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Topology Optimization for Elastic Analysis of 3D Structures using Evolutionary Methods

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Abstract. This work aims at the study and application of Topology Optimization (TO) in problems of elasticity for the determination of the final configuration in 3D structures using the criterion of minimum compliance, seeking the minimization of volume and maintaining its rigidity. The methods of evolutionary structural optimization employed are: a) Solid Isotropic Material with Penalization (SIMP); b) Evolutionary Structural Optimization (ESO); c) Smoothing Evolutionary Structural Optimization (SESO) and d) Sequential Element Rejection and Admission (SERA) which are based on the systematic and gradual removal of the elements following a set of sensitivity criteria. For this, all the methods are implemented in Matlab software and numerical examples are presented to demonstrate the ability of the proposed methods to solve 3D topology problems, presenting the difference in computational time among them.

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Keywords: Topology Optimization, SIMP, ESO, SESO, SERA

1 Introduction

The topological structural optimization aims to "best" distribution of material in the solution domain. This article approaches the evolutionary structural optimization methods in an elastic analysis for three-dimensional structures. Researchers have investigated this topic and several scientific articles have been published with the aim of providing the theoretical foundations for this problem. We highlight here the codes in Matlab by Sigmund et al. [1], using SIMP (Solid Isotropic Material with Penalization) model, Andreassen et al. [2] proposed 88-lines as an extension of the code proposed by Sigmund. Challis [3] presents a compact implementation of the level-set method for statically loaded structures. These studies demonstrate the minimization of compliance for a linear elastic analysis in a 2D domain, using finite quadrilateral bilinear elements. Huang and Xie [4] also published a Matlab code for 2D compliance minimization using the Bi- Evolutionary Structural Optimization (BESO) method. The Sequential Element Rejection and Admission (SERA) method was implemented in Matlab for topology optimization of structures and compliant mechanisms by Loyola et al. [5].
In the last decade, researches related to 3D topology optimization using Matlab code have addressed the evolutionary optimization methods as SIMP method, including extensions for multiple load cases, continuation strategy, synthesis of compliant mechanisms and heat conduction problems, considering a compliance minimization problem, such as presented by Liu and Tovar [6] and the BESO method by [4]. In addition, we have for the 3D methodologies the paper proposed by Zegard and Paulino [7] and [8], respectively, for ground structure and the tool called "*TOPslicer*" both developed in Matlab to generate suitable outputs for additive manufacturing. Gebremedhen et al. [9] in your article provided a stress-based topology optimization mathematical model for three-dimensional optimization problem using SIMP.
Langelaar [10] article presented a topology optimization formulation that includes a simplified additive manufacturing (AM), the procedure demonstrated involves compliance minimization, eigenfrequency maximization and compliant mechanism design. Zuo and Xie [11] presented a 100-line Python code for general 3D topology optimization, developed for the compliance minimization with a volume constraint using the BESO method extensions to multiple load cases where nonlinearities are considered. Borrvall and Petersson [12] considered large-scale topology optimization of elastic continua in 3D, parallel computing is used in combination with domain decomposition and the equilibrium equations was solved by a preconditioned conjugate gradient algorithm and the optimization part is solved using sequential convex programming.
This article investigates the SESO by Simonetti et al. [13], ESO proposed by Xie and Steven [14] and reviewed by Ghabraie [15] and SERA methods in comparison with the SIMP method, including the extensions for multiple load cases and the synthesis of compliant mechanisms proposed by [6]. In this sense, a novelty of this investigation occurs in the implementation of SESO and SERA in Matlab code to perform the 3D topological optimization procedure.
The remainder of the article is organized as follows: Section 2 gives a definition of the problem of minimum compliance and the different optimization methods implemented in this article, Section 3 explains in detail the formulation of the hexahedron (8 node) finite element, Section 4 presents the numerical examples and at Section 5 is presented the conclusions.

