6.96\times10^{-11}$ |
| Earth 1 | $2.13\times10^{-9}$ |
| Earth 2 | $5.55\times10^{-9}$ |
| Moon | $2.63\times10^{-11}$ |
| Mars | $2.91\times10^{-5}$ |
| Jupiter | $2.38\times10^{-10}$ |
| Saturn | $8.86\times10^{-12}$ |
| Uranus | $1.56\times10^{-13}$ |
| Neptune | $4.70\times10^{-14}$ |
| Ceres | $4.13 \text{ x } 10^{-17}$ |
| Pallas | $6.22 \text{ x } 10^{-17}$ |
| Vesta | $9.12 \text{ x } 10^{-17}$ |

The next figures show the main results. Figure 2 considers the external orbits. The orbits start with semi-major axis of 18.0 km and goes up to 50.0 km. The initial and final phase of the curve show the expected behavior of an increase in the duration of the lifetimes with the distance, representing the fact that the spacecraft is moving away from the perturbing bodies, so their effects decrease. Then, there is a maximum lifetime of around 30 days, at the semi-major axis of 32.0 km, following a fast decrease and then stabilization around 10 days of lifetime after the semi-major axis of 33.0 km. Those values are good enough for several types of missions, because it allows several days of observation without any maneuver required. It is important to emphasize the effects of the radiation pressure at this point. Neglecting this force would give larger lifetimes, but those results would be unrealistic, since this force is always present. To have an idea of the differences, figure 3 shows similar results, but excluding the solar radiation pressure. Note that the general forms of the curves are similar, but the maximum is 180 days for the orbits, which is the time limit used in the simulations. The lifetimes are stabilized near the same values, but it occurs in earlier stages, with respect to semi-major axis. Extending the integration time the results showed that the maximum lifetimes goes up to 18,000 days. This maximum is located at the same value of the semi-major axis, which is 32 km. A similar test was made to see the importance of the flattening of the central body in those orbits, and the results showed that the lifetimes are not modified. It means that, for exterior orbits, this force is negligible, as far as lifetimes are concerned. The reason is the large distance of those orbits from the central body.

Another test verified the influence of the sense of the orbit, and the results showed that direct and retrograde orbits have the same lifetimes. Figure 4 shows the trajectory of the spacecraft with maximum lifetime (30 days). The oscillations have short amplitudes, which makes this trajectory very interesting for practical missions.

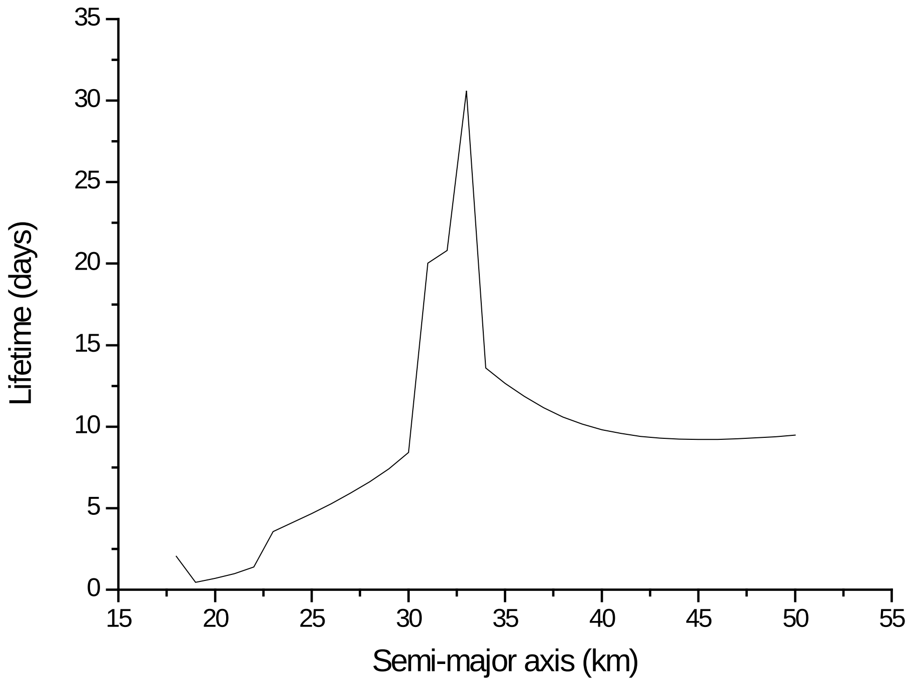



Fig. 2. Duration of the orbits (days) as a function of the semi-major axis (km) for external orbits (18.0 to 50.0 km) using a limit of 2.0 km for the oscillations.

