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# DYNAMIC INSTABILITY OF WIND TOWERS

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**Abstract.** In this work the dynamic instability of a coupled tower-blade wind turbine system is investigated numerically. For this, the linear Euler-Bernoulli theory is used to describe the coupled tower-blade system and considering the lateral acceleration of the nacelle at the top of the tower, which is the base of the flexible blade. The Galerkin method is applied to obtain a set of linear ordinary differential equations of dynamic equilibrium which are, in turn, solved using the runge-kutta method. First, the coupled eigenvalues are obtained considering several blade rotational speeds and both the veering and instability phenomena are studied. Second, an external harmonic load is applied to the system and the linear dynamic behavior is analyzed. A parametric analysis is performed to observe their influence on the linear oscillations of the system. Obtained results show that the tower-blade system can show instability or veering regions mainly due to blade rotation and, it is possible to observe jumps or large amplitude oscillations due to external loads

**Keywords:** Dynamic instability; Wind turbine; Coupling, Veering, Tower-blade

# 1 Introduction

Currently, several countries invest in various kinds of energy sources, this is because nonrenewable energy is running out (such as oil and coal); in the case of wind, Brazil has a high potential, being a source that will get more investments over the years.

The present work fits in the structure analysis aiming to study the dynamic stability of the wind tower. Wind towers may present in their structure dynamic instability due to the coalescence effect of the tower and shovel system, a phenomenon that occurs because the tower and blade present natural frequencies that approach or join. The dynamic study of wind towers has been widely approached, such as Chen et al. [1] showed that when considering a coupled tower and blade system, and another considering the blade and nacelle masses at the top of the wind tower, the coupled system presents a much larger displacement at the top of the tower in relation to the uncoupled case. Murtagh et al. [2] demonstrated that increasing the rotational effect of the blades increases the natural frequencies. Increasing the rotational frequency decreases the tower's time responses and the blades have greater rigidity when this happens. Kang et al. [3] made a dynamic analysis of a coupled blade and tower experimentally and analytically. They found the eigenvalues of the dynamic system and showed that the frequencies coalesce as the vibration mode of the tower and blade approaches. With the experimental analysis validated the numerical and experimental results, showing good agreement between them.

Through the study it is expected to prove that instability occurs when the natural frequencies of the system approach or join, analyzing the displacement of a point over time, showing the instability of displacement. The objective is to study the dynamic instability of the wind tower, to obtain responses in the time of displacement and to make parametric analyzes of the tower-blade system for instability.

# 2 Mathematical Formulation

Consider a wind tower of circular section, height $H$, Young´s modulus $E_T$, moment of inertia $I_T$, cross section $A_T$ and density $\rho_T$, while the blade has length $L$, Young´s modulus $E_B$, moment of inertia $I_B$ and cross section $A_B$ and density $\rho_B$. The tower has vertical axis $z$ and transverse displacement field $v$, while the blade has vertical axis $x$ and transverse displacement field $u$ as seen in Fig.1. The rotation speed of the blade is $\Omega$, the centrifugal force on the blade is $F_c$, while the mass of the nacelle is $M_o$.

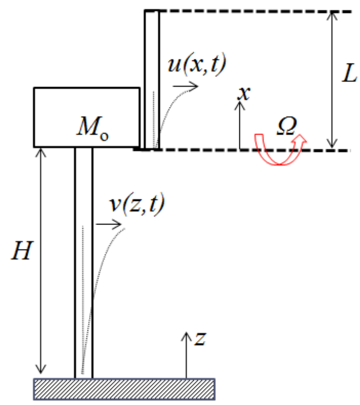



Figure 1 – Wind Tower Modeling.

In this work, the formulation is based on Kang et al. [3], Eq. (1) and Eq. (2) show, respectively, the dynamic equilibrium equations of coupled tower-blades system, considering the Euler-Bernoulli linear theory for beams.

$$E_T I_T \frac{\partial^4 v(z,t)}{\partial z^4} + \rho_T A_T \frac{\partial^2 v(z,t)}{\partial t^2} + \left[ (Mo + \rho_B A_B L) \left. \frac{\partial^2 v(z,t)}{\partial t^2} \right|_{z=H} + \int_0^L \rho_B A_B \frac{\partial^2 u(x,t)}{\partial t^2} dx \right] \delta(z-H) + c_T \frac{\partial v(z,t)}{\partial t} = 0 \tag{1}$$

$$E_B I_B \frac{\partial^4 u(x,t)}{\partial x^4} + \rho_B A_B \frac{\partial^2 u(x,t)}{\partial t^2} + \rho_B A_B \left. \frac{\partial^2 v(z,t)}{\partial t^2} \right|_{z=H} - \frac{\partial}{\partial x}\left( Fc(x) \frac{\partial u(x,t)}{\partial x} \right) + c_B \frac{\partial u(x,t)}{\partial t} = 0 \tag{2}$$