2 Optimization problem formulation

2.1 Problem Statement – Minimum Compliance

A TO problem can be defined as a binary problem in which the objective is the best distribution of material in the solution domain, obeying certain criteria. The topological optimization problem analyzed in this article is the classical binary formulation for compliance, which minimizes the work performed by external forces subject to prescribed volume. The mathematical formulation of this problem is as follows.

$$\begin{aligned} &\underset{X}{\text{minimize}} \quad C(X) = U^T KU \\ &\text{subject to} \quad \text{X= } x_i \ , \ x_i = 1 \text{ or } x_i = 0, \ \forall \ i = 1,2,...,N \\ &\qquad\qquad\quad F = KU \\ &\qquad\qquad\quad V(X) = \sum_{i=1}^{N} x_i V_i - V^* \leq 0 \end{aligned} \tag{1}$$

where the compliance $C(X)$ is the objective function; X is the finite elements of the design and thus vector of binary design variables as in a common discrete problem; x_i is the e-th design variable with candidate values of either 1 and 0, respectively, for solid element and void element. N is the total number of elements; F and U are the global force and displacement vectors, respectively; K is the global stiffness matrix; $V(X)$ is the total volume of the structure with V_i being the volume of structure in iteration i ; V / V_i is the imposed value of the volume constraint.

Due to the variation of energy caused by the removal of the element from the structure, the design variable will be updated by the derivative of the objective function that represents sensitivity of the element and is given by equation

$$\frac{\partial C(X)}{\partial x_i} = -u_i^T(x) \ x_i (E_0 - E_{\min}) \ u_i(x) \tag{2}$$

Thus, low sensitivity values can be removed from the structure without abruptly reducing the overall rigidity of the system. In this paper, the value of 10^{-6} is adopted to avoid singularity of the finite element stiffness matrix.

2.2 Approach Optimization for different methods

Using the ESO and SESO approaches that differ only in their heuristic of removal of inefficient elements, this problem can be written as:

$$\begin{aligned} &\text{Minimize C = U}^T KU = \sum_{i=1}^{NE} U_i^T K_i U_i \\ &\text{subject to } KU = F \\ &\qquad\quad V(X) = \sum_{i=1}^{NE} x_i V_i - V^* \leq 0 \\ &\qquad\quad X = \{x_1 \quad x_2 \quad x_3 \quad ... \quad x_N\}, \quad x_i = 1 \, ou \, x_i = 10^{-9} \end{aligned} \tag{3}$$

The SERA method proposed by Rozvany et al. [16] uses a "virtual material" without using intermediate densities or interpolation of the power law. Thus, the sensitivity number for "real" and "virtual" material are sorted separately during the optimization process. Thus, TO problem using SERA can be written as:

$$\begin{aligned} &\text{Minimize C = U}^T KU = \sum_{i=1}^{N} U_i^T K_i U \\ &\text{subject to } KU = F \\ &\qquad\qquad V(\rho) = \sum_{i=1}^{N} \frac{\rho_e V_e}{V_0} \leq V^* \rho_e \\ &\qquad\qquad \rho_e = \ \rho_{\min}, 1 \end{aligned} \tag{4}$$

where *Ve* corresponds to each finite element volume, *V** is the prescribed volume fraction. The density (ρ) is the vector of design variables and it is discrete in the SERA method, so density can only be zero or one.

In Eq. (3) and (4) the loading and the geometry play an important role in the performance of the structure. We apply the TO formulation in Eq. 3 based on the ESO and SESO approach whose design variables are each of the finite elements. Eq. (4) defines the TO procedure via SERA and have as design variables the material density. It should be noted that these methods differ only in their heuristics of addition and removal of elements.
The implementation of TO using the SIMP method is expressed mathematically as:

$$\begin{aligned} &\text{Minimize } \ C = U^T KU = \sum_{i=1}^{N} x_i^p U_i^T K_i U_i \\ &\text{subject to } KU = F \\ &\qquad \frac{V(x)}{V_0} = f \\ &\qquad 0 < x_{\min} < x_e < 1 \end{aligned} \tag{5}$$

