EXPERIMENTAL AND NUMERICAL ANALYSIS OF WING PLATE MODEL.

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Abstract. The development of thin plate theory was due to the evolution of engineering that continually needed to improve the mode of analysis of elements in a plate. In 1888 Augustus Edward Hough Love (Weston-Super-Mare, 1863 - 1940) used Kirchhoff's hypothesis to determine a two-dimensional mathematical model for the determination of stresses and deformations in thin plates subjected to forces and moments, assuming a surface plane. Average can be used to represent a three-dimensional plate in two-dimensional form (LOVE, 1897). As the flat plate theory has been refined by adding new methods of analysis and theories, the approximation of equations by a discrete point system in spacetime has become a fundamental necessity, the most common methods being: 1. Volume Method Finite; 2. Finite Element Method and 3. Finite Difference Method. Equations can be written in different forms depending on the coordinate system, such as Cartesian, cylindrical, spherical, curvilinear, orthogonal, and non-orthogonal curvilinear. The present work had as main motivation the comparison between two (2) different methods of analysis of flat plates of thickness $t / a \ll 1$, where “t” is the thickness and “a” the largest dimension of the plate, with the configuration Free-Free-Embossed Edges (LLLE). Thus, the objectives of this work are the assembly of an analysis system using accelerometers (Model MPU 6050) in meshes (5x8 points) to verify the displacement of x, y and z coordinates, spread over forty (40) points forming a mesh, and two (2) points with Geokon @ 4150 vibrating string sensors horizontally and vertically.With these sensors it was possible to verify the plate displacement dimension for both methods, as well as the difference between the experimental analysis methods and their applicability in other projects.

Keywords: Kirchhoff-Love; Linear Elasticity Theory; Flat Plate

1 Introduction

The development of the slab plate theory took place through the evolution of engineering that continually needed to improve the mode of element analysis. In 1744 Leonhard Euler (Basel, 1707-1783) publishes the book “Calculus of Variations” the first approach to theory. buckling analysis method, which applies the variational calculus to the theory of elasticity, specifically for bending a rod subjected to an axial load. Euler also verified several problems of linear vibrations in plates of different shapes using the analogy of two perpendicular taut string systems. (GAUSTSCHI, 2008). In the mid-1750s Leonhard Euler and Daniel Bernoulli (Groningen, 1700-1783) Johann Bernoulli's son and Jaques Bernoulli's nephew perfected the theory of buckling analysis published by Euler by exchanging the rope network for a mesh of beams, the model of Euler-Bernoulli's beam is a simplification of the linear theory of elasticity that provides a means of calculating the deflection characteristics of a beam under a given load (static or dynamic), which consists of a fourth order linear partial differential equation. Daniel also produced important work in the field of probabilities and economic policy, created the concept of moral hope and applied it to insurance, studied vibration, and was a precursor in the field of partial differential equations. (CANNON, 1981). German physicist Ernst Florens Friedrich Chladni (Wittenberg, 1756 - 1827) described various works of plate vibration analysis and discovered the free vibration modes in his experiments using evenly distributed powder which formed regular patterns after vibrations were introduced known today as modal analysis. Chladni described his experiments in "Theorie des Klanges" and "Die Akustik" respectively published in 1781 and 1802 in Leipzig (WALLER, 1961). In 1809 Chladni showed at the French Academy of Sciences that through his theory he could determine the frequencies corresponding to these vibration patterns, this presentation began the competition to receive works on the mathematical theory of plate vibration, so in this period the mathematics Marie -Sophie Germain (Paris, 1776 - 1831) submitted her study entitled "Reserches sur la theory des surfaces élastiques," published in 1811, she used (following Euler's earlier work on elastic curves) an approximation of tension energy, and after some setbacks for the correct definition of Equation 1.1.1 in 1816 won the prize of the French Academy of Sciences (SZILARD, 2004).

$$\frac{\partial^2 z}{\partial t^2} + k^2 \left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = 0 \tag{1.1}$$

The mathematician Siméon Denis Poisson (Pithiviers, 1781-1840) tried to determine the correct value of the constant k^2 in the differential plate vibration equation (Equation 1.1.2). Assuming that the plate particles were located in its central plane, however, he erroneously concluded that the constant is proportional to the square of the plate thickness and not to its cube (SZILARD, 2004).

