XL CILAMCE
IBERO-LATIN AMERICAN
CONGRESS ON
COMPUTATIONAL
METHODS IN
ENGINEERING
NOVEMBER
11-14, 2019
Praiamar Natal Hotel & Convention
Natal, RN-BRAZIL

AUXETIC MATERIAL DESIGN THROUGH STRUCTURAL OPTIMIZATION APPROACH

Cícero Ribeiro de Lima
Paula Fregnani
cicero.lima@ufabc.edu.br
paula.fregnani@ufabc.edu.br
School of Engineering, Modelling and Applied Social Sciences, UFABC
Alameda da Universidade, s/nº, São Bernardo do Campo, SP, 09606-045, Brazil

Abstract. Unlike conventional materials, an auxetic material has negative Poisson's ratio, that is, it increases the size of its cross-section when is under traction and decrease when is compressed. The auxetic structural behavior can provide many benefits, since the structural material may acquire notable mechanical properties, such as increased resistance to impact. Due to this improved impact absorption capacity, the auxetic materials have great potential for use in the aerospace structures. In this work, parametric optimization has been applied to develop a re-entrant structure (periodic cell) that simulates the behavior of the auxetic material microstructure. Thus, the auxetic behavior arises from the mechanism deformation of geometric configuration of the re-entrant structure. The main motivation of this work is to contributes with a more systematic methodology to design of the auxetic structures, make it independent of the designer expertise. Some 3D re-entrant auxetic structures of the literature are adopted as initial design domain and a parametric optimization is carried out to obtain optimized dimensional configurations for the geometry of the auxetic structure. Dimensional parameters of the domain, such as angles, wall thickness, height, and width, are considered as design variables in the optimization problem, in which the objective function is formulated to maximize de behavior of the auxetic structure (negative Poisson's ratio). Computational simulations of finite element models are carried out to evaluate the optimized auxetic structures.

Keywords: Parametric optimization, Auxetic structure, Negative Poisson's ratio, Metamaterial

1 Introduction

Structures with auxetic behavior have negative Poisson's ratio, that is, they expand laterally when tensioned and contract all sides when compressed. The auxetic structural behavior can provide many benefits, since the structural material may acquire notable mechanical properties, such as increased resistance to impact. It is demonstrated that when the auxetic material is hit by an object, the material flows to the region of impact to reinforce that region, instead of escape way from the impact zone as observed in the conventional materials (Fig. 1).

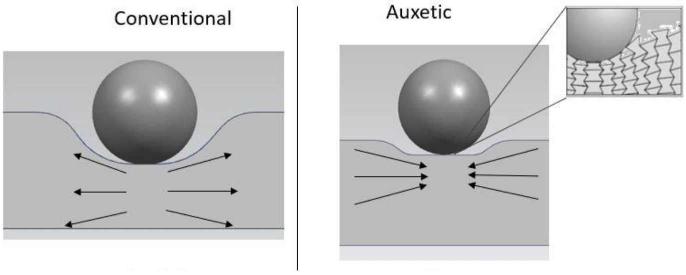


Figure 1. Conventional and auxetic material

In the last decades, the importance of this metamaterial has gained notorious prominence in the scientific community, aiming at engineering applications. For instance, due to the improved impact absorption capacity, the auxetic materials have great potential for use in the aerospace structures [1]. Also, mechanical properties such as shear resistance and fracture toughness can be significantly improved in a material with negative Poisson ration, making it a potential to be used as vanes for aircraft gas-turbine engines [2].

Auxetic materials have been employed in the development of high-tech products in many fields, such as intelligent expandable actuators, shape morphing structures and implantable biomedical devices. An example of this can be found in [3], which applies auxetic shape-memory alloys for developing deployable satellite antennas. A practical application of an auxetic structure is shown in [4], which presents auxetic nails that become thinner when knocked, making it easier to entry into a hole, and become fatter when pulled out, making it more difficult to remove.