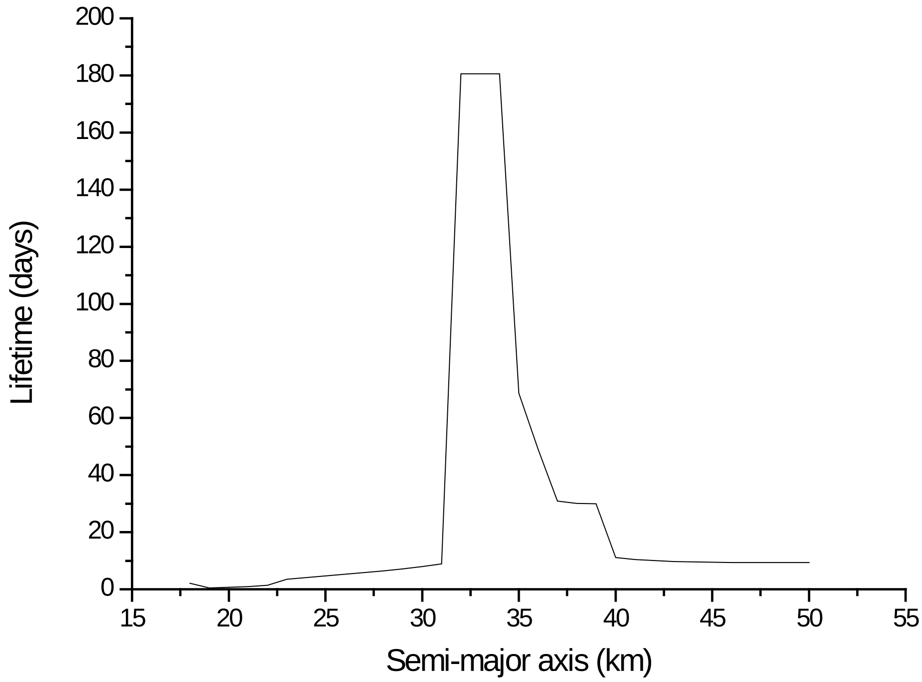



Fig. 3. Duration of the orbits (days) as a function of the semi-major axis (km) for external orbits (18.0 to 50.0 km) using a limit of 2.0 km for the oscillations and neglecting the solar radiation pressure.

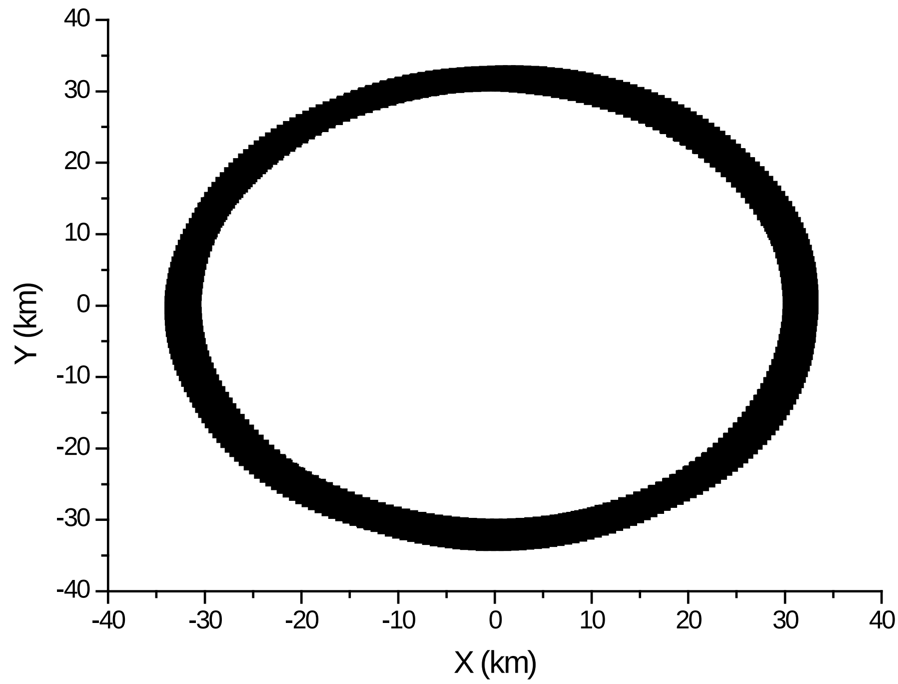



Fig. 4. Trajectory of the spacecraft with maximum lifetime (720 hours).

The evolution of the curve representing the lifetime of the orbits, when leaving the region close to the bodies, can be understood based in the definition of the "distance-stability" used in the present paper. The definition is based in the variations of the distance of the Spacecraft to the body Alpha. The periapsis ($r_p = a(1-e)$) and apoapsis ($r_a = a(1+e)$) of the orbit determines the regions of stability, where $a$ is the semi-major of the orbit and e is the eccentricity. The evolution of those two quantities determine the duration of the orbit. Regarding the perturbations received by the spacecraft, they are third-body perturbations coming from two different bodies and the solar radiation pressure. The third-body perturbation does not affect the semi-major axis, but changes the eccentricity. So, when orbits with larger semi-major axis are considered, even smaller variations in the eccentricity generates orbits that reach the limit of oscillations defined, since the limit is a fixed value. It means that the external part of the curve shown in Fig. 2 is expected. The lifetimes increase with the semi-major axis, because the perturbations decrease. These results emphasize the importance of the region near 32.0 km of semi-major axis, since it is an island of stability that was not expected. This region is very interesting to place the spacecraft, if the goal is to minimize the oscillations of the orbit. Another advantage is that those orbits are much closer to the bodies to be observed, compared to the other regions of stability.