$$\frac{\partial^2 z}{\partial t^2} + \lambda^2 \left(\frac{\partial^6 w}{\partial x^4 \partial y^2} + \frac{\partial^6 w}{\partial x^2 \partial y^4} \right) = 0 \tag{1.2}$$

Engineer and scientist Claude Louis Navier (Dijon, 1785-1836) developed the first correct differential equation for plates, subject to distributed lateral loads (x, y). Since Navier first integrated the isolated findings of his predecessors and the results of his own investigations into a unified system, the publication of his study was a milestone for the development of modern structural analysis (SZILARD, 2004). Navier used the hypotheses for treatment of bending in Bernoulli beams with the addition of two-dimensional stress and stress actions. In 1823 he published the correct definition of the governing differential equations on plates subjected to static lateral stress p_z (x, y) as in Equation 1.1.3:

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)= p_z(x,y) \tag{1.3}$$

In equation, 1.1.3 denotes D as the flexural stiffness of the plate, which is proportional to the thickness cube and w (x, y) represents the flexed central surface. To solve certain boundary value problems in rectangular plates, he introduced a method that transforms the plate differential equation into an algebraic equation, based on the use of double trigonometric series introduced by Fourier in the same decade. Navier also developed a valid differential equation for buckling plates subjected to uniformly distributed compression forces along the contour (SZILARD, 2004).

In 1850 Gustav R. Kirchhoff (Berlin, 1824 - 1887) developed the first complete theory of plate flexions, based on Bernoulli's assumptions for beams, and deriving the same differential equations for plate flexions as Navier (SZILARD, 2004).).

After Kirchhoff the “equivalent shear forces” replaced the torsional moments at the plate edges, all boundary conditions could now be declared as a function of displacements and their derivatives with respect to x or y. Later, Kirchhoff was regarded as the founder of the extended plate theory, which takes into account the combination of forces. In analyzing plates with large deflections, he found that nonlinear terms could no longer be ignored. His other contributions are the development of a plate frequency equation and the introduction of the virtual displacement method to solve various problems (SZILARD, 2004).

In 1888 Augustus Edward Hough Love (Weston-Super-Mare, 1863 - 1940) used Kirchhoff's hypothesis to determine a two-dimensional mathematical model for the determination of stresses and deformations in thin plates subjected to forces and moments, assuming a surface plane. Average can be used to represent a three-dimensional plate in two-dimensional form (LOVE, 1897).

Stephen P. Timoshenko (Shpotovka, 1878 - 1972) contributed significantly among other subjects with the application of circular plate flexion analysis, also studied circular plate solutions, calculation of frequencies and modes of vibration in circular plates and the formulation of elastic stability. (TIMOSHENKO, 1961)

It is noted that the development of hypotheses for the determination of a two-dimensional mathematical model for the determination of stresses and deformations in a plate took more than 150 years, and even with Love's determination in 1888, other researchers consolidated the theory. of plates with the addition of other variables such as flexural stiffness, deformation and analysis for circular plates.

In the late twentieth century (1994) Wang and Xiang "Buckling And Vibration Of Annular Mindlin Plates With Internal Concentric Ring Supports Subject To In-Plane Radial Pressure" analyzed circular Mindlin plates with radially loaded concentric ring holders, where the solution of this The problem was based on the RayleighRitz approach and subsequent determination of buckling factors. (LIEW, K & XIANG, 1994).

In 2010 in the article “An exact analytical solution for freely vibrating piezoelectric coupled circular / annular thick plates using Reddy plate theory” written by Hosseini-Hashemi and Es'haghi discusses the calculation of natural frequencies and displacements for thick circular plates with different conditions. and based on Reddy's third-order strain theory. (HOSSEINI-HASHEMI, SHAHROKH & ES'HAGHI, 2010).