where $\boldsymbol{N}$ is the number of elements to discretize the design domain and $\boldsymbol{p}$ is the penalization factor, $V(x)$ and V_0 are the material volume and design domain volume, respectively, and f is the prescribed volume fraction. The material density is used as a continuous design variable.
One of the main differences in the formulations for ESO, SERA, SESO and SIMP is the volume. ESO, SERA and SESO reduce the volume to find the optimum topology solution, whereas SIMP finds the optimum topology solution to the given volume specified a priori. Thus, the volume in SIMP remains constant during the optimization procedure and in the other methods it decreases in a hyperbolic way.
ESO, SERA e SESO result in mean compliances that are close but highly dependents on the selected parameters, mesh size and filter radius. The final mean compliance from the SIMP method is higher than the one from other methods because it converges to a local optimum with elements of intermediate densities.

3 Finite Element Analysis

3.1 Equilibrium equation

Following the SESO method given by [13] and the generalized Hooke's law, defined for three-dimensional constitutive matrix, an isotropic element *i* is interpolated from void to solid as:

$$D_i(x_i) = E_i(x_i) D_i^0, \quad x_i \in \ 0,1 \tag{6}$$

with $E_i(x_i) = x_i * E_0$ and E_0 is the elastic modulus of the solid material. The SESO method is based on the calculation of the sensitivity of the structural system, discretized by the Finite Element Method, when a finite element is drawn from the approach space, as shown in Fig. 1. The idea is then to remove p% the less sensitive finite elements of the mesh according to this sensitivity and give back (1-p%) of these elements taking into account a rate of removal of the same. This procedure continues until the volume constraint imposed on the problem is met.

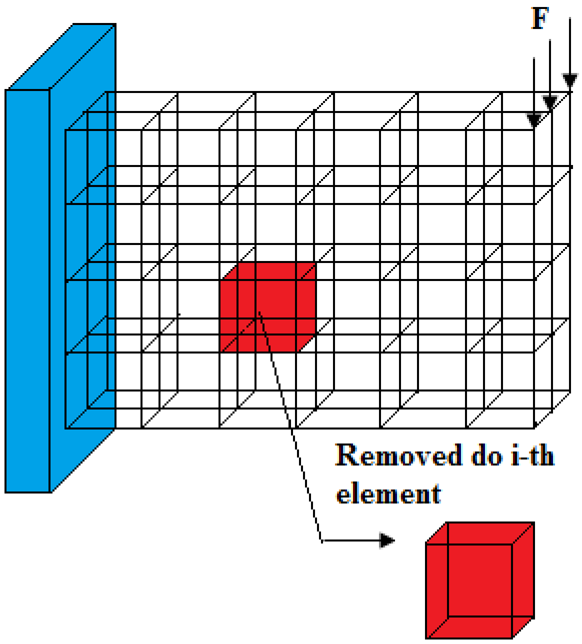


Figure 1 - Removal of hexahedral finite element from mesh

In the ESO, SESO and SERA methods, the calculation of the sensitivity of the system when removing a finite element from the approach space is used to impose modifications in the domain making the problem discrete (0 or 1). In the SIMP method, the sensitivity is calculated locally with the introduction of a continuous intermediate density values distribution function in order to avoid the binary nature of the problem.

The SIMP method is based on a heuristic relation between (relative) element density x_i and element Young's modulus E_i given by:

$$E_i(x_i) = E_{\min} + (x_i)^p (E_0 - E_{\min}), \qquad x_i \in [0,1] \tag{7}$$

where $E_{\min}$ is the elastic modulus of the void material, which is non-zero to avoid singularity of the finite element stiffness matrix, proposed by [6].

In the SERA method, the global stiffness matrix K is assembled from the element stiffness matrices K_e, which are obtained multiplying the element isotropic stiffness matrix K_0 by the density of the element, since Young's moduli are assumed to depend linearly on the density variable e is given by:

$$K_e(\rho_e) = \rho_e K_0, \qquad \rho_e \in \{\rho_{\min}, 1\} \tag{8}$$

These design variables are discrete in the SERA method, so density can only be zero or one. Nevertheless, in order to avoid obtaining a singular stiffness matrix, a non-zero lower bound is assigned to density ($\rho_{\min}$), by [5].