Figure 5 shows the results considering orbits between both smaller bodies. The lifetimes of the orbits first increase with the semi-major axis, because the spacecraft increases its distance from the body Gamma, which is the closest perturbing body. Then, there is a group of peaks of duration, because the spacecraft stays in middle distances from both perturbing bodies. It means that their gravity forces are acting in opposite directions, so helping to keep the spacecraft in a more "distance-stable" orbit. Those peaks reach the maximum at the allowed time for the integration, which is 180 days. Finally, the lifetimes decrease again, when the spacecraft gets closer to the body Beta. The reason for the alternation of values, in this figure and in similar ones, comes from the geometry of the bodies. Some values of the semi-major axis generate smaller distances between the perturbing and perturbed body, increasing the perturbation, so decreasing the lifetime of the orbits. Figure 6 shows the same results, but excluding the solar radiation pressure from the model. The difference is that there are more oscillations in the curve now. It means that the phenomenon of oscillations comes from the gravity fields of the smaller bodies, and the solar radiation pressure has the effects of reducing those oscillations. These oscillations are probable caused by resonances in the orbital motion [16 and 20].

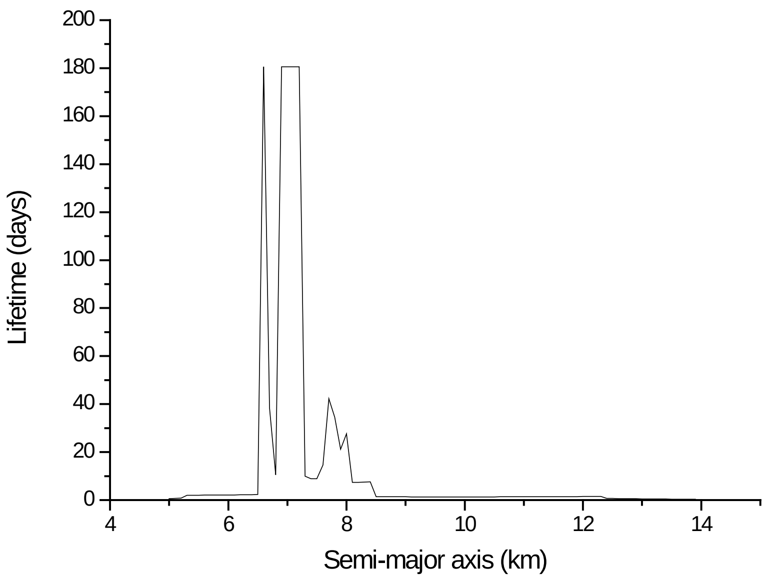



Fig. 5. Duration of the orbits (days) as a function of the semi-major axis (km) for intermediate orbits (5.0 to 13.0 km).

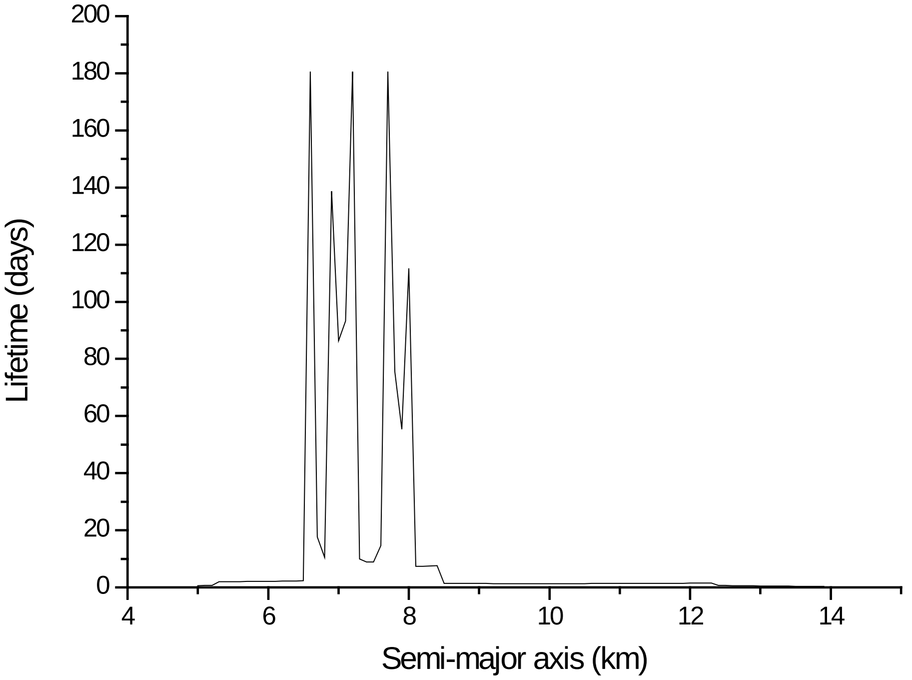



Fig. 6. Duration of the orbits (days) as a function of the semi-major axis (km) for intermediate orbits (5.0 to 13.0 km) without solar radiation pressure.