In 2011 the publication “Analytical Bending and Stress Analysis of Variable Thickness FGM Auxetic Conical / Cylindrical Shells with General Tractions” published by M.Shariyat and M. Alipour made the buckling analysis of viscoelastic circular plates with functional gradation and full sensitivity analysis. to evaluate the effects of various parameters on buckling load. (SHARIYAT, M & ALIPOUR, 2011)

Another far-reaching publication in 2011 was the work of Mazhari and Shahidi, “Analysis of post buckling behavior of circular plates with non-concentric hole using the Rayleigh – Ritz method”, where post-buckling behavior of homogeneous circular plates was determined. with concentric bore submitted to a uniform radial load using the Rayleigh-Ritz method, also using the nonlinear Von-Karman theory. (MAZHARI, EMAD & SHAHIDI, ALIREZA, 2011).

As flat plate theory has been refined by adding new methods of analysis, matter, and theory, so the approximation of equations by a system of equations at discrete points in time has come as a fundamental necessity, the most common methods being: 1 Finite volume method; 2. Finite Element Method and 3. Finite Difference Method. Equations can be written in different forms depending on the coordinate system, such as Cartesian, cylindrical, spherical, curvilinear, orthogonal, and non-orthogonal curvilinear. The choice of coordinate type depends on the type of flow to be studied.

The finite volume method uses the integral form of the equations, where the domain is divided into contiguous control volumes and the conservation equations are applied to each one. Surface and volume integrals are approximated by quadrature formulas, so any type of mesh should be conservative to facilitate programming. However, it is difficult to obtain high order because they have two levels of approximation, interpolation and integration (EYRNARD & GALLOUET, 2000).

In the finite element method we use the same concept studied in finite volumes, except that the equations are multiplied by a weight function before being integrated into the whole domain. The domain should be divided into discrete elements that can be quadrilateral or triangles. (REDDY, 2006).

In the 18th century Euler introduced a finite difference method for solving partial differential equations, easily used in simple geometries with the approximations of the derivatives obtained by Taylor series expansion or polynomial approximation. Today this method is the dominant approach to the numerical solution of partial differential equations, so this method was chosen because of its simplicity because the geometry used is simple (GROSSMANN & HANS-G. ROOS, 2007).

In addition to the theories of flat plate analysis, there are other methods for stress and flexural analysis on plates using vibrating string technology. In 1928 Andre Coyne (1891 - 1960) patented a force gauge using the vibrating string principle based on the acoustic indicator principle described as the first commercial use of the vibrating string sensor in France (ASCE, 2000). The principle of operation of vibrating rope (VW) sensors is based on the change of rope frequency, which is stretched on a support, depending on the physical parameters of the wire and the environment in which oscillations occur. A vibrating string sensor has an interesting field measurement technique developed for magnetic center determination of units in accelerators. The important advantages of proper construction of vibrating string sensors are inherent in long term stability, high accuracy and resolution, good reproducibility and small hysteresis (ARUTUNIAN, 2006).

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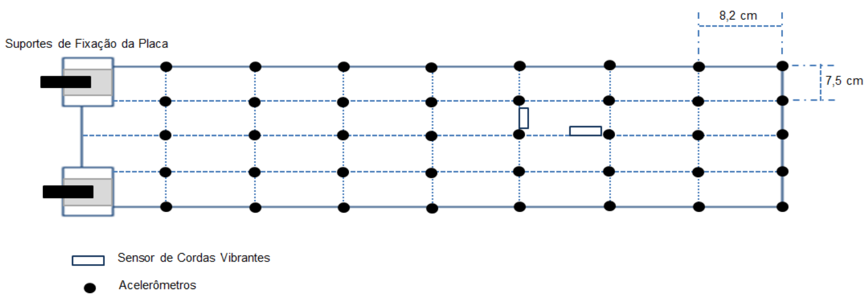


Figura 1 - Wing representation using a flat plate, top view.

So from the assembled and instrumented plate we can stimulate the movement using a ruler, all the movements studied here had the same amplitude. Figure 2 represents the side view of the instrumented board and the board side marking system.

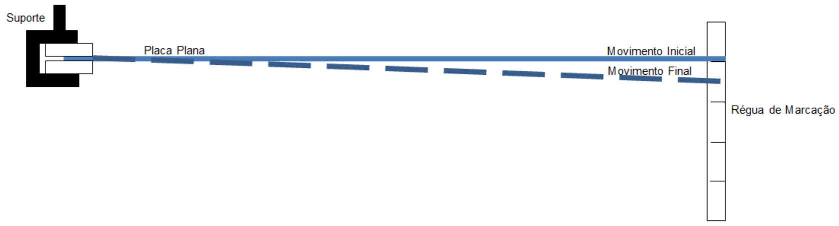


Figura 2 - Wing representation using a flat plate, side view.