The first purposely developed auxetic materials are like special foams, which its cellular structures can be produced by control of pressure and temperature in their manufacture [5, 6]. The auxetic behavior is not inherent in the chemical composition of the materials, that is, this behavior can also be provided by the configuration of their microstructure. There are no records of purely auxetic materials that do not depend on their microstructure configuration. They are usually found in truss or beam arrangements so that the interaction between them proposes an unconventional behavior. Thus, the challenge arises of creating auxetic materials from elements with conventional materials, arranging them so that the structure presents the desired properties (negative Poisson's ratio, for example).

This arrangement is commonly known as man-made structures, in which conventional material is applied to generate a re-entrant structure, which has the auxetic behavior. The auxetic behavior arises from the mechanism deformation of the reentrant structure, in which each interconnected member rotates when is loaded to produce expansion of the structure in horizontal and vertical directions [7]. Figure 2 illustrates a 3D man-made auxetic macrostructure.

The design of the reentrant structures is traditionally based on the experience of engineers. However, using optimization techniques to design auxetic structures with preferred performance is arising in this field. In the other words, the auxetic structure can be designed by the means of structural optimization methods [8], in order to generate the geometry that gives the best performance to the desired behavior. So, the objective of this work is to explorer the application of parametric (size) optimization to develop a re-entrant structure (periodic cell) that simulates the behavior of the auxetic material microstructure.

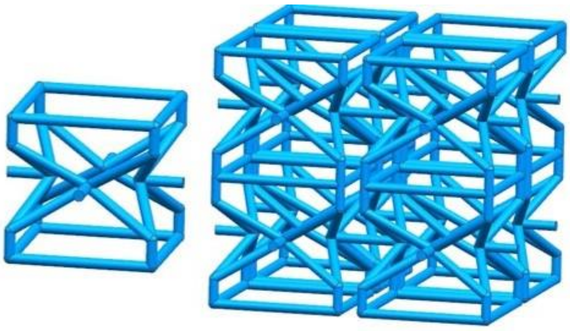
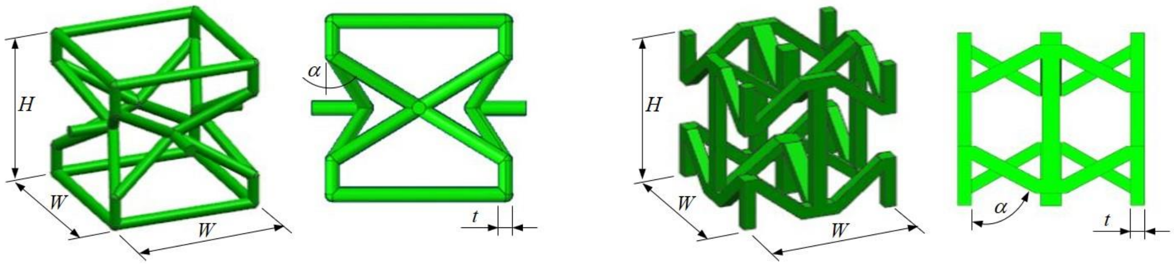


Figure 2. Man-made auxetic structure

Next sections are organized as follows. Section 2 describes the optimization problem and its design variables (parameters to be optimized). Section 3 shows the numerical implementation by using a finite element and optimization software. Section 4 presents some results. Finally, Section 5 gives the conclusions.

2 Optimization problem

Most of design of various successful optimized auxetic structures of the literature still remains in 2D. So, to contribute in providing more 3D auxetic structures with superior performance, two well-known 3D re-entrant auxetic structures, found in the literature [9, 10], are adopted in this work as initial design domain for the optimization procedure. The goal is to find a set of dimensional parameters (width - *W*, height - *H*, inclination angle - α, and thickness wall - *t*) that improve auxetic behavior of the structure. Figure 3 shows the geometric models, generated by a CAD software, and the parameters (*W*, *H*, α, and *t*) of the auxetic structure to be evaluated by a systematic design methodology.



(a) (b)

Figure 3. Geometric models: (a) geometry 1; (b) geometry 2

The optimization problem is given to maximize the Poisson's ratio (ν) of the microstructure, which is defined as the ratio between the negative transverse and longitudinal strains (ε). In other words, the objective to be achieved here is to maximize a transversal output displacement (u_{out}). Thus, the objective function can also be represented by following equation [11]

$$\nu = -\frac{\varepsilon_{transv}}{\varepsilon_{load}} = -\frac{u_{out}}{u_{in}} \tag{1}$$

where u_{in} is the input displacement generated by the external load applied to the auxetic structure.