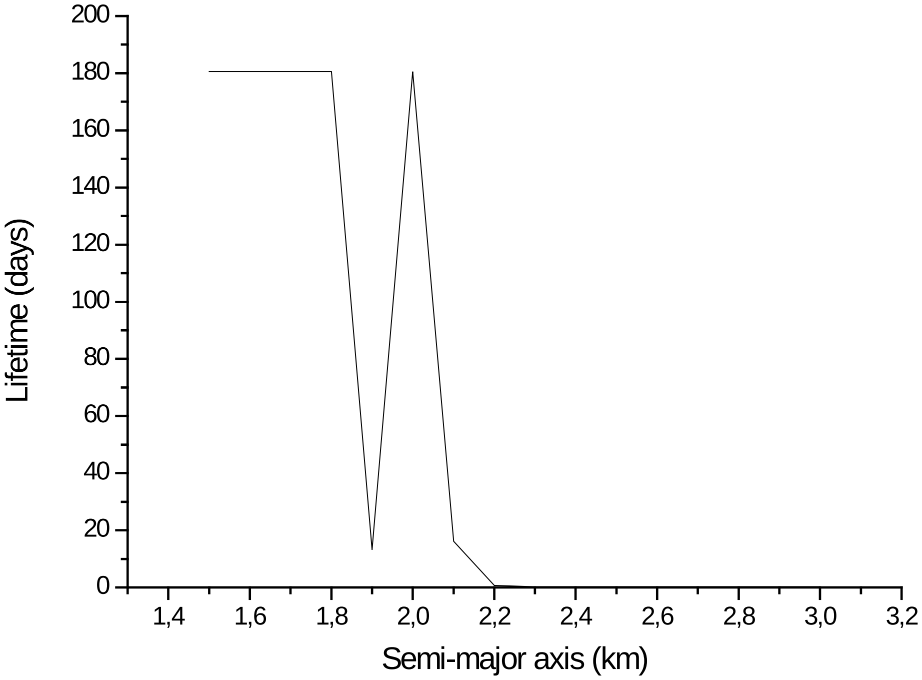



Fig. 7. Duration of the orbits (days) as a function of the semi-major axis (km) for internal orbits (1.5 to 3.0 km).

Figure 7 shows the results for the internal orbits, the ones below the orbit of Gamma. The lifetimes alternate from 180 days, the time limit for the simulations made here, with orbits that are destroyed almost immediately, due to the strong perturbation of Gamma. So, this type of study gives a good view of the best orbits to be used by a spacecraft visiting this triple system, from the aspect of orbits that have a limited oscillation in its distance from the main body. Retrograde orbits and situations where the spacecraft starts its motion at the apoapsis of the orbit were also studied, for all cases, with similar results.

Simulations were made using the same orbits, but removing the effects of the solar radiation pressure. The results are exactly the same. It means that the solar radiation pressure is a small force in this orbital region, due to the proximity to the two bodies Alpha and Beta. At those shorter distances the gravity of the bodies dominates the dynamics.

To understand better the dynamics involved in that system, and not only finding the trajectories, the accelerations suffered by the spacecraft are monitored during the numerical integrations, as done in references [19, 22-31]. Their cumulative effects are computed by the integral index defined before (Eq. 1).

The goal of using this technique is to obtain the information about the contribution of all the forces at the end of one trajectory. Figure 8 shows the integrals per unit of time (m/s) of each component x-y-z and the total magnitude (Eq. 1) as a function of the semi-major axis of the orbit of the spacecraft for exterior orbits. The bottom plot is a closer view of the top plot. It is clearly noted that the minimum perturbation level, which is the minimum magnitude of the vector formed by the three components of the integral of the accelerations, is located exactly at the semi-major axis that gives the maximum lifetime for the orbit. It is noted that the z-component is always zero, while the x and y components have small values around the range 32-35 km, with the minimum at 33 km of semi-major axis. It means that we found the best location for the spacecraft, from the point of view of minimum oscillations. This minimum point is a result of the evolution of the geometry. The y-component depends on the locations of both perturbers. Sometimes they are in the same semi-plane with respect to the x-axis, sometimes they are in the opposite semi-plane. The y and x components depend on the relative position of the perturbing bodies and the solar radiation pressure. So, after considering all those factors, there are regions of minimum total perturbations, which give longer lifetimes for the trajectories.

The addition of the $J_2$ term of the gravity field of the primary does not change those curves, since this orbital region is far from the primary. The consideration of retrograde orbits also does not modify the results. Therefore, those plots are omitted in the present paper.

Next, Fig. 9 shows the same results, but excluding the solar radiation pressure from the dynamical model. The plot at the right side is once again a closer view of the one shown at the left side. It is clear that the minimum regions have smaller magnitudes at this time, which justifies the longer lifetimes of the orbits when this force is not included. The same physical explanations made before applies here, and are not repeated.

