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OPTIMIZATION OF THE NATURAL FREQUENCIES OF EULER-BERNOULLI BEAMS

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Abstract. Various types of engineering structures are subject to periodic loading such as offshore platform parts and wind turbine blades. One of the main causes of failure in these structures is due to the resonance effect, when the frequency of external loading coincides with some natural frequency of the structure. Therefore, the maximization of natural frequencies is an increasingly sought-after topic in the design of these components. In this paper a genetic algorithm is developed to maximize natural frequencies of Euler-Bernoulli beams. Genetic algorithms are stochastic search methods, which are based on biological concepts of adaptation, natural selection, fitness and evolution, to solve optimization problems. A beam population is created, each of them discretized in a mesh of cylindrical elements with different diameters, initially random. The natural frequencies of the beam are found by the Finite Element Method, and the one with the highest natural frequency creates a new generation of offsprings. In each offspring is applied a mutation scheme that changes the diameter of any random element, making the entire population change. So over the generations the algorithm finds out the best diameter combination that maximizes the natural frequency of the beam. Results present different shapes are obtained for several boundary conditions and different natural frequencies maximized.

Keywords: Parametric Optimization, Genetic Algorithm, Finite Element Method

1 Introduction

Optimization techniques are increasingly used in engineering projects, as they allow structures with unconventional formats, allowing cost reduction, space and increased efficiency. Since the resonance phenomenon is one of the main causes of failures in structures subject to periodic loads, such as submerged structures and components of rotor designs, several researchers have sought techniques to increase the natural frequency of these structures. An interesting result was found by Picelli et al. [1] who used the bi-directional evolutionary structural optimization (BESO) to maximize the first natural frequency of acoustic-structure systems. A different approach was proposed by Wang et al. [2], who used a homogenization-based topology optimization method for natural frequency optimization in a cantilevered plate with a honeycomb structure increasing the first natural frequency and reducing weight and, using the differential evolution optimization, Roque and Martins [3] maximized the first natural frequency for a functionally graded beam.

For optimization problems whose the derivative of the objective function is not known or is difficult to compute, derivative-free algorithms are utilized. Usually these algorithms are metaheuristic because the random sampling employed in these algorithms ensures an evenly distributed evaluation of the objective function (Hofmeister et al. [4]). Genetic algorithms (GA) are stochastic search methods, which are based on biological concepts of adaptation, natural selection, fitness and evolution, relying on Darwin's principle of survival of the fittest to solve optimization problems. Due to their robustness and efficiency, GAs are widely used in various optimization problems in robotics, control systems, logistics problems and design of structures.

In 1859 Darwin [5] offers an explanation of the origins of biological diversity and its underlying mechanisms. In what is sometimes called the macroscopic view of evolution, natural selection plays a central role. Given an environment that can host only a limited number of individuals, and the basic instinct of individuals to reproduce, selection becomes inevitable if the population size is not to grow exponentially. Natural selection favours those individuals that compete for the given resources most effectively, in other words, those that are adapted or fit to the environmental conditions best. This phenomenon is also known as survival of the fittest (Eiben et al. [6]).

In biology, the visible features of an individual, as the color of hair, could be determined by genes, which are segments of a chromosome. Each individual has a phenotype (observable features) which is influenced by the genotype (complete heritable genetic identity). A chromosome is an organized package of DNA found in the nucleus of the cell. In simple terms, the chromosome contains the combination of genes of an individual. Each individual has a unique combination of genes. If this combination evaluates favourably (in the phenotype), the individual will survive and generate offsprings; otherwise, if evaluates unfavourably, then the individual will die without offsprings. Favourable combination of genes can be propagated to the offsprings. Some fails occurs in the reproduction, causing random changes in the offspring's genotype. This process is known as mutation and causes variation in the next populations. The best ones survives and reproduces and specie evolves. Relying in these principles, GA mimics the evolution process treating variables of the physical world as genes and using the concepts of mutation and survival off the fittest to find what is the best variables to optimise some problem. Each gene can be treated as a string or a number. The combination of these genes is called chromosome.

Several authors used GAs together with the finite element method (FEM) to solve structural optimization problems. For example: Lin [7] developed a GA optimization approach to search for the optimal locations to install bearings on the motorized spindle shaft to maximize its first-mode natural frequency. Most recently, Elrehim et al. [8] used GA to a geometrical structural optimization study for a deck concrete arch bridges. Also using GA in conjunction with FEM, Walker and Smith [9] minimised a weighted sum of the mass and deflection of fibre reinforced structures and Bhandary et al. [10] proposed a procedure to determine the factor of safety of a slope.