The accelerometers were mounted to reproduce the plate through 40 (accelerometer) distributed points of homogeneous form obeying the same spacing between the sensors. Note that the sensors on the edge of the board set on the side have an offset value of 0, but sensors along the body have offset values larger the distance to the crimped point.

2.1 Plate definition

According to (REDDY, 2016) a plate is a structural element with large dimensions compared to its thickness and are subject to loads that cause bending, deformation as well as stretching. In most cases, the thickness is no larger than 1/10 of the smallest dimension in the plane. For (TIMOSHENKO, 1964) the flexural properties of a plate depend greatly on its thickness compared to other dimensions. Already (CHAVES, 1997) defines plate as the structural element where t is a value much smaller than the other dimensions (length and width) (1.3.1), it is noted that regardless of the author the approximation of the plate concept remains if equal. In Figure 4 - Thick plate thickness t presents the variables of a flat plate, being the thickness (t), the length (b) and the width (a). The thickness of the plate that defines its smallness, fitting it into one of three possible categories:

(1) Thin plates with small deflections;
(2) Thin plates with large deflections;
(3) Thick plates.

$$\frac{t}{a} << 1 \tag{1.4}$$

Depending on the ratio (t / a) between the thickness (t) and the smallest dimension (a) measured in the medium plane, the plate can be classified as (CHAVES, 1997).

Very Slender → $t / a < 1/80$;
Slender → $1 / 80 \leq t / a \leq t / 5$;
Thick → $t / a > 1/5$;

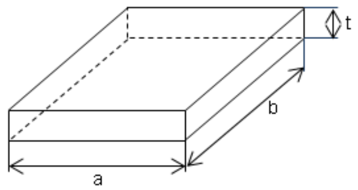
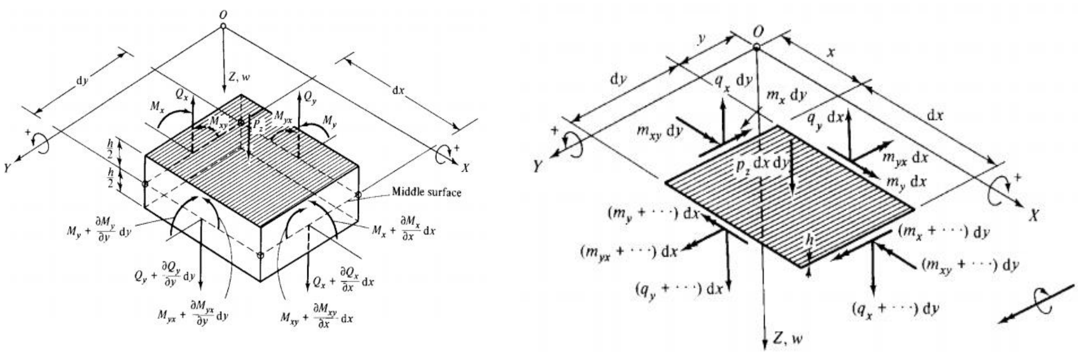


Figura 3 - Thin plate with thickness t.

A mathematically accurate stress analysis of a thin plate - subjected to loads that normally act on its surface - requires the solution of three-dimensional differential equations, but in most cases the classical Kirchhoff plate theory which yields results is used. sufficiently accurate without the need to perform a complete three-dimensional stress analysis. Figure 5 - Considering a 3D element, we assign the positive internal forces and moments to the faces close to the plate element thus making it a 2D element. To satisfy the element's equilibrium, negative internal forces and moments must act on its distant sides (SZILARD, 2006).



(a) (b)

Figure 4 - In (a) a three-dimensional element that after assigning internal forces and moments the faces becomes a 2D element (b) (SZILARD, 2006).

Kirchhoff's Plate Theory equation allows for a simplification of the problem, so it is possible to determine the basic differential equations of the plates given to the hypotheses below (SZILARD, 2004):

1. Plate material shall be considered elastic-linear, homogeneous and isotropic according to Hooke's law;
2. Transverse displacements are small compared to the t-value of the plate;
3. There is no deformation in the middle plane of the plate (Neutral Surface);
4. The flat and perpendicular section of the middle surface remains flat, perpendicular and undeformed with respect to that surface after flexion.