A stress constraint (σ_{max} = 20 MPa) is applied to the junctions, where the members of re-entrant structure are connected and rotate about them when is loaded (critical regions), as illustrated in Fig 4.

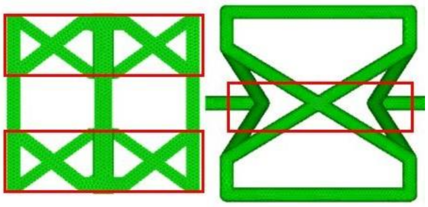


Figure 4. Regions for applying stress constraint regions

Moreover, upper and lower limits (box constraints) are specified for each dimensional parameter (design variables W, H, α, and t), according the project requirements. Thus, the optimization problem can be formulated as follows

$$\begin{aligned}
&\underset{W,H,\alpha}{\text{Maximize}} && \nu \\
&\text{Subject to:} && \boldsymbol{K}\,\boldsymbol{u} = \boldsymbol{f} \qquad (\textit{equilibrium equations}) \\
& && \sigma \leq \sigma_{\max} \\
& && t = t_f \\
& && \alpha_{\text{low}} \leq \alpha \leq \alpha_{\text{upp}} \\
& && W_{\text{low}} \leq W \leq W_{\text{upp}} \\
& && H_{\text{low}} \leq H \leq H_{\text{upp}}
\end{aligned} \tag{2}$$

where $\boldsymbol{K}$ is the stiffness of the structure, $\boldsymbol{u}$ and $\boldsymbol{f}$ are the displacement field and the applied load, respectively.

3 Numerical implementation

The optimization procedure is carried out numerically by using the optimization tools of the OPTISTRUCT, which is one of commercial software used in industry for structural analysis and optimization designs, and also available for academic purposes.

In this case, the algorithm of the OPTISTRUCT applies a deterministic optimization, based on gradient methods [8], to perform the required parametric optimization. So, calculation of derivatives of the objective function in relation to design variables (sensitivity analysis) is evaluated at each step of the optimization algorithm to define the optimized search direction. The derivatives (gradients) of the objective function are calculated implicitly from the gradients of the equilibrium equations of structure and using the adjoint method [12].

The geometric models, shown in Fig. 3, is discretized by a finite element (FE) mesh with 94,800 tetrahedral elements, and FE analysis is carried out to evaluate the deformation (strain) of the auxetic structure, which is measured in terms of the nodal displacements. Figure 5 shows the FE model, including the applied load (F_{in}) and output displacement regions.

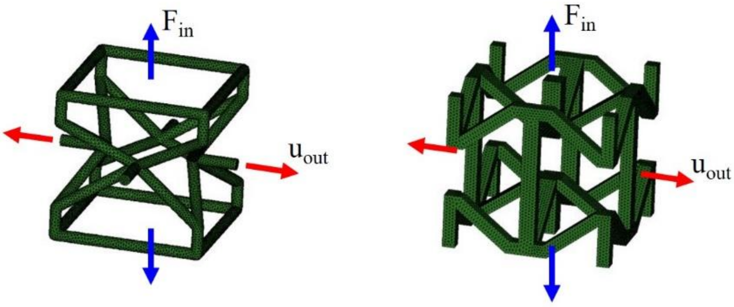


Figure 5. FE model with tetrahedral elements

Successively FE analyses are carried out in the iterative process of the optimization procedure, in order to solve the optimization problem of Eq. (2). In each iteration, an improved set of design variables (dimensional parameters) is update by the optimizer of the OPTISTRUCT to improve (maximize) the objective function. The iterative process is stopped as convergence is achieved for the objective function value and, thus, an optimized solution is plotted.