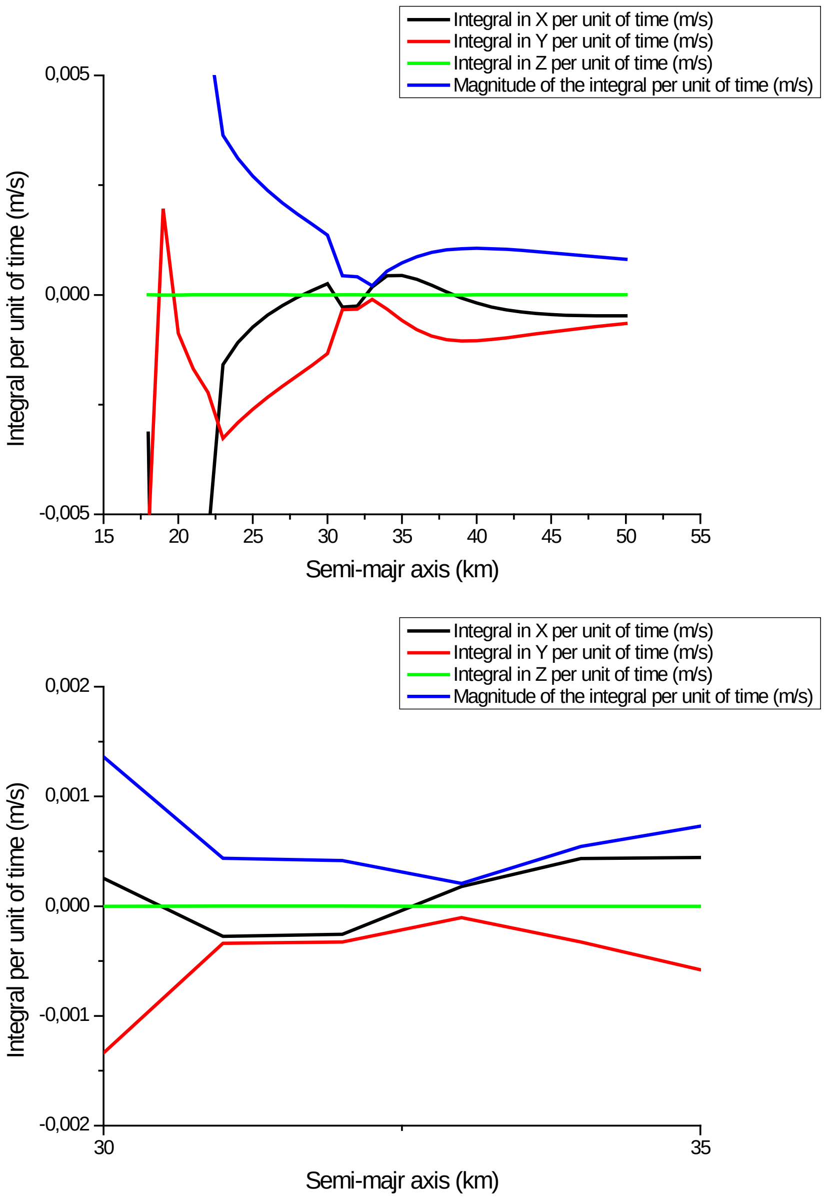



Fig. 8 Integrals per unit of time (m/s) as a function of the semi-major axis of the orbit of the spacecraft for exterior orbits. The bottom plot is a closer view of the top one.

