

## **A NUMERICAL INVESTIGATION ON THE INTERACTION BETWEEN GLOBAL AND LOCAL BUCKLING MODES IN CASTELLATED BEAMS**

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**Abstract.** The present work aims to investigate the interaction between lateral torsional buckling and compression 'tee' local buckling in castellated beams. Finite element method (FEM) is used to perform a parametric analysis for Litzka-beams subject to pure bending moment and considering combinations of flange-to-web width and thickness ratios and unbraced lengths. To account for the possibility of different yield strengths, non-dimensional local and global slenderness are used to assess the behavior in a comprehensive manner. The responses for beams having different combinations of slendernesses are compared and the relative strengths are discussed. The FEM ultimate bending moments are compared to those calculated according to the current design recommendations for castellated beams, showing that these may either over or underpredict actual capacities. Finally, a direct strength method approach is tested for the prediction of the nominal bending strength.

**Keywords:** Buckling interaction, Direct strength method, Castellated beams

## 1 Introduction

Castellated beams are steel I-section beams with hexagonal openings located at fixed distances along the web. The fabrication starts with a zigzag cut and then stagger the two halves and weld them back together. These structural members are subjected to special stability conditions due to their geometry when compared to plain webbed beams. It is important to understand their behavior and strength when interaction between lateral torsional buckling (LTB) and compression 'Tee' local buckling (TB) modes occurs.

Lateral torsional buckling resistance of castellated beams was the topic of a previous study conducted by Sonck [1], but it did not take into consideration the possibility of a local buckling mode. Ellobody [2] investigated the behavior of normal and high strength castellated steel beams under lateral torsional and distortional buckling modes considering initial imperfections and material nonlinearities and concluded that the interaction of these modes decreases the failure load of slender castellated steel beams. Oliveira et al. [3] studied the elastic local buckling critical stress of castellated beams subjected to pure bending considering the interaction between flange and web. Bezerra [4] proposed a procedure for determination of nominal resistance for lateral torsional buckling in castellated beams.

The present work aims to investigate the interaction between LTB and TB in castellated beams. The scope of the study is limited to laterally unbraced Litzka-type beams, which parameter are presented in Fig. 1, subjected to pure bending, with geometric initial imperfection and considering the material nonlinearity.