In this work, the shape of Euler-Bernoulli beams discretized into a series of finite elements are optimized to maximize the natural frequencies (NF) of the beam. Once the derivative of the NF is difficult to compute, a genetic algorithm is developed to find the best combination of diameters of the mesh of finite elements that maximizes the NF. Two-node beam cylindrical elements with different diameters are used to compute the eigenvalues of the structure and the genetic algorithm find out the optimal shape that maximizes the eigenvalue. The codes for the FEM and the GA optimization technique are developed in MATLAB programming platform. Different shapes of beams are found from different boundary conditions and different natural frequency maximized.

2 Formulation

2.1 Finite Element Model

Considering an one-dimensional, two-noded kth beam element with modulus of elasticity E, the moment of inertia of cross section I_k, the mass density ρ, the length l_e and the cross-sectional area A_k is shown in Fig. 1. Each element of the mesh has a different cross-sectional area. The generalized coordinates at each node are w, the total deflection, and θ, the total slope. This results in a element with four degrees of freedom which enable the expression for w and θ to contain two undetermined parameters each, which can be replaced by the four nodal coordinates.

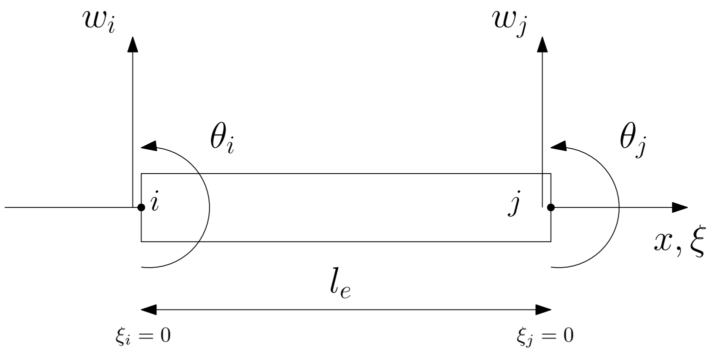


Figure 1. Beam element.

Considering the non-dimensional coordinate ξ, the displacement w can be written in matrix form as follows:

$$w = [\,\mathbf{N}(\xi)\,]\ \{\mathbf{v}\}_k, \tag{1}$$

in which

$$[\,\mathbf{N}(\xi)\,] = \left[\begin{array}{cccc} 1 - 3\xi^2 + 2\xi^3 & (\xi - 2\xi^2 + \xi^3)\,l & 3\xi^2 - 2\xi^3 & (-\xi^2 + \xi^3)\,l \end{array}\right] \tag{2}$$

and

$$\left\{\,\mathbf{v}\,\right\}_k^T = \left[\begin{array}{cccc} w_i & \theta_i & w_j & \theta_j \end{array}\right]. \tag{3}$$

The stiffness and mass matrices of the beam element k are given by, respectively:

$$[\,\mathbf{K}\,]_k = \frac{1}{2}\int_0^1 \frac{EI_k}{l^3}\,[\,\mathbf{N}''(\xi)\,]^T\,[\,\mathbf{N}''(\xi)\,]\,d\xi, \tag{4}$$

$$[\,\mathbf{M}\,]_k = \frac{1}{2}\int_0^1 \rho\,[\,\mathbf{N}(\xi)\,]^T\,[\,\mathbf{N}(\xi)\,]\,A_k\,l\,d\xi. \quad (5)$$

The global stiffness $[\mathbf{K}]$ and mass $[\mathbf{M}]$ matrices are obtained from the element matrices $[\,\mathbf{K}\,]_k$ and $[\,\mathbf{M}\,]_k$ through the finite element assembly. The equation for free vibration a undamped system can be express as (Wu [11]):

$$[\mathbf{M}]\,\{\ddot{v}(t)\} + [\mathbf{K}]\,\{v(t)\} = 0, \quad (6)$$

where $\{v(t)\}$ is the global displacement vector. Assuming that the displacement vector $\{v(t)\}$ takes the form:

$$\{v(t)\} = \{u\}e^{j\omega t}, \quad (7)$$

where $\{u\}$ is the amplitude vector of $\{v(t)\}$, ω is the natural frequency of the system, t is time and $j = \sqrt{-1}$. Substituting Eq. (7) into Eq. (6) gives:

$$([\mathbf{K}] - \omega^2\,[\mathbf{M}])\{u\} = 0. \quad (8)$$

Equation (8) is in the form of a generalized eigenproblem. Its solution includes the eigenvalue ω_r and the corresponding eigenvector $\{u\}_r$ (with $r = 1, 2, \ldots$), in which are also called the rth natural frequency and the rth mode shape for a vibrating system (Wu [11]).