4 Numerical Examples

To verify the differences in responses and processing time, the methods SESO, SERA were implemented in Matlab language in the existing code developed by [6], which was created for the SIMP method. All numerical examples were processed on a notebook Core i3-2370, 2.400GHz CPU.

4.1 EXAMPLE 1 – 3D cantilever beam – Force on the centroid line

The 3D-SESO algorithm will be compared to the ESO, SERA and SIMP algorithms. For this comparison it should be noted that the filter scheme is a heuristic technique for overcoming the checkerboard and mesh dependency problems in topology optimization. Thus, the long cantilever shown in Fig. 2 is selected as a test example because it involves a series of bars broken during the evolutionary procedure of the topology optimization. Young's modulus E = 210.0 GPa and Poisson's ratio v = 0.3 are assumed. The design domain is a discretized prismatic structure with 80x20x2, totalizing 3200 of eight-noded cubic elements, formulation proposed by [6]. This structure is fully constrained at one end and a distributed vertical load of F = 100N is applied downwards in the center of the free edge.

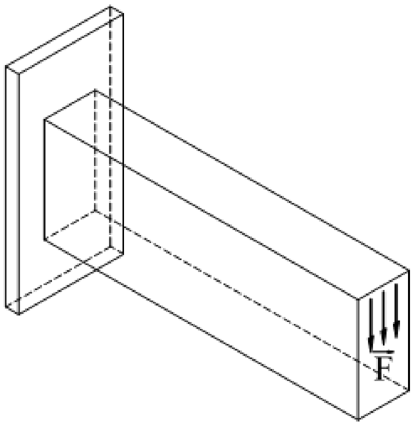
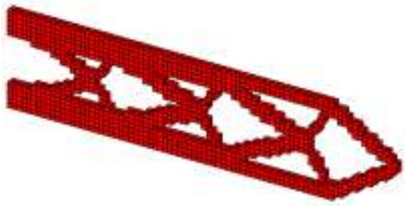
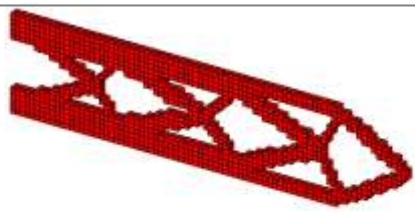
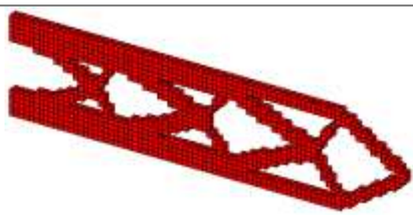
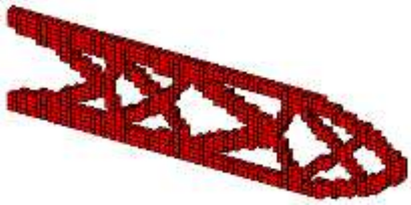


Figure 2 - Topology optimization of 3D cantilever beam – (a) Design domain

The present problem is analyzed with the performance of the mesh independence filter. Table 1 lists the used parameters and solutions obtained of the various topology optimization algorithms. It can be seen that the topology optimization algorithms produce very similar topologies except with SIMP design that shows some grey areas of intermediate material densities. The topologies presented in Table 1 show cleaner definitions of the members and are more useful to practical use.

Table 1 - Comparison of topology optimization methods with a mesh-independency filter

Method	Cost (seconds)	Volume	Settings	Compliance
SESO	539.4	0.50		793.1
SERA	456.3	0.50		794.6
ESO	538.4	0.50		794.3

SIMP	546.7	0.50		986.6

The ESO, SESO and SERA methods presented similar topologies. However, the SESO method converged with a computational cost greater than the other three methods but has the lowest value of the objective function. For this optimal analysis, it is observed that the SIMP method has a different topology for the same volume and has the highest value for the objective function.