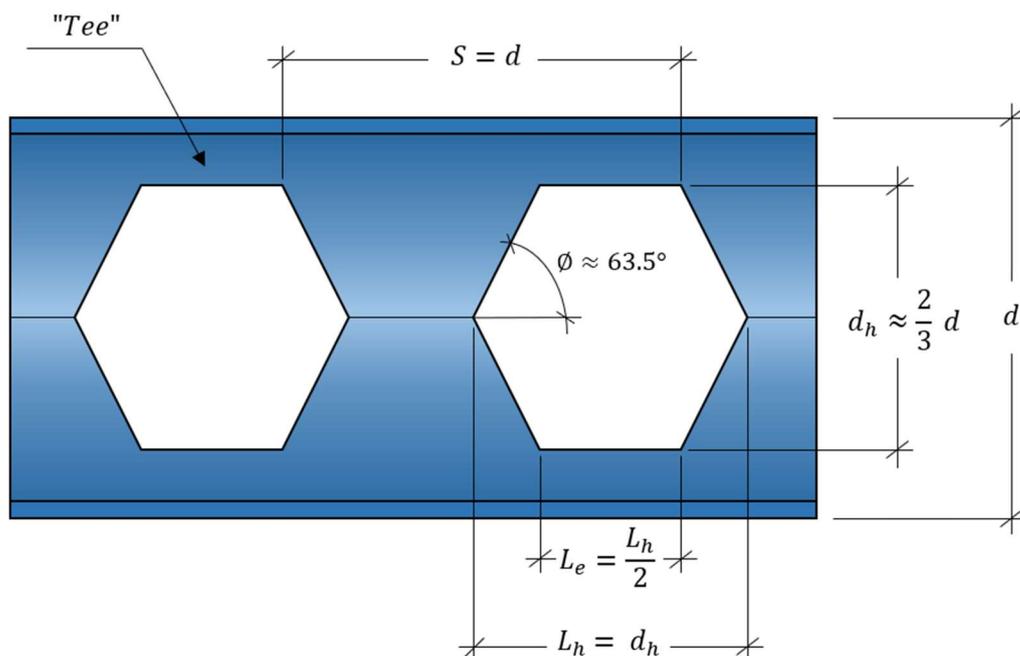


Figure 1 - Litzka-type beams parameters

## 2 Direct Strength Method (DSM) and Design Guide 31

### 2.1 DSM

The modern Direct Strength Method [5], present in the North American Specification for the Design of Cold-Formed Steel Structures AISI NAS S100 [6], is employed to assess the strength of plain webbed members under pure compression or pure bending, considering the interaction between buckling modes. In fact, the fundamental concept behind DSM consists in the fact that the member strength can

be determined directly as a function of elastic buckling loads for the different modes – i.e. local, global and distortional – and yield strength. In the present work, focus is given on the procedure recommended in AISI code to analyze beams subjected only to interaction between global and local buckling. The DSM approach is not applicable to cold-formed steel members with holes, but the present work analyzes how the method behaves for castellated beams.

To use DSM equations to determine beam strength, it is necessary to determine associated critical moments, which can be done using Finite Strip Method (FSM), Finite Element Method (FEM), Generalized Beam Theory (GBT) or theoretical elastic buckling solutions. For the specific case of a castellated beam, FEM seems to be more appropriate, as it allows the introduction of discontinuities such as holes and openings along the member length.

The first step of the method is to determine the nominal Euler moment (global buckling),  $M_{ne}$ , using the Eq. (1), that correlates the critical global buckling moment,  $M_{cre}$ , and the moment at yielding,  $M_y$ , which the ratio  $M_y$  to  $M_{cre}$  is related to global slenderness:

$$\begin{aligned} \text{If } M_{cre} < 0.56M_y & \quad Mn_e = M_{cre} \\ \text{If } 2.78M_y \geq M_{cre} \geq 0.56M_y & \quad Mn_e = \frac{10}{9}M_y\left(1 - \frac{10M_y}{36M_{cre}}\right) \\ \text{If } M_{cre} > 2.78M_y & \quad Mn_e = M_y \end{aligned} \quad (1)$$

Then, the nominal local moment,  $M_{nL}$ , must be calculated using Eq. 2. This equation correlates the nominal global moment calculated in the previous step and critical local buckling,  $M_{crL}$ , as follows:

$$\lambda_{L,DSM} = \sqrt{\frac{M_{ne}}{M_{crL}}} \quad (2)$$

$$\begin{aligned} \text{If } \lambda_L \leq 0.766 & \quad Mn_L = Mn_e \\ \text{If } \lambda_L > 0.766 & \quad Mn_L = Mn_e\left(1 - 0.15\left(\frac{M_{crL}}{Mn_e}\right)^{0.4}\right)\left(\frac{M_{crL}}{Mn_e}\right)^{0.4} \end{aligned} \quad (3)$$

Where  $\lambda_{L,DSM}$  is the local slenderness for the DSM and  $M_{crL}$  is the critical local moment.