2.2 Genetic Algorithm

In this work, we are interested in find what is the best combination of diameters of cylindrical elements that makes an Euler-Bernoulli beam have the highest value of some natural frequency. In this problem the genes are the diameters and the fitness is the natural frequency. An individual of the population is a beam discretized into n cylindrical elements which have diameters between D_{min} and D_{max}. Since D_k is the diameter of the kth element, the c chromosome of a beam discretized into n elements can be written as:

$$c = [\,D_1\;D_2\;\cdots\;D_{n-1}\;D_n\,]\,. \quad (9)$$

Changing any diameter D_k through mutation, the cross-sectional area A_k and the moment of inertia I_k are modified, consequently, the Eqs. (4) and (5) are changed, then the eigenvalues that solves the Eq. (8) also changes. Considering E and ρ constant throughout the beam, the rth natural frequency (NF) is a function that depends only on the c chromosome vector. We developed a GA that find out which chromosome c maximizes $\omega_r(c)$. NF is calculated using the finite element method (FEM), and the fittest individual generates offsprings, which in turn also mutate. The discretized circular cross section beam is shown in the Fig. 2.

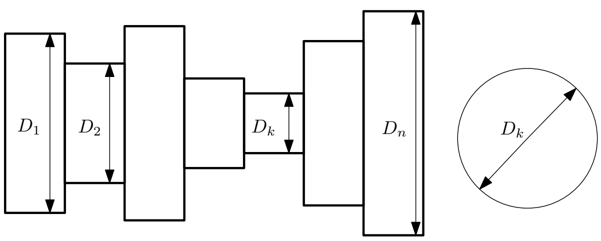


Figure 2. Discretization throughout the beam.

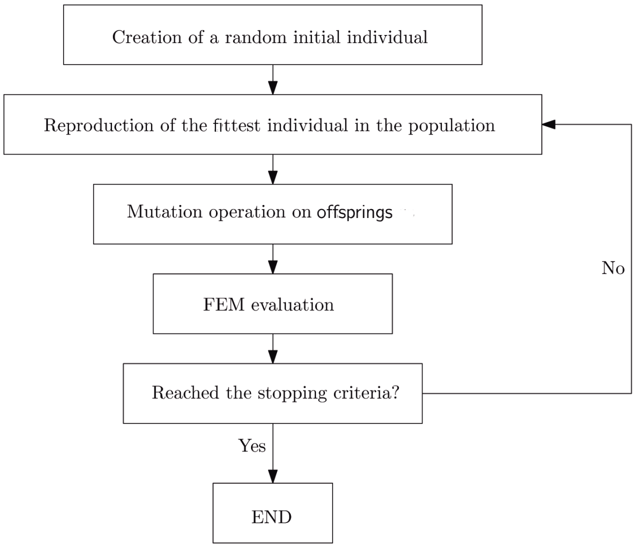


Figure 3. Flowchart of the Genetic Algorithm.

Figure 3. shows an overview of the GA. The first individual is a beam discretized into elements with random diameter values. Offsprings are created and a mutation scheme changes some gene of the offsprings. The mutation scheme chosen for this work is to add some random number in range $[-0.2, 0.2]$ to the diameter of a random beam element. FEM evaluation of the eigenvalues occurs and the code find what is the fittest individual in the current population. To the next generation, the individual with the highest value of NF will generate the offsprings, making population increasingly fit.

The stopping criteria chosen for this work is to check if the code has been 100 consecutive generations consecutive with max fitness of the beam population unchanged. When this occurs, the algorithm stop and plots the optimized shape of the beam.

3 Numerical Results

3.1 Cantilever Beam

For example, consider a cantilever beam with $L = 1$, $\rho = 1$ and $E = 1$, discretized into 5 finite elements, initially presenting random diameters in the range $[0.05, 1]$. Using random number generation functions, the first individual is showed in the Fig. 4:

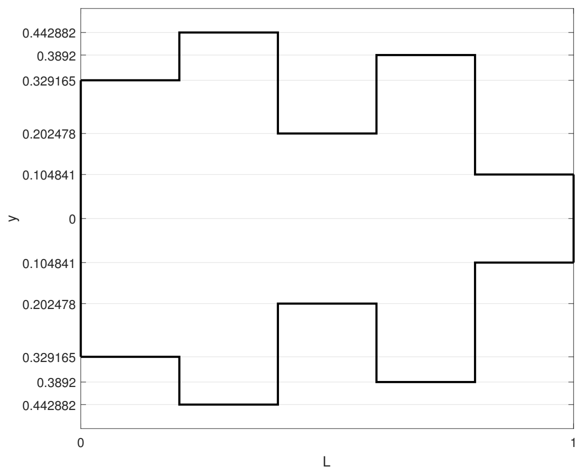


Figure 4. The random initial beam

This beam can be expressed as the following chromosome vector:

$$c_{\text{initial}} = [\, 0.6583 \;\; 0.8858 \;\; 0.4050 \;\; 0.7784 \;\; 0.2097 \,]\,,$$

where each element of the vector c is a gene that represents the diameter of a beam element. In other words, the vector c is the genotype and the Fig. 4 is the phenotype. From these initial individual, 3 offsprings are created and the mutation scheme modifies some random diameter. Initial population is represented in Table 1 in which each line is the chromosome corresponding to an individual of the population, where the individuals 2, 3 and 4 are the offsprings and the bold genes are those who mutated.