4.2 EXAMPLE 2 – 3D cantilever – Force on the lower edge

The Fig. 3 shows a 3D cantilever as an example with a filter radius of 1.5 and target volume fraction of 30%. The design domain is a discretized prismatic structure with 60x20x4, totalizing 4800 of eight-node cubic elements. This structure is fully constrained at one end and a distributed vertical load of F = 100N is applied downwards on the lower free edge.

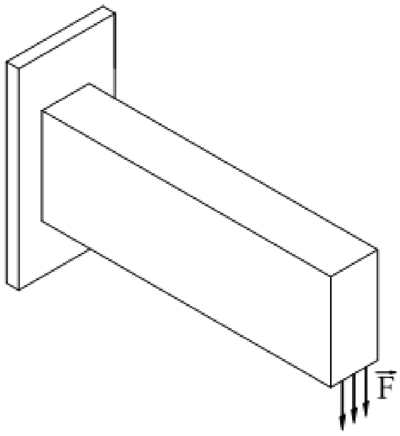


Figure 3 – Initial design domain

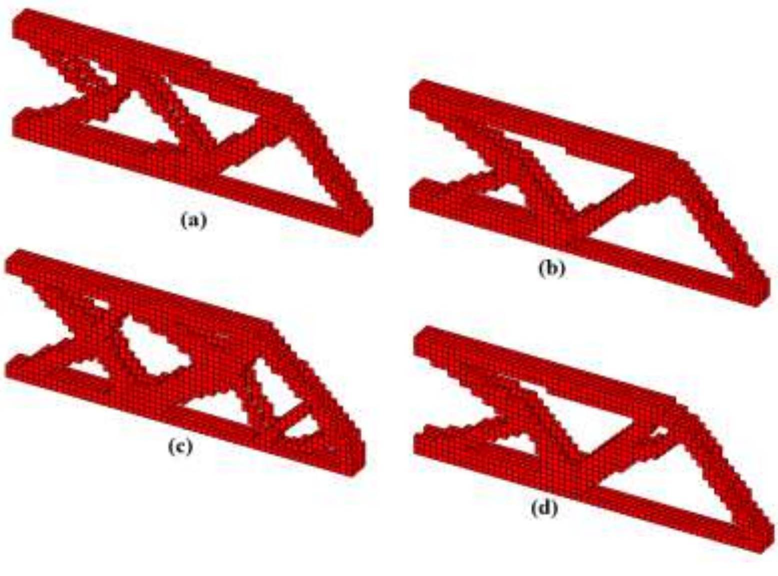


Figure 4 – 3D cantilever – Optimized settings – (a) SERA,
(b) SESO, (c) SIMP and (d) ESO.

The Fig. 4 summarizes the results of the optimal settings for the SERA, SESO, SIMP and ESO methods respectively in figures 4a, 4b, 4c and 4d. It is noteworthy that the lowest computational cost for this problem was via SESO method with 805.9 seconds while the highest computational cost was via SERA method with 918.6 seconds. In addition, SESO has optimum solutions close to ESO and SERA and different from the solutions presented by the SIMP method that presented the highest compliance value ($C = 1325.8$), approximately 66% higher than the values of the other methods analyzed in this article.

5 Conclusions

This article presented four classical topological optimization based in compliance procedures applied for three-dimensional elastostatic problems. Some methods developed (ESO, SESO and SERA) for structural optimization are evolutionary and the obtained results are compared with compliance minimization resulted with the deterministic SIMP method. A free code, presented in [6], was used to introduce the overmentioned methods, in which a hexahedron finite element is used for discretizing design domains and the elastic analysis is used for computation of the objective function for each method. From the results, it was clear that the developed model can generate optimal topologies that can sustain applied loads under the boundary conditions defined. In addition, with the results presented, it is clear that the SESO, SERA and ESO methods have the closest optimal configurations, lower compliance and lower computational cost than the SIMP method. Worth highlighting that the SIMP method presented is lower computational cost than the other methods when the prescribed volume low, that is, the convergence to the optimum in the SIMP method is faster than in the other models present.

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