4 Results

The geometric configuration of the re-entrant auxetic structures (1 and 2), shown in Fig. 3, are adopted as start for the optimization procedure. The adopted parameters for both initial structures are W = 25 mm (width), H = 25 mm (height), and t_f = 2 mm (thickness wall). The initial inclination angle (α) for structures 1 and 2 are 30º and 50º, respectively. The region of the input force and the desired output displacement, as well as fixed supports configuration (boundary conditions) are shown in the Fig. 6.

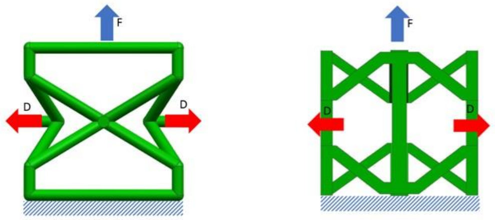
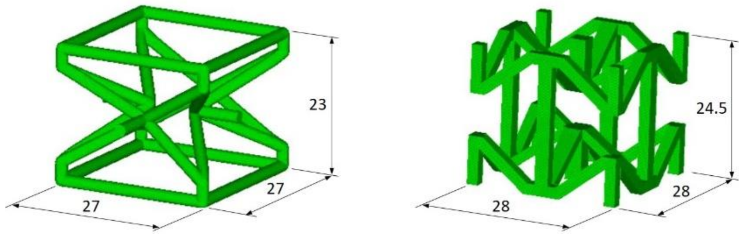


Figure 6. Boundary conditions (F: applied load; D: output displacement)

Finite element linear static analyses are carried out to evaluate the displacement field of the auxetic structure. In this work, the value of the input force (F_{in}) is equal to 16 N. Consistent units are employed. The elastic modulus and Poisson's ratio of the base material utilized in the computational simulations are 50 MPa and 0.4, respectively. Fig. 7 presents the results obtained after few iterations of the parametric optimization performed by the OPTISTRUCT software.



(a) (b)

Figure 7. Optimized set of parameters: (a) geometry 1 (α = 46º); (b) geometry 2 (α = 50.5º)

Table 1 summarizes quantitatively the improvement found for the auxetic behavior of both initial structures. As can be seen, the negative Poisson´s ration (ν) of both structures has increased considerably. For instance, the proposed optimization methodology makes the value of ν of the geometry 1 rises from 0,39 to 0,56, which provides an improvement about of 44% in relation to the initial configuration of the structure (Fig. 3).

Table 1. Results obtained for the auxetic structures

	Geometry 1			Geometry 2		
	Limits	Initial	Final	Limits	Initial	Final
H	20-35 mm	25 mm	23 mm	20-35 mm	25 mm	24.5 mm
W	25-35 mm	25 mm	27 mm	20-35 mm	25 mm	28 mm
α	30°-50°	30°	46°	20°-60°	50°	50.5°
t_f	2	2	-	2	2	-
ν	-	-0,39	-0,56	-	-0,55	-0,66
improvement	44%			20%		

To illustrate the optimization procedure, Fig. 8 describes the evolution of the iterations performed by the optimization software (OPTISTRUCT), as well the convergence curve of the objective function. The FE analysis and optimization solution carried out in each iteration has been solved in approximately 8 minutes by using a notebook Core i7 (7th generation) with 8GB RAM.

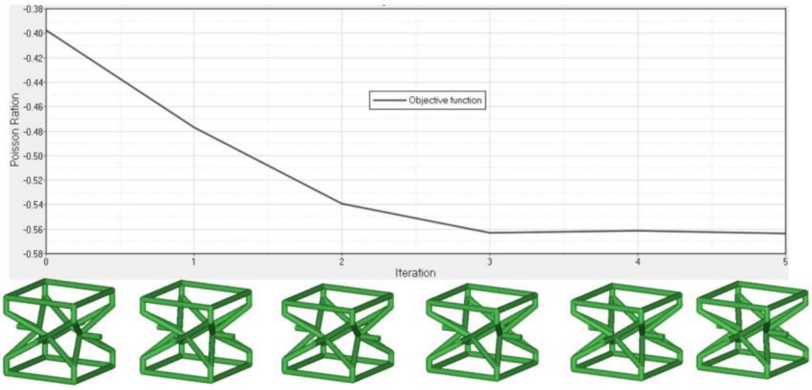


Figure 8. Evolution of the optimization procedure at each iteration.

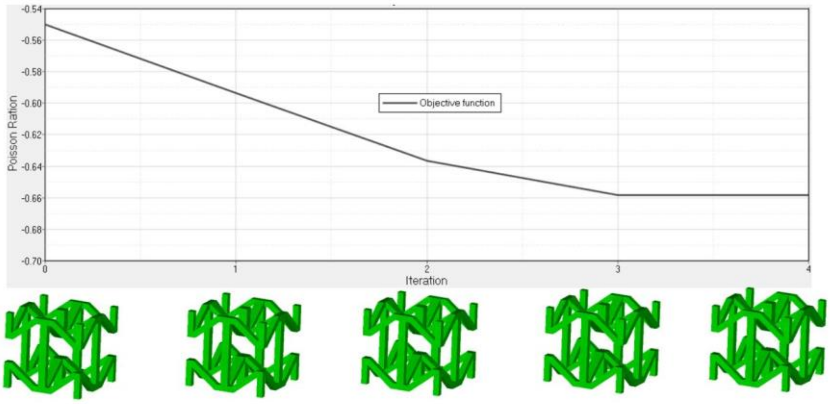


Figure 9. Evolution of the optimization procedure at each iteration.

5 Conclusion

A systematic design methodology, based on parametric optimization, has been applied to obtain structures with optimized auxetic behavior. The commercial software (OPTISTRUCT) has aided to perform the FE analyses and the iterative optimization procedure promptly. According to the obtained results, the dimensional parameters have the great influence on the behavior of structure, in which an optimized set can provide 3D auxetic structures with superior performance. For future work, the optimized unit re-entrant cells (microstructure) will be utilized to obtain macrostructures with auxetic behavior by applying a periodic repetition of this cell over a domain. Moreover, non-liner finite element analysis may be employed to achieve more accurate models and simulate the auxetic structure under large deformation. Additive manufacturing and experimental testing could also be considered to evaluate the auxetic behavior of optimized structures.

Acknowledgements

The authors would like to acknowledge the support by FAPESP, CAPES, and CNPq.

References

[1] M. Mir, M. N. Ali, J. Sami, and U. Ansari. Review of mechanics and applications of auxetic structures. Advances in Mat. Sci. and Eng., 1–17, 2014.
[2] Y. Prawoto. Seeing auxetic materials from the mechanics point of view: a structural review on the negative Poisson's ratio. Comput. Mater. Sci. 58 140–53, 2012.
[3] F. Scarpa, S. Jacobs, C. Coconnier, M. Toso and M. D. Di. Auxetic shape memory alloy cellular structures for deployable satellite antennas: design, manufacture and testing. EPJ Web of Conf. 6 27001, 2010.
[4] X. Ren. Auxetic nail: Design and experimental study. Composite Structures, 184, 288–298, 2018.
[5] R. S. Lakes. Foam structures with a negative Poisson's ratio. Science 235, 1038–1040, 1987.
[6] K. E. Evans. Tailoring the negative Poisson's ratio. Chem. Ind. 20, 654–657, 1990.
[7] Y. Liu and H. Hu. A review on auxetic structures and polymeric materials. Scientific Research and Essays, 5(10), 1052–1063, 2010.
[8] S. S. Rao. Engineering optimization: Theory and practice. John Wiley & Sons, 1996.
[9] G. Imbalzano, P. Tran, T. D. Ngo, P.V. Lee. Three-dimensional modelling of auxetic sandwich panels for localised impact resistance. Journal of Sandwich Structures and Materials, 19(3), 291–316, 2017.
[10] X. Ren, R. Das, P. Tran, T. D. Ngo, Y. M. Xie. Auxetic metamaterials and structures: A review. Smart Materials and Structures, 27(2), 1–38, 2018.
[11] C. R. de Lima, G. H. Paulino. Auxetic structure design using compliant mechanisms: A topology optimization approach with polygonal finite elements. Advances in Engineering Software, 129, 69–80, 2019.
[12] P. W. Christensen, A. Klarbring. An Introduction to Structural Optimization, Springer, 2009.