Where the damping of the tower and blade are respectively $c_T$ and $c_B$, where $c_T = 2\xi_T\, \omega_T\, A_T\, \rho_T$ and $c_B = 2\xi_B\, \omega_B\, A_B\, \rho_B$. The term $\xi$ is the critical damping for the tower and $\xi_B$ for the blade, whilst $\omega_T$ is the tower's natural frequency and $\omega_B$ is the paddle's natural frequency. In Eq. (1), $\delta$ corresponds to the Dirac delta. The harmonic displacements of the tower and blade are given by:

$$v(z,t) = V(z) e^{-i\omega t} \text{ and } u(x,t) = U(x) e^{-i\omega t} \tag{3}$$

The displacement $V(z)$ and $U(x)$ are given by:

$$V(z) = \sum_{j=1}^{n} b_j\, \psi_j(z) \text{ and } U(x) = \sum_{j=1}^{n} a_j \phi_j(x) \tag{4}$$

Where $\Psi(z)$ is the tower shape function and $\phi(x)$ is the blade shape function, $a_j$ and $b_j$ are constants. The field displacements for the tower and blades, are given in by (Kang et al. [3]).

$$\psi_j(z) = 1 - \cos\left[\frac{(2j-1)\pi z}{2H}\right] \text{ e } \varphi_j(x) = 1 - \cos\left[\frac{(2j-1)\pi x}{2L}\right] \tag{5}$$

In addition to the form equations presented in Eq. (5), the field displacement can also be described as in Eq. (6) and Eq. (7) (Rao [4]).

$$\psi_j(z) = \cos(\beta_j z) - \cosh(\beta_j z) - \frac{\cos(\beta_j H) + \cos(\beta_j H)}{\text{sen}(\beta_j H) + \text{senh}(\beta_j H)} \left( \text{sen}(\beta_j z) - \text{senh}(\beta_j z) \right) \tag{6}$$

$$\phi_j(x) = \cos(\beta_j x) - \cosh(\beta_j x) - \frac{\cos(\beta_j L) + \cos(\beta_j L)}{\text{sen}(\beta_j L) + \text{senh}(\beta_j L)} \left( \text{sen}(\beta_j x) - \text{senh}(\beta_j x) \right) \tag{7}$$

The values for $\beta_j H$ and $\beta_j L$ are constants for the free vibration of cantilever beams and can be found in (Rao [4]). The centrifugal force of the blade in Eq. (2) is described by:

$$Fc(x) = \int_x^L \rho_B A_B s \Omega^2 ds \tag{8}$$

This work also considered an external harmonic force, $F_v$, applied to the tower looking to evaluate the time response of the system, for this the force was represented according to Eq. (9), where $P$ represents the amplitude of force at the top of the tower and $\Omega_f$ represents the frequency of the force.

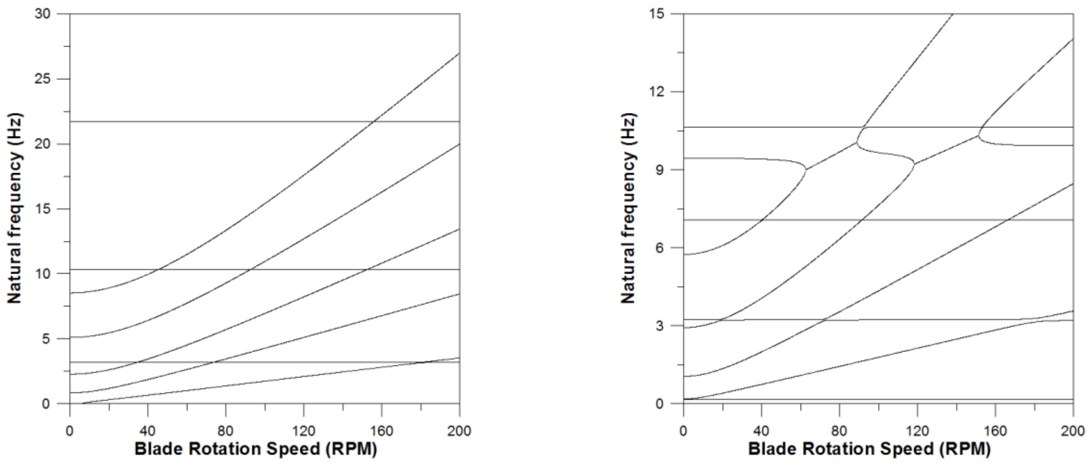
$$F_v = \frac{P z^2 \cos(\Omega_f t)}{H^2} \tag{9}$$

The Galerkin method is applied to obtain a set of ordinary dynamic equilibrium equations, in free vibrations, it allows to obtain the natural frequencies of the system and, when forced externally, it can be integrated numerically using the Runge-Kutta method.

# 3 Numerical Results

## 3.1 Natural Frequencies

A tower with height $H$ = 46 m, density $\rho_t$ = 7850 kg/m³, internal diameter of 1.49 m and external diameter of 1.50 m and Young´s modulus $E_t$ = 210 GPa, while the blade with length $L$ = 22 m, density $\rho_B$ = 2770 kg/m³, thickness 0.1 m, width 0.5 m, Young´s modulus $E_b$ = 69 GPa with zero damping of both tower and blade and no external load, and mass of nacelle $M_o$ = 30000 kg. The natural frequency values of the coupled system for these conditions are shown in Fig. 2. Were analyzed five modes of vibration, and coupled system results in two eigenvalues. Where natural frequencies approach or come together, they correspond respectively to the phenomena of veering and coalescence, where dynamic instability occurs, that is, they will cause large displacement vibrations. It is at these points that the responses in time must be analyzed to prove instability.