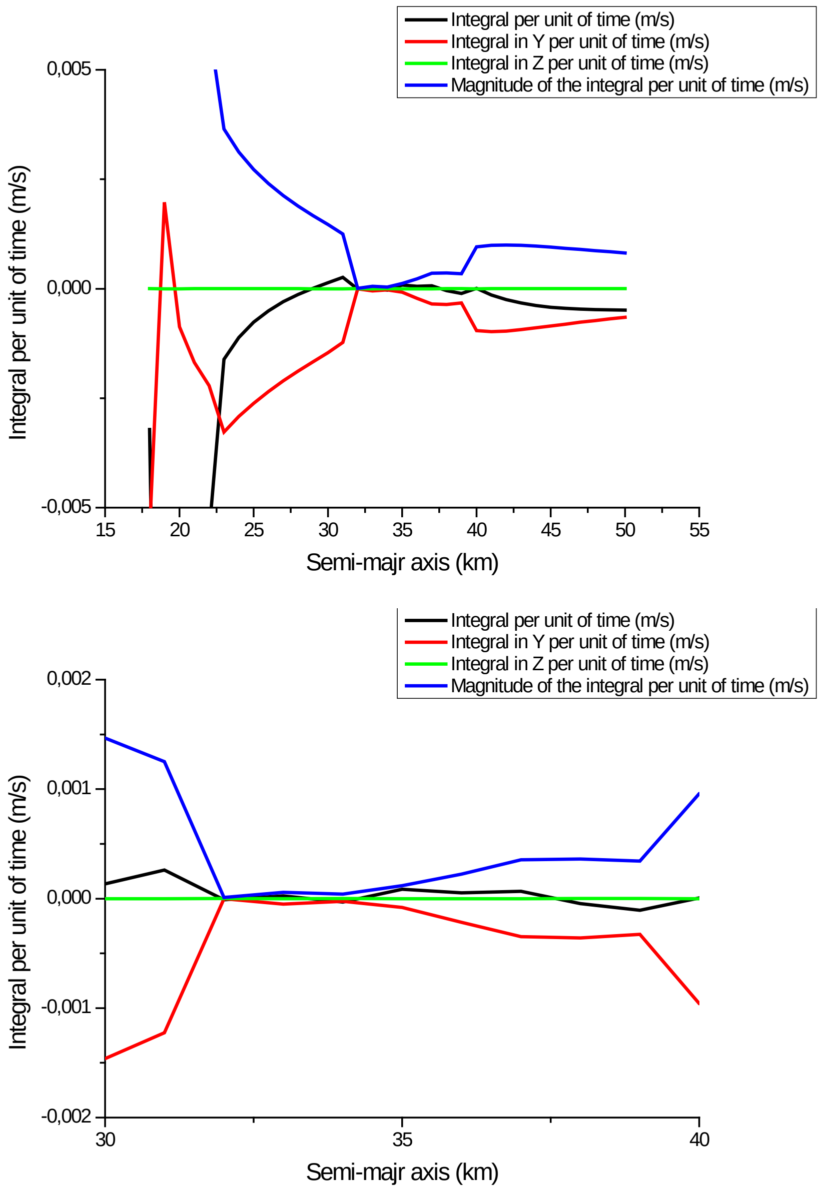
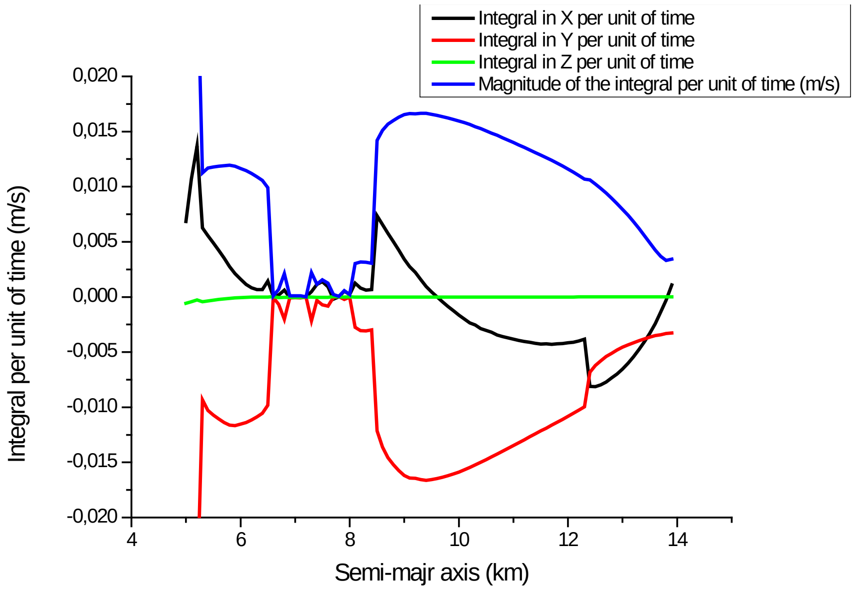



Fig. 9 Integrals per unit of time (m/s) as a function of the semi-major axis of the orbit of the spacecraft for exterior orbits, excluding the solar radiation pressure from the dynamical model. The bottom plot is a closer view of the top one.

The same study is now realized for the intermediate orbits, which are the ones in the middle of the two satellite bodies. Figure 10 shows the results, which are equivalent to the ones shown in the previous figures. It confirms the minimum found near semi-major axis of 7 km, for the same physical reasons explained before. The zoom of this region shown in the plot on the right side shows this fact in detail. The same plots made without considering the solar radiation pressure, shown in Fig. 11, indicates the region near semi-major axis if 7 km shows now more oscillations. This figure explains physically the stabilization effect of the solar radiation pressure shown in Figs. 5 and 6.



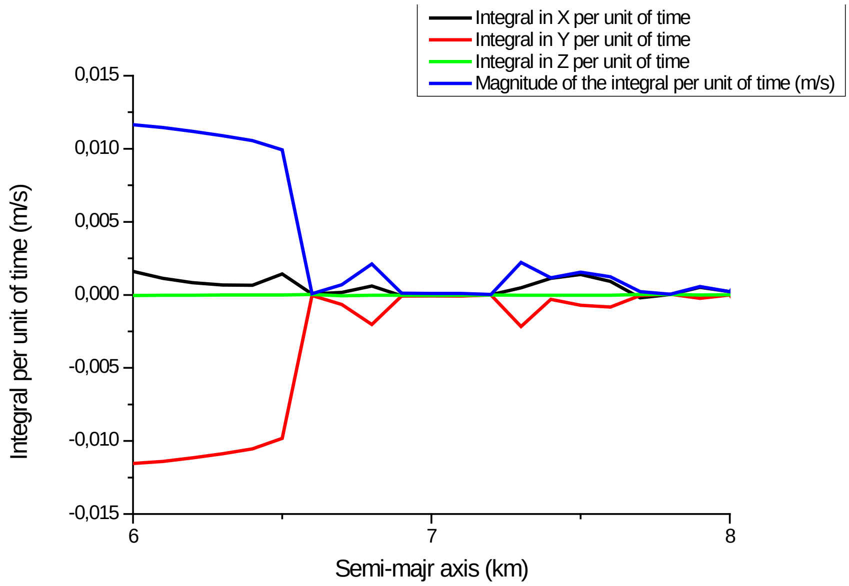




Fig. 10 Integrals per unit of time (m/s) as a function of the semi-major axis of the orbit of the spacecraft for intermediate orbits. The bottom plot is a closer view of the top one.