Table 1. Initial population

Individual	Genes					ω_1
1	0.6583	0.8858	0.4050	0.7784	0.2097	0.7135
2	0.6583	0.8858	**0.4059**	0.7784	0.2097	0.7690
3	0.6583	0.8858	0.4050	**0.7010**	0.2097	0.7716
4	0.6583	**1.0000**	0.4050	0.7784	0.2097	0.7192

Individual 3 has the highest value of the first natural frequency ω_1, so he will create the offsprings for the next generation. The code will stop when there are 30 consecutive generations with the max fitness unchanged. The final population is given in Table 2.

Table 2. Final population

Individual	Cromossomes					ω_1
1	1.0000	1.0000	0.7145	0.3710	0.1412	2.1539
2	1.0000	1.0000	**0.9025**	0.3710	0.1412	2.0724
3	1.0000	1.0000	**0.6016**	0.3710	0.1412	2.1135
4	1.0000	1.0000	0.7145	0.3710	**0.2860**	1.7343

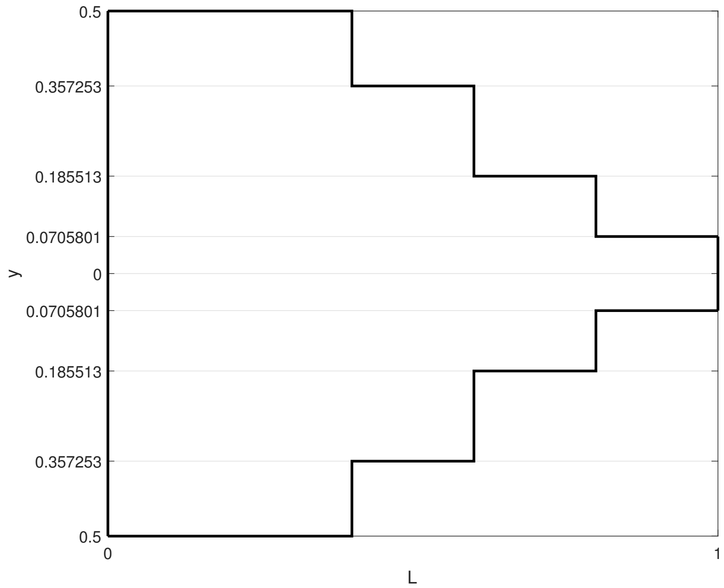


Figure 5. Optimal shape of a cantilever beam discretized into 5 elements.

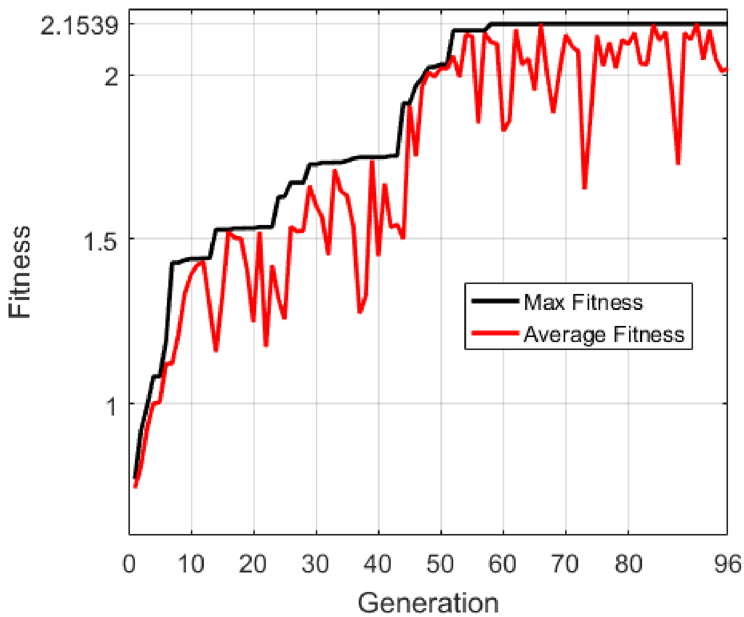


Figure 6. Maximum and average fitness of each generation of the first eigenvalue optimization of a cantilever beam discretized into 5 elements.