2.2 Vibrantig Wire Sensor

A vibrating string sensor has as its main components, which are: the vibrating string, the sensor body (where external force is applied), and the sensor coil with its analog circuit.

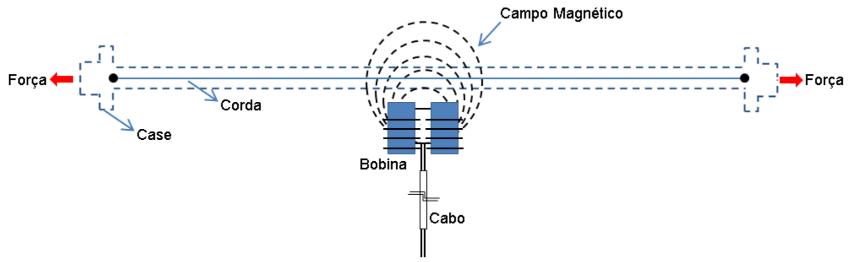


Figure 5 - Vibrant wire sensor and its components.

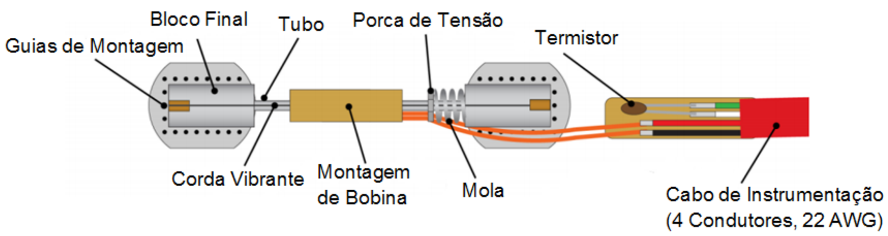


Figure 6 - Geokon 4150 series vibrant wire sensor

Vibrating wire sensors are based on the rope oscillation behavior which, having a length L, linear mass density ρ, and intended by a longitudinal force T, will have an oscillation frequency f, given by the equation (SILVA, 2002):

$$f = \frac{1}{2L}\left(\frac{T}{\rho}\right)^{\frac{1}{2}} \tag{1.5}$$

Variation of any of the three parameters (T, ρ or L) will change the oscillation frequency. This change can be “felt” by the analog circuit installed near the swinging rope (Figure 14).

In most common sensors the force applied to the rope is the variable parameter, changed by an external force acting on the sensor body. The sensor coil is used to initiate rope oscillation by inserting an electric pulse. Next, the coil itself is used to capture the vibration frequency of the rope (SILVA, 2002). The sensor output signal is a sine wave whose frequency f can be measured by a data acquisition system; which analyzes the collected data, associating them to the measured quantity (force, pressure, displacement, temperature and etc ...). Figure 15 illustrates the sensor connection to the signal conditioning and acquisition system.

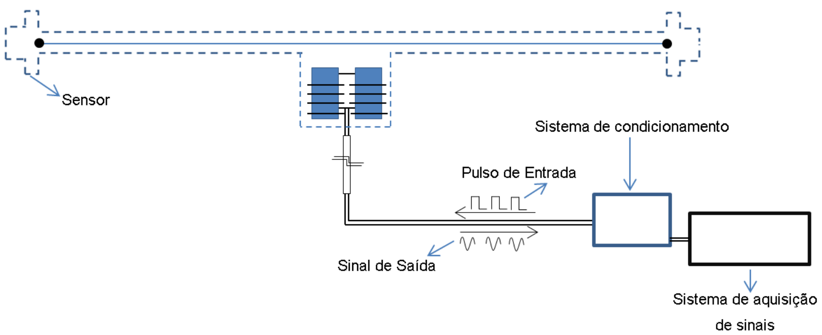


Figura 7 - Sensor connection to signal conditioning and acquisition system.

The main parameters that influence the measurement are related to the characteristics of the rope (Material used, diameter, density), the mechanical properties of the sensor body, the sensitivity of the magnetic circuit and the characteristics of the signal conditioning circuit. The signal conditioning and acquisition system performs two essential functions: measuring the signal period of the sensor and sending current pulses to the sensor, exciting the vibrating rope.