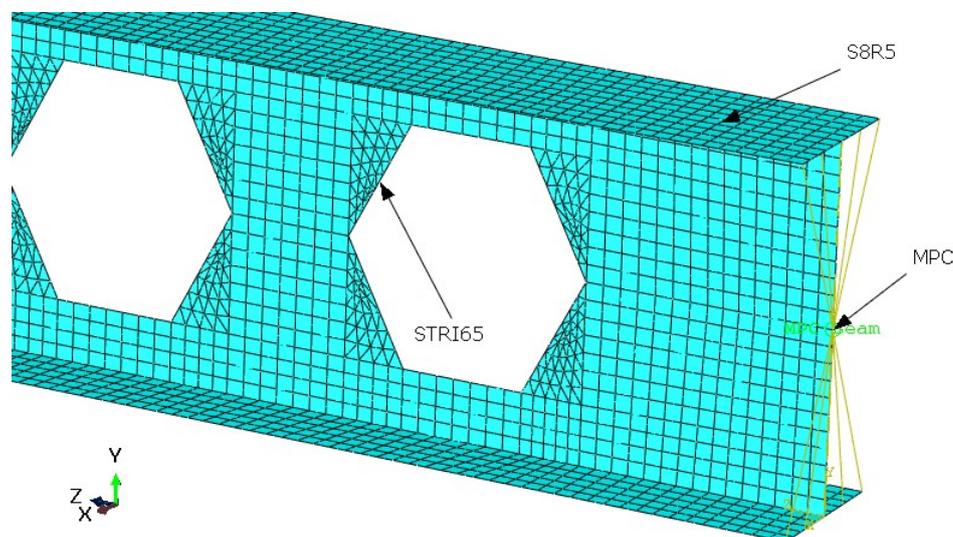
## 2.2 Design Guide

AISC Design Guide 31 – Castellated and Cellular beam design (DG) [7] is a document recently published by the American Institute of Steel Construction (AISC) that provides the state of the practice for the design of castellated beams in the United States. The guide conforms to the 2016 AISC Specifications [8] and covers the new failure mechanisms that differ from that of plain-webbed beams such as Vierendeel mechanism, web post buckling, buckling of compression tee and lateral-torsional buckling. For the present work purpose, ultimate moments are calculated only for LTB and the compression ‘tee’ local buckling. The DG presents sections to assess the LTB and TB separately, but doesn’t mention the interaction between modes.

## 3 Finite Element modeling

In this study, the general-purpose FE software package ABAQUS is used to perform a parametric analysis for Litzka-beams subject to pure bending moment and considering combinations of flange-to-web width and thickness ratios and unbraced lengths. The models were developed using quadrangular 8-node thin shell elements with reduced integration and 5 degrees of freedom per node (S8R5) and 6-

node triangular thin shell elements, using 5 degrees of freedom per node (S8R5) as shown in Fig. 2. A mesh convergence study, with a margin of tolerance of approximately 2%, was performed for all FE models presented herein. A multipoint constraint, which imposes a relationship between two or more degrees of freedom, was employed to ensure the application of the pure bending.



**Figure 2 - Mesh elements and multipoint constraint**

### 3.1 Validation

The modeling techniques employed in this work were validated by comparing the critical load and buckling mode obtained from the FE simulation with the experimental results from a previous experimental study of castellated beams web buckling in shear in a three-point bending test results developed by Redwood & Demirdjian [9]. The 10-7 specimen from their study was selected for the comparison. In this experiment, rollers were used at both ends and five points on the upper flange were laterally braced. The experimental critical load was obtained using the Southwell plot and is reported in Table 1. Figure 3 shows that the buckling mode found using the numerical model is the same as the one in the mentioned study. It can also be seen that a difference of 1.2% was obtained and, therefore, it can be considered validated.

Table 1. Validation results

Model	Critical Load (kN)	Error (%)
Redwood & Demirdjian	91.7	-
Current work	92.8	1.2