Note that the entire population evolves, as illustrated in Fig. 6. Figure 5 shows the best individual obtained by the algorithm. With only five elements, the FEM evaluation is not accurate. As the number of elements increases, the number of genes to be modified increases, the beam aspect and FEM accuracy improves. Initially, the problem of this work is a parametric optimization problem, but with many elements in the discretization, GA ends up finding the optimal shape of the beam. This is solve a problem of shape optimization with parametric optimization.

Consider a cantilever beam with the same properties as the previous beam, now discretized into 120 elements. Changing the number of offsprings generated to 29 and modifying the stopping criteria to cease when there are 100 consecutive generations with max fitness unchanged, the following format with optimized FNF is shown in Fig. 7:

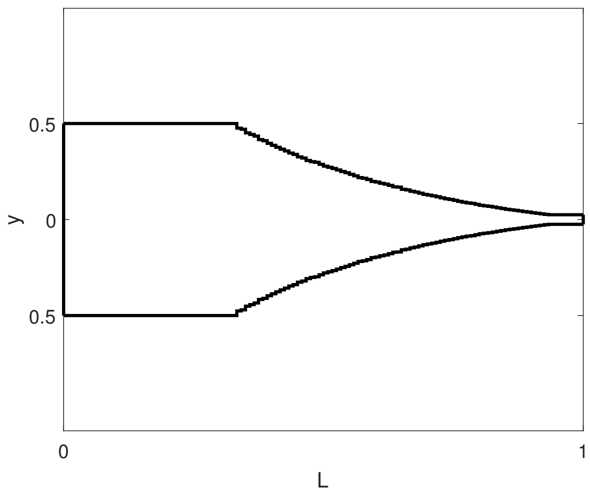


Figure 7. Optimal shape of a cantilever beam.

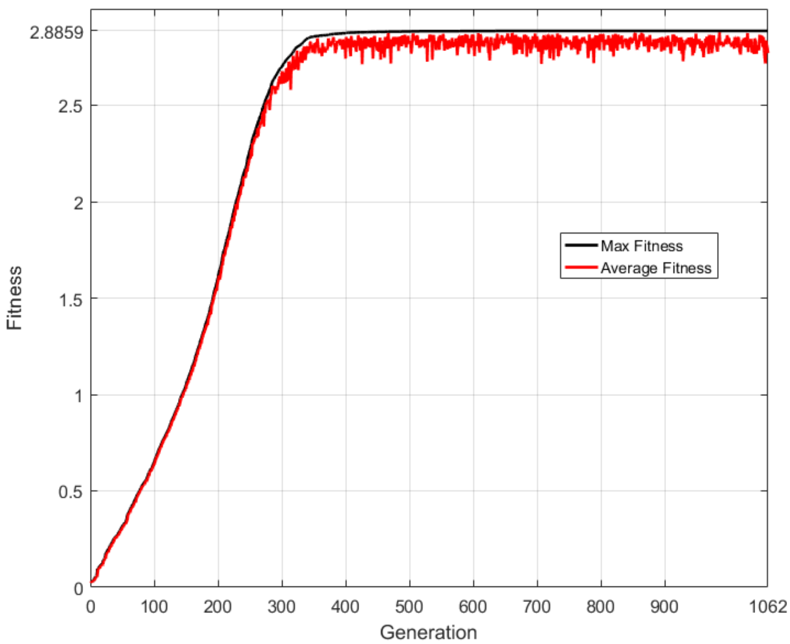
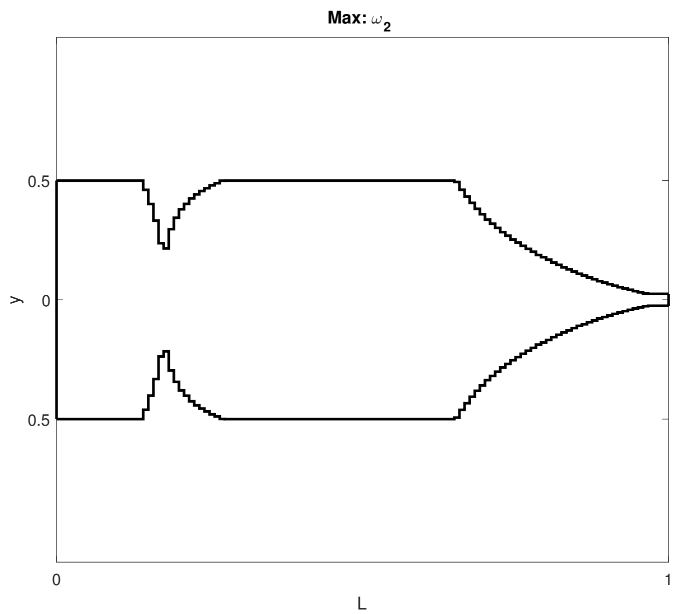
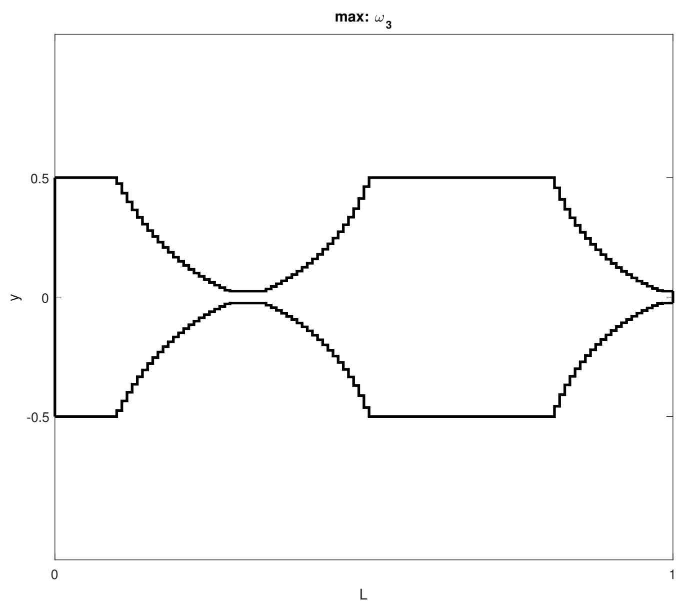