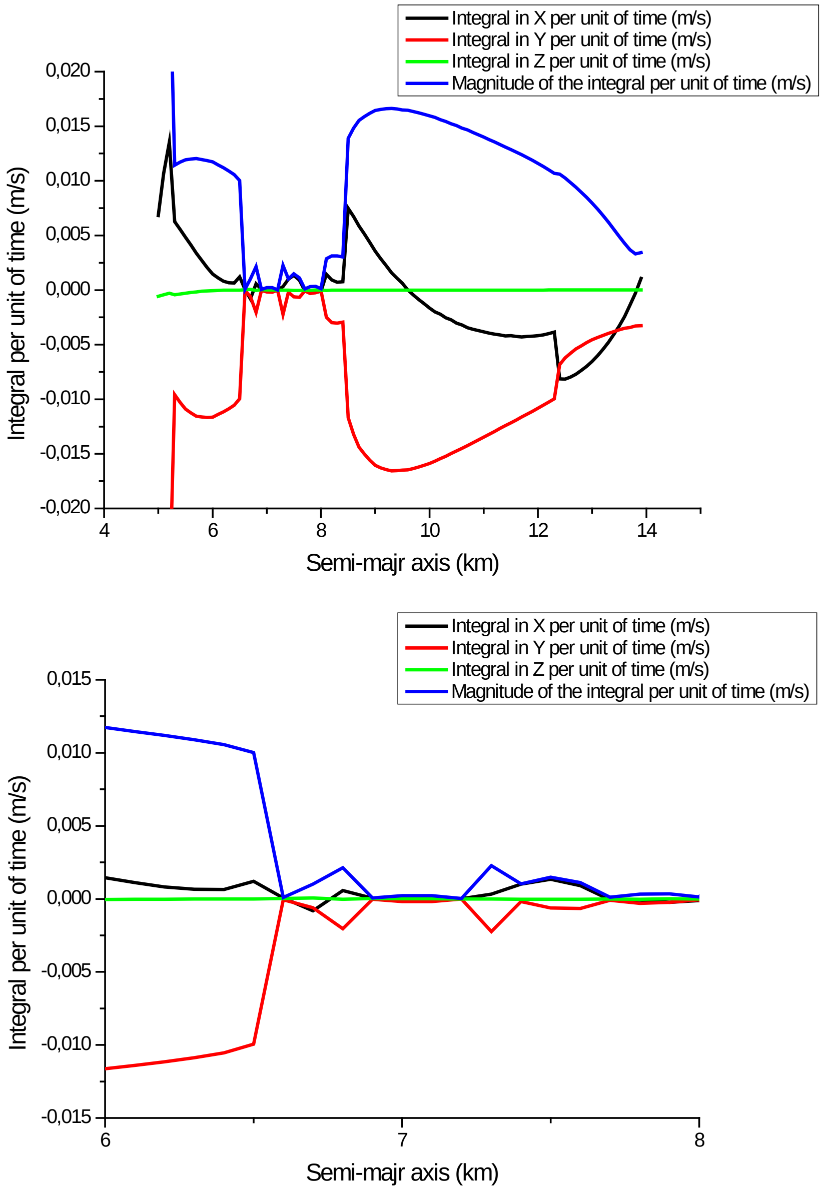



Fig. 11 Integrals per unit of time (m/s) as a function of the semi-major axis of the orbit of the spacecraft for intermediate orbits, excluding the solar radiation force from the mode. The bottom plot at the right side is a closer view of the top one.

This study is completed with the results shown in Fig. 12, for the internal orbits. It confirms that the regions of minimum perturbations give longer lifetimes for the orbits. The exclusion of the solar radiation pressure does not change the plots this time, since this region is dominated by the gravity of the bodies.

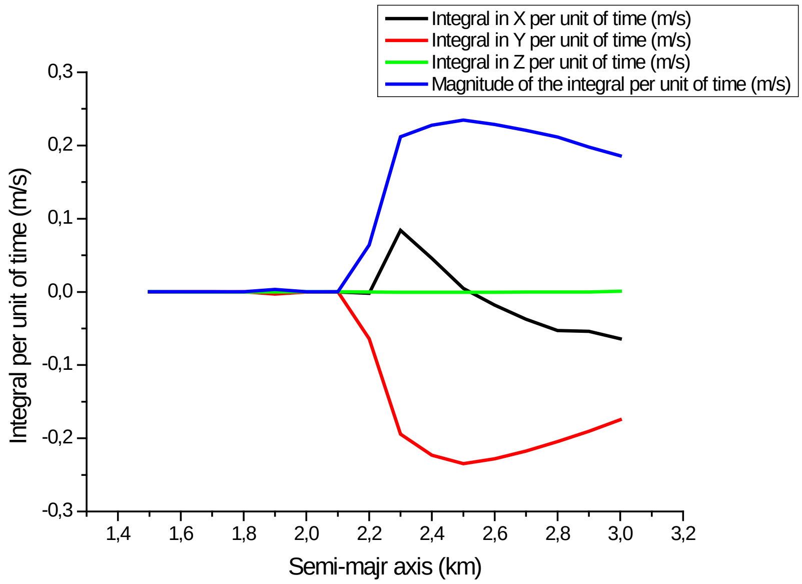



Fig. 12 Integrals per unit of time (m/s) as a function of the semi-major axis of the orbit of the spacecraft for internal orbits.