A fixed number of periods of the sensor signal establishes the frequency of the excitation pulses, in this project the sensor has a frequency output in the range of 600-1000 Hz, and the 25 - 50Hz variation excitation signal with the generation of approximately one pulse every 20 cycles of the sensor signal.

3 Experimental Method

3.1 Experimental analysis

In this step, Geokon ™ 4150 vibrating string sensors were attached to the plate to capture the displacement of the horizontal and vertical axis. When starting the experiment we found that there is a natural decay of the plate and it is an additive of errors at the time of reading the data.

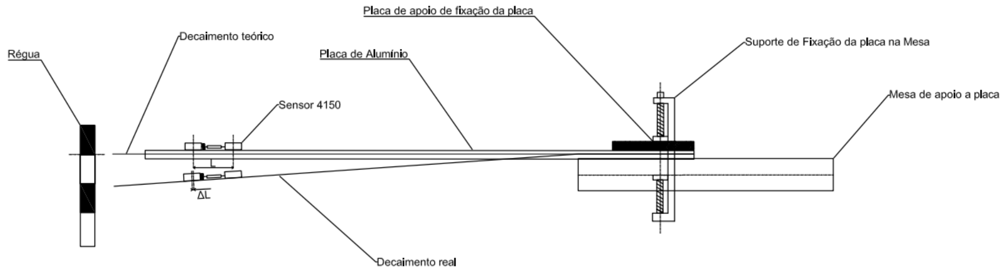


Figure 7 - Assembly of the vibrating string experiment and accelerometer loop

The first step was to analyze the data from the stationary and completely straight plate, so without the actual decay, in these first analyzes we used a frequency of 60Hz or 1 measured per second. It was noted that small vibrations like a person walking around the experiment. drastically interfered in the experiment, it can be seen that in points 1 and 3 of the figure below movements related to the positioning of the plate, while in items 2 and 4 there is a small movement related to walking next to the experiment. From register 5200 we noticed a stabilization in the experiment and calculated the initial calibration value of Delta_H equal to 0.0158 and to Delta_V 0.0147 with changes after the third decimal place.

Thus through the data analysis we obtain that the highest value of Delta L, showing the value of increase in L in the vibrating string sensor correlating with the increase and decrease movements of the plate angle. Note that for plate angle change motions a resolution of 60 Hz (1 reading per second) is too large for this type of analysis, so the resolution value has been decreased to 200 Hz (1 reading every 0, 3 seconds), to perform this type of reading, it was necessary to change the Campbell CR6 equipment to Campbell CDM305VW equipment, all assigned to the project by Campbell Scientific do Brasil Ltda.

The difference in equipment shows us a sufficient resolution to distinguish the flexing motion of the plate, even if there is residue generated by the sensor. In the comparison below we can verify the data generated by both methods on the same board at the same time.