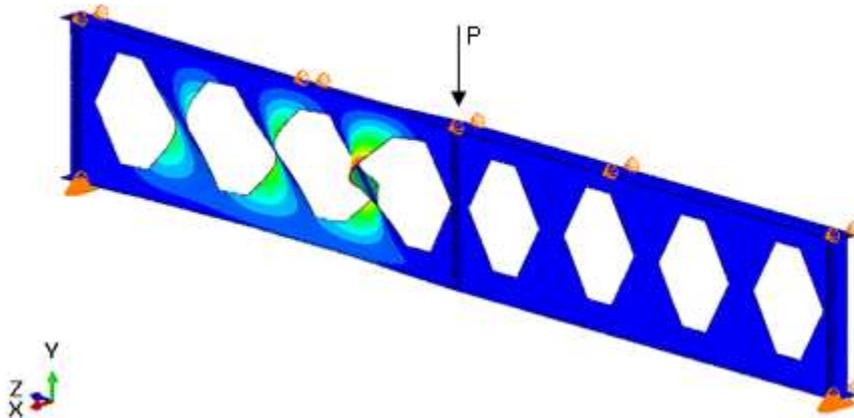


Figure 3- Boundary conditions, load and first buckling mode of the validation model

### 3.2 Parametric Study

For the purpose of evaluating the interaction between lateral torsional buckling and compression ‘tee’ local buckling, a parametric study was conducted varying parameters such as the unbraced length and the flange thickness. To perform this study, linear buckling and fully non-linear analyses were carried out in order to obtain the critical bending moments for both local ( $M_{crL}$ ) and global ( $M_{crG}$ ) buckling modes and the ultimate bending moment ( $M_u$ ), respectively. For the non-linear analysis, an initial geometric imperfection factor was assumed as a combination of the global and local buckling shapes, with amplitudes assumed as  $L/1000$  and  $H/500$ , respectively, where  $L$  is the beam length and  $H$  is the beam overall depth.

To account for the possibility of different yield strengths, non-dimensional local ( $\lambda_L$ ) and global ( $\lambda_G$ ) slendernesses were used to comprehensively assess the behavior. Another parameter used to evaluate the strength of each model is the ratio of ultimate bending moment to yield bending moment,  $\chi$ , the so-called relative strength. These parameters are defined in Equations 4 through 6.

$$\lambda_G = \sqrt{\frac{M_y}{M_{crG}}} \quad (4)$$

$$\lambda_L = \sqrt{\frac{M_y}{M_{crL}}} \quad (5)$$

$$\chi = \frac{M_u}{M_y} \quad (6)$$

In the model, a bilinear stress-strain curve without strain hardening and with a Young’s modulus of 200 GPa was adopted. The Poisson’s ratio was 0.3 and the yield stress was 690 GPa. A web thickness of 4.3 mm and beam depth of approximately 220 mm was assumed for all models.

To assess the influence of the global slenderness on the bending strength of the beam, the unbraced length was varied for a given cross-section (Models 1 2 and 7 in Table 2), i.e. keeping the local slenderness unchanged. On the other hand, to evaluate the influence of the local slenderness on the

strength, the flange thickness was varied for a given span (Models 2 to 6 in Table 2).

Table 2 - Geometric parameters for the parametric study

Model	Unbraced Length (mm)	Flange thickness (mm)
1	2444	4.9
2	3110	4.9
3	3110	4.3
4	3110	3.75
5	3110	3.35
6	3110	2.65
7	4000	4.9

## 4 Results and discussion

The yield moment ( $M_y$ ) is calculated using strength of materials principles, considering the neutral axis at the center of the castellated beam cross section, as shown in Table 3. This table also presents the results obtained from the eigenvalue analysis, the global and local slendernesses for each case studied.

Table 3 - Yield moment, global and local critical moments and slendernesses

Model	$M_y$ (kN.m)	$M_{crG}$ (kN.m)	$M_{crL}$ (kN.m)	$\lambda_G$	$\lambda_L$
1	85.9	121.0	132.0	0.84	0.81
2	85.9	76.7	132.0	1.06	0.81
3	77.1	67.0	101.7	1.07	0.87
4	69.1	58.3	79.6	1.09	0.93
5	63.4	52.0	65.3	1.10	0.99
6	53.5	41.1	42.1	1.14	1.13
7	85.9	48.0	132.0	1.34	0.81

From fully non-linear analysis, it was possible to obtain not only the ultimate moments for each model but also the failure modes, as shown in Table 4. The Table also presents the ultimate moments calculated using the Design Guide 31 and the DSM approaches, as well as the relative capacities for each method.