(a) (b)

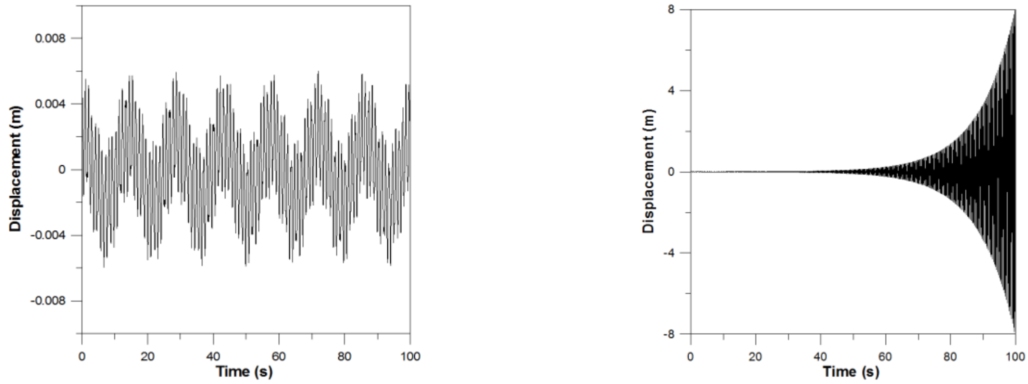
Figure 2 – Frequency values for coupled tower-blade system. (a) Considering Eq (5). (b) According to Eq. (6) and Eq. (7).

## 3.2 Response in time

All responses in time were made considering the analysis at the highest point of the blade, therefore, being $x = L$, and the fact that the blade is coupled to the tower adds this displacement with that of the tower so that $z = H$. In addition, Eq (5) was considered, therefore, the natural frequencies shown in Fig. 2a. The blade rotation speed values of 50 rpm and 73.5 rpm, respectively corresponding to Fig. 3a and Fig. 3b, were used. The blade rotational speed values were chosen to exemplify displacement before, during and after the natural frequency approximations shown in Fig. 2a. For the processing of the time response in these cases two vibration modes were used and because of this we chose the value of 73.5 rpm, which occurs when the first two vibration modes are approached. It is noticed that the displacement remains periodically at frequencies of 50 rpm but, with 73.5 rpm it grows indefinitely, becoming unstable when coupling occurs. So what makes it unstable is not to increase the rotational speed of the blade, but rather as the natural frequencies approach.

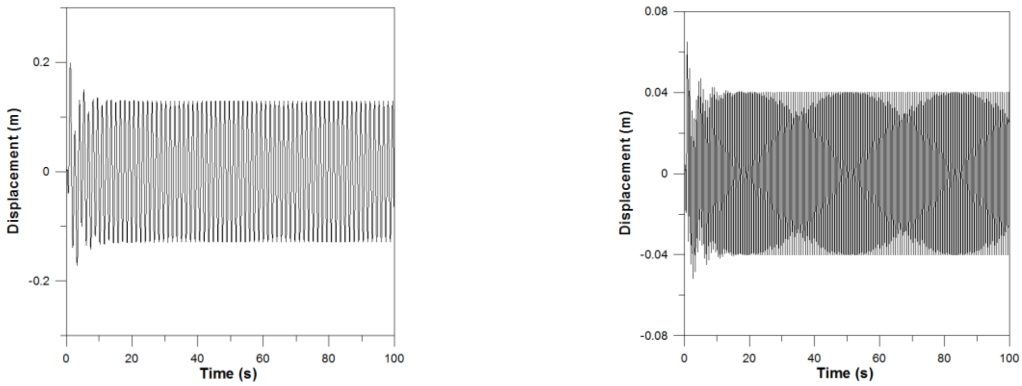
## 3.3 Response in time with force applied to the tower

The time response was analyzed for two different cases: $P$ = 10000 N with $\Omega_f = \omega_{T1}$ (Fig. 4a) and $P$ = 10000 N with $\Omega_f = 2\omega_{T1}$ (Fig. 4b) considering 1% of the damping coefficient in all modes, where $\omega_{T1}$ is the first natural frequency of the tower. Comparing the time response figures with force, it is observed that the application of force demonstrates greater displacement with the value of smaller $\Omega_f$, that is, with the value of the frequency of application of force in the largest tower, the displacement decreases.




(a) (b)

Figure 3 – Tower time response in coupled tower-blade system. (a) Ω = 50 rpm. (b) Ω = 73.5rpm.




(a) (b)

Figure 4 – Tower time response in coupled tower-blade system. (a) Considering $P$ = 10000 N and $\Omega_f = \omega_{T1}$. (b) Considering P = 10000 N and $\Omega_f = 2\omega_{T1}$.

# 4 Conclusion

It was possible