Figure 8. Optimal shape of a cantilever beam.

Figure 8 shows the maximum and the average fitness of each generation. The increase of the number of generations necessary to reach the stopping criteria in comparison to the five elements beam is because the increase in the number of genes to be optimized, so the greater the number of generations needed. When we choose other natural frequency to be maximized as objective function, the shapes obtained are different, as illustrated in the Fig. 9.



(a)



(b)

Figure 9. Different shapes obtained of a cantilever beam with (a) second and (b) third natural frequency maximized.

3.2 Clamped-Clamped Beam

Consider a clamped-clamped cylindrical beam, with $E = 1$, $\rho = 1$, $D = 1$, and $L = 1$ as shown in Fig. 10

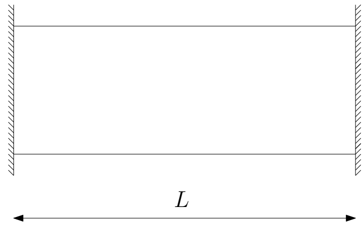


Figure 10. Clamped-Clamped.

Using the genetic algorithm to maximize the first three natural frequencies, we found the shapes shown in Fig. 11 and Table 1 shows the increase in the natural frequencies. Notice that increase is very significant.

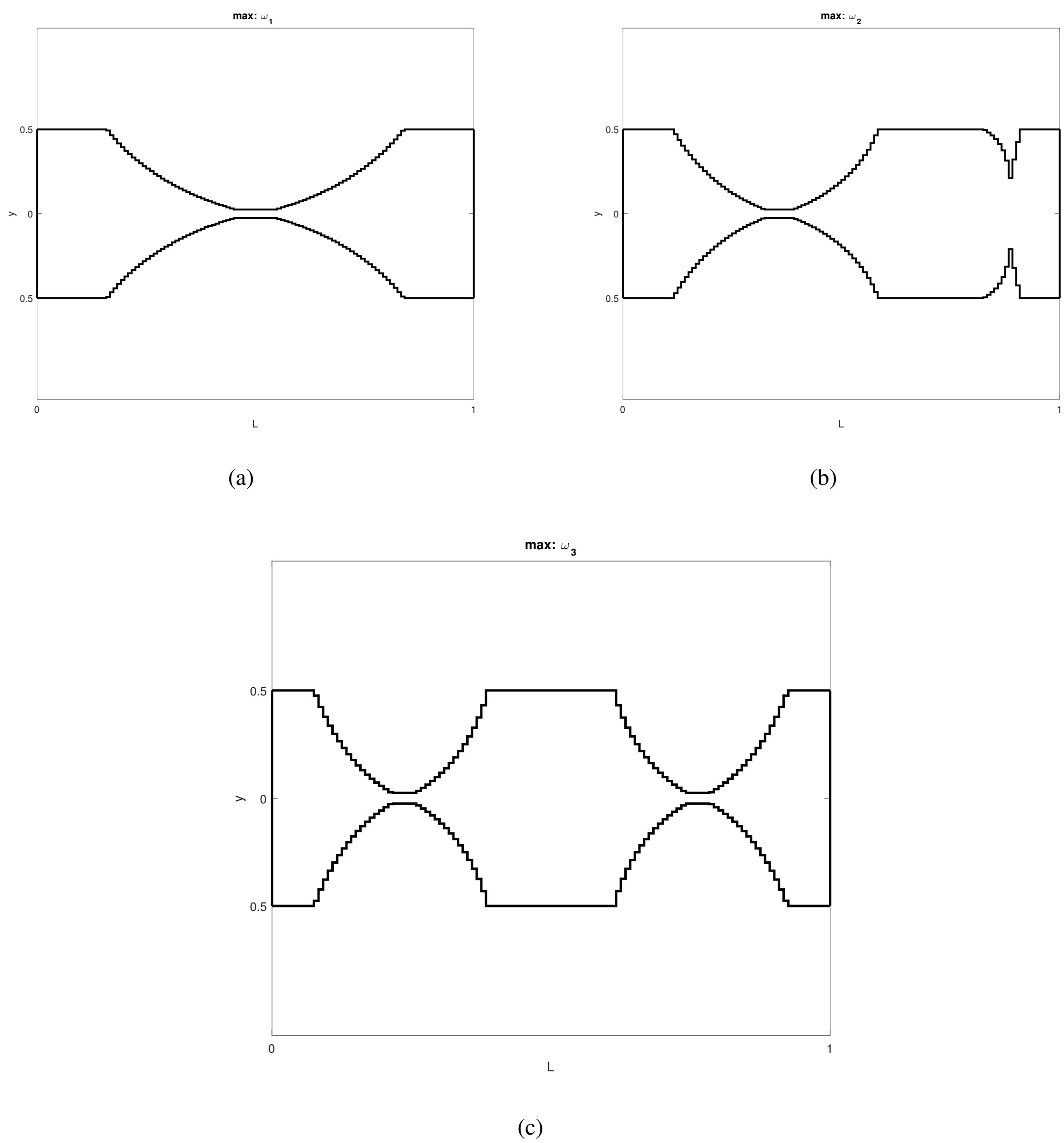


Figure 11. Different shapes obtained of a clamped-clamped beam with (a) first (b) second and (c) third natural frequency maximized.

Table 3. Comparison between optimized shapes and a prismatic beam (Clamped-Clamped)

Natural Frequency (rad/s)	Prismatic Beam	Optimized Shape	Increase
ω_1	5.5933	11.9581	113.79%
ω_2	15.4182	23.3542	51.47%
ω_3	30.2258	53.4267	76.76%

3.3 Hinged-Hinged Beam

Now, consider a hinged-hinged beam with the same properties, as shown in the Fig. 12:

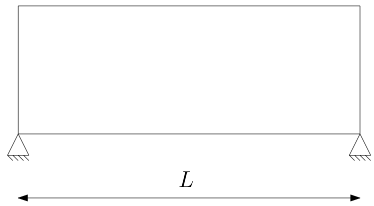


Figure 12. Hinged-Hinged Beam

An interesting result occurs: the cylindrical shape, illustrated in Fig. 13, optimises all the first three natural frequencies . The optimal values of natural frequencies are $\omega_1 = 2.4674\ rad/s$, $\omega_2 = 9.8696\ rad/s$ and $\omega_3 = 22.2066\ rad/s$.

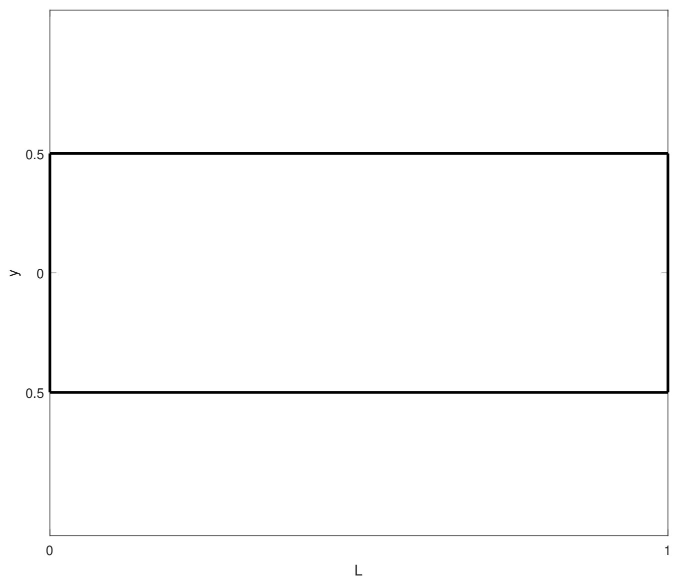


Figure 13. Optimized shape of a hinged-hinged beam that maximizes ω_1, ω_2 and ω_3.