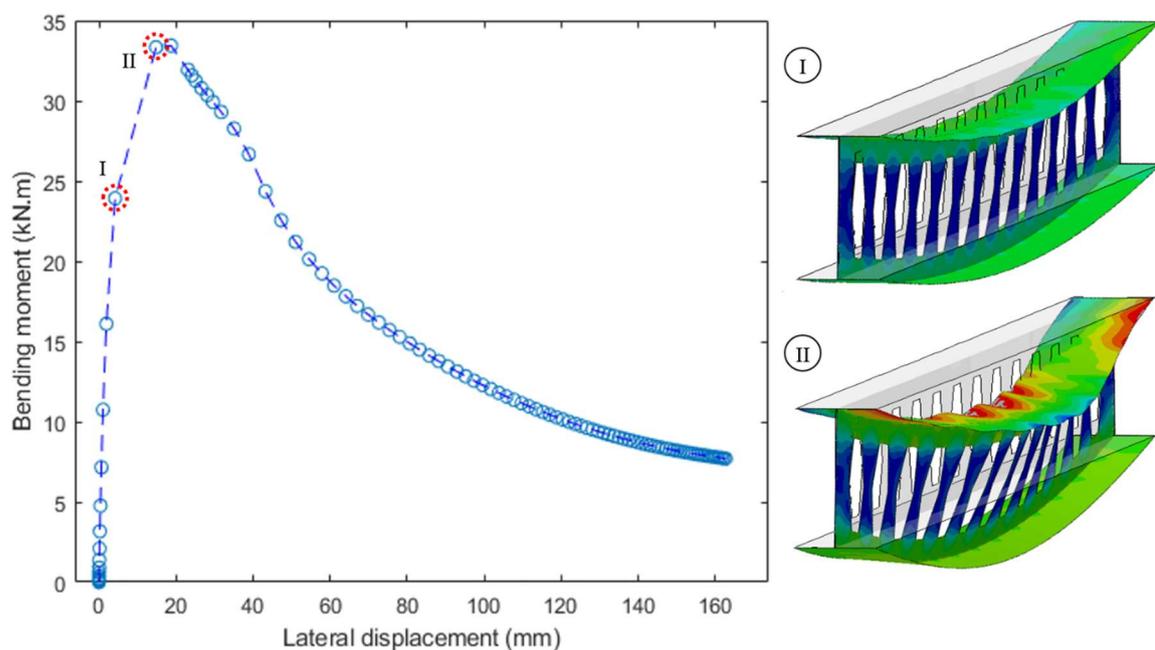
Table 4 - Ultimate moments, ratio of ultimate bending moment to yield bending moment, and failure modes

Model	$M_{uFEM}$ (kN.m)	$M_{uDSM}$ (kN.m)	$M_{uDg}$ (kN.m)	$\chi_{FEM}$	$\chi_{DSM}$	$\chi_{DG}$	Failure modes
1	77.9	76.6	63.4	0.90	0.89	0.73	Y - LTB - TB
2	61.2	65.8	49.9	0.71	0.76	0.58	LTB/Y - TB
3	53.8	61.4	44.7	0.69	0.79	0.57	LTB - Y - TB
4	46.8	53.6	40.9	0.67	0.77	0.59	LTB - Y - TB
5	41.7	47.4	38.2	0.65	0.74	0.60	LTB/TB/Y
6	31.0	35.2	33.3	0.57	0.65	0.62	LTB/TB - Y

7	41.9	48.0	37.6	0.48	0.55	0.43	LTB - Y - TB
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\* FEM, DSM and DG refer to Finite Element Method, Direct Strength Method and Design Guide 31. In the Failure modes column, Y means yielding failure mode, '/' means interaction between failure modes and '-' means that the follow failure mode happened in sequence.