These results bring the question of testing a different limit for the stability of the orbits, to take into account that the limits should increase with the semi-major axis. Figure 13 shows the evolution of the lifetimes of the orbits as a function of the semi-major axis, this time considering the limit of oscillations of 5% of the semi-major axis, instead of a fixed value in kilometers. It means that the orbit is considered "not stable" if the radius vector reaches values 5% above or below the semi-major axis of the orbit. There is an increase in the lifetimes, and there is no stabilization. It is one more choice available for the mission designers, with lifetimes going up to 360 hours (12 days), if this type of limit is accepted. This point is particularly important if orbits with larger semi-major axis are considered.

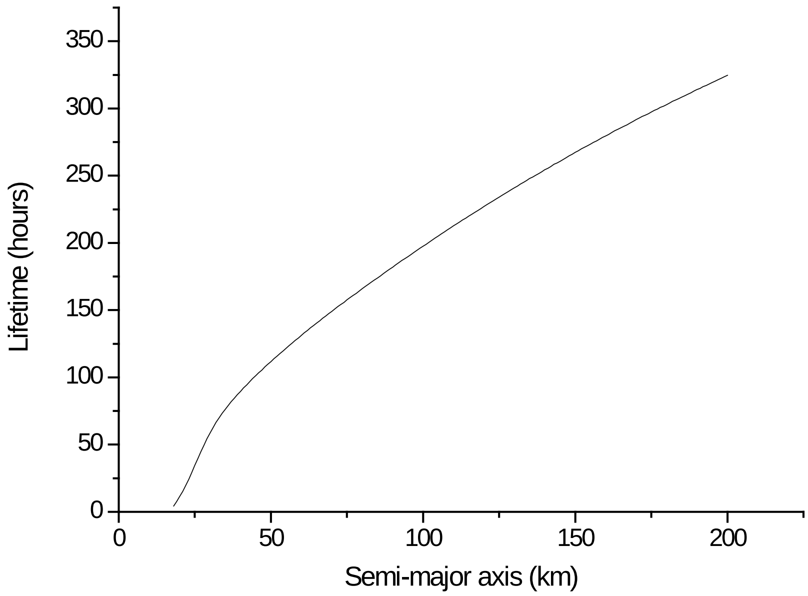



Fig. 13. Duration of the orbits (hours) as a function of the semi-major axis (km) for external orbits (20.0 to 200.0 km) using a limit of 5% of the semi-major axis for the oscillations.

After studying the central body, the same type of study is made for orbits around the two smaller bodies of the system. The dynamics used here consider the same forces used in the previous case, modifying only the initial conditions of the spacecraft. The results showed that orbits around Beta have very short durations, below the limit of two hours, which means that the spacecraft does not complete a full revolution before reaching the limit.

The same study was made for the body Gamma, and “distance-stable” orbits around this body, as defined in the present paper, does not exist. It means that the perturbations are too strong to allow the existence of orbits with low amplitude oscillations. It happens due to the very small distance between the bodies Alpha and Gamma and it is also an expected result that is confirmed by the present study. It means that, in order to study Gamma, it is necessary to find orbits around Alpha that has passages near Gamma, but not to close that generates a Swing-By effect that makes the spacecraft to leave the triple system.

## 4. Conclusions

A general study was made for the triple asteroid $2001SN_{263}$, from the point of view of searching for orbits that are more stable, in the sense of keeping the distance Spacecraft-main body inside a fixed limit. It was defined the concept of "distance-stable orbits", which are the orbits that remain inside given limits for the amplitude of oscillations. The three regions of the space were considered: internal orbits, which are below the orbit of Gamma; exterior orbits, which are above the distance Beta-Alpha; and intermediate orbits, which are those orbits staying between the orbits of both smaller bodies of the system.

The results mapped regions where the orbits can live up to 720 hours (30 days), from the maximum integration time used of 180 days. It means that they are good choices to place a spacecraft for a mission that requires limited oscillations for the Spacecraft-Alpha distance. Those results are much longer if the solar radiation pressure is not included in the model, reaching up to 1800 days, which indicates the strong effects of this force

Different values for the limit of oscillations and integration time were used, as well as different geometries for the position of the bodies. The existence of very long duration orbits was shown, in the case of internal orbits with both smaller bodies on the opposite side of the orbit with respect to the spacecraft.

Orbits around Beta have very short lifetimes, even with flexible limits of oscillations, and “distance-stable” orbits around Gamma were not found.

The effects of each body in the orbits are also studied, by measuring the integral of the forces over the time for each specific orbit. They quantify the contribution of each body in the destruction of the orbits. They also gave a physical explanation of the regions of maximum lifetimes for the orbits, by showing that those regions corresponds to the regions of minimum perturbations,

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