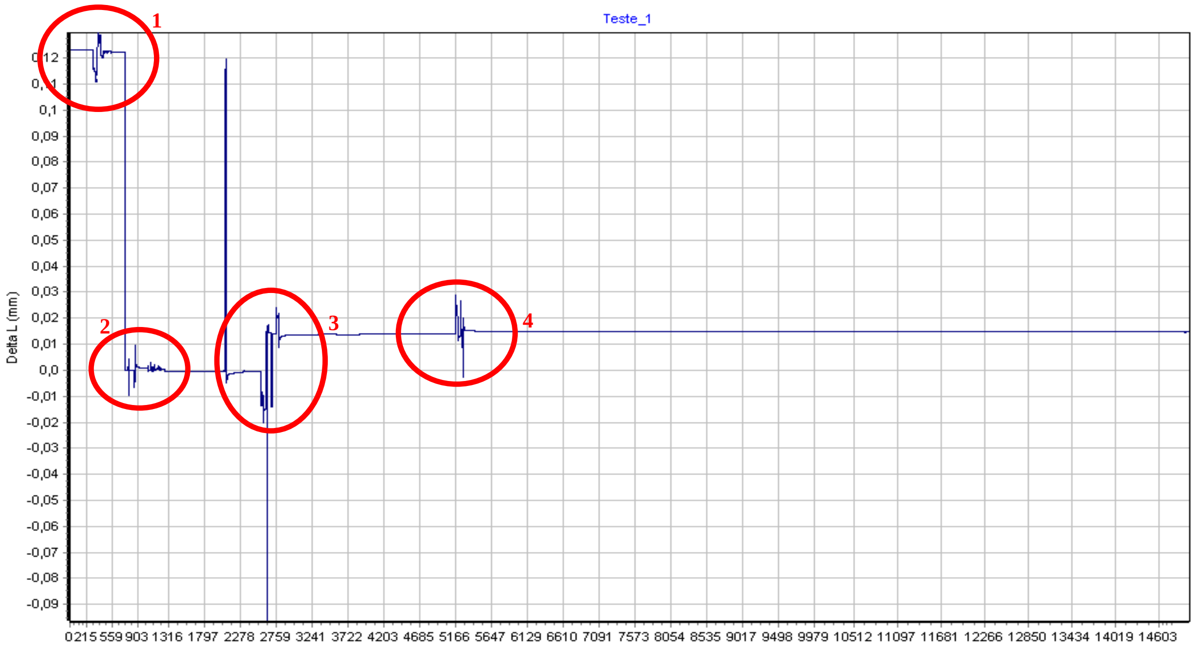


Figure 8 - Analysis of Vibrant String Data

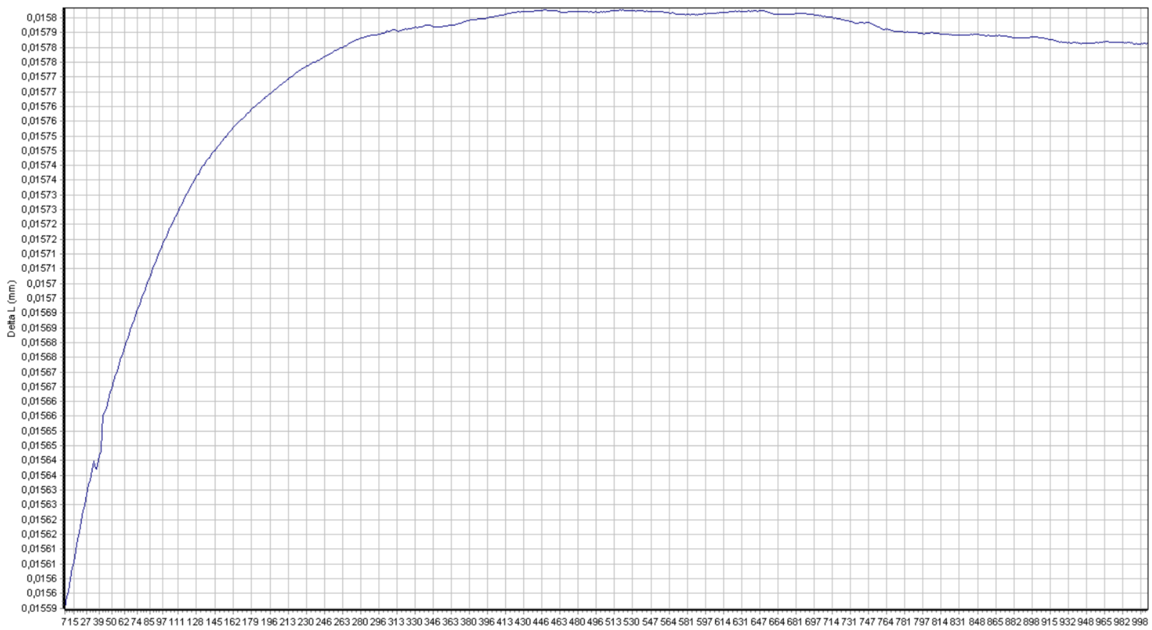


Figure 9 - Plate Offset Variation

4 Conclusion

We can evaluate that both plate displacement analysis methods can be used, however some considerations should be made. Firstly the vibrating string sensor operates at a considerably high speed up to 333Hz, so it is necessary to use equipment with the above mentioned processing capacity and noise analysis technology, the use of the accelerometer mesh depends on the analysis speed to be used. measured the maximum speed generated is 12Hz, so it is necessary to analyze the data at the same step in time.

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