According to Tables 3 and 4, it can be noticed that the Design Guide 31 is conservative for all cases tested in this work, except for model 6, where both global and local slendernesses are higher than 1 and the failure mode noticed by the authors was an interaction of LTB and TB followed by yielding. It also can be observed that model 5, for which both slendernesses are close to 1, failed by an interaction of LTB, TB and yielding. Figure 4 shows the failure mode of model 5 in the bending moment versus lateral displacement graph, where it is possible to note, in two distinct moments (I and II), the interaction between LTB, TB, with a smooth deformation of the upper flange, and, in red and gray, the upper flange yielding.



**Figure 4 – Lateral displacement versus bending moment graph and deformed model 5 in two distinct moments of the analysis**

Comparing the FEM results to the DSM results, it is important to mention that DSM ultimate bending moments were overpredicted in all cases analyzed in the present work, except for the model 1, where the  $M_{uDSM}$  was slightly lower than the  $M_{uFEM}$ . This analysis indicates that the current DSM approach for cold-formed steel beams cannot be extended for castellated beams. It also can be observed that, as  $\chi$  is closer to 1, the failure mode starts with yielding of the material. Figure 5 shows how the relative strength decreases as the beam becomes slenderer and it can also be seen that DG underpredicts the strength for the numerical models presented.

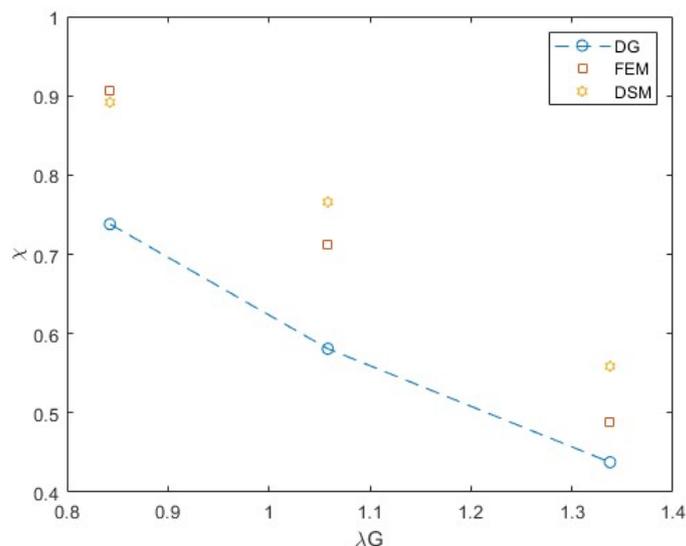


Figure 5 -  $\lambda_G$  versus  $\chi$ ,  $\lambda_L = 0.81$

## 5 Conclusions

The present work investigated the interaction between lateral torsional buckling and compression ‘tee’ local buckling in castellated beams. The finite element method was used to perform a parametric analysis for Litzka-type beams subject to pure bending moment and considering combinations of flange-to-web width and thickness ratios and unbraced lengths. Non-dimensional local and global slenderness were used to account for the possibility of different yield strengths and to assess the behavior in a comprehensive manner.

Linear buckling analyses were made to obtain critical moments and buckling modes and the respective local and global modes were applied to the fully non-linear analyses as a factor of initial imperfection and a bilinear stress-strain curve without strain hardening was implemented to achieve the ultimate moments for each model. The ultimate bending moments from finite element analyses were compared to ultimate bending moments from DSM and from DG.

It was observed that the Design Guide 31 underpredicts the strengths for all the models in the present work. On the other hand, the DSM overpredicted the strengths for all the models with the exception of model 1, which was governed by yielding. Finally, it may be concluded that DG can predict the strength of the studied castellated beams with safety and also further research on DSM should be done to predict the behavior of the cross sections present in this study.

## Acknowledgements

The first author would like to acknowledge Mr. João Braga, Mr. Daniel Linhares, Ms. Gabriela França and Mr. Christovam Weidlich for their support during the development of this study.

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