3.4 Clamped-Hinged Beam

For a clamped-hinged beam, the algorithm find the shapes shown in Fig. 15. Table 4 shows the increase in the natural frequencies. We observed that the algorithm succeeded in maximizes natural frequencies in any boundary condition.

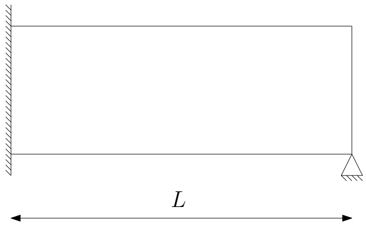
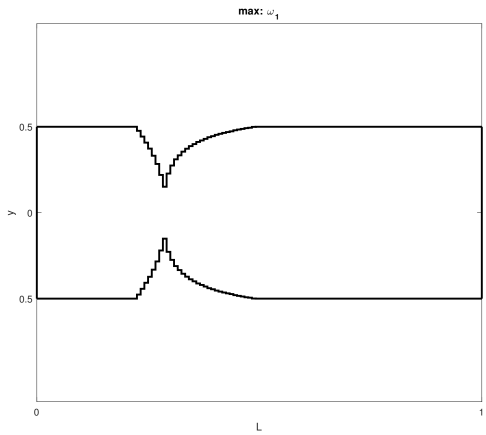
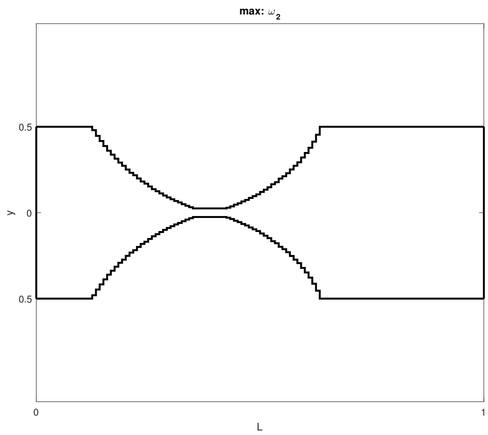


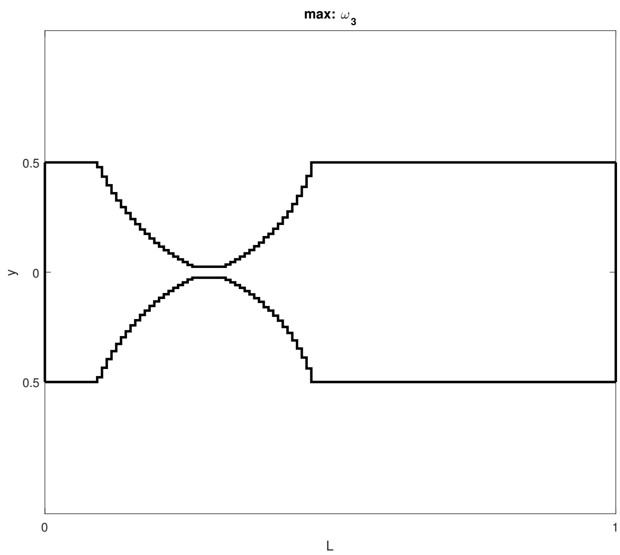
Figure 14. Clamped-Hinged Beam



(a)



(b)



(c)

Figure 15. Different shapes obtained of a clamped-hinged beam with (a) first (b) second and (c) third natural frequency maximized.

Table 4. Comparison between optimized shapes and a prismatic beam (Clamped-Hinged)

Natural Frequency (rad/s)	Prismatic Beam	Optimized Shape	Increase
ω_1	3.8546	3.9192	1.6759%
ω_2	12.4912	19.7206	57.876%
ω_3	26.0619	36.1026	38.526%

4 Conclusion

This paper uses a genetic algorithm together with the finite element method to optimise the shape of beams to maximize eigenvalues in classical boundary conditions. Initially, a finite element is formulated using the Rayleigh-Ritz method to find polynomial equations to approximate the displacements and slopes and natural frequencies are obtained from the equations of motion, solving a generalized eigenvalue problem. A customized GA is developed to find out the best combination of diameters of the mesh of finite elements that maximizes some eigenvalue. A cantilever beam with domain discretized into 5 elements is used to be a simple example about how the code works and a graphical representation of the evolution of the population is illustrated. Are obtained optimal shapes for 4 classical boundary conditions: cantilever, clamped-clamped, hinged-hinged and clamped-hinged for the first three natural frequencies. Results shows a great increase in the natural frequencies maximized.

5 Permission

The authors are the only responsible for the printed material included in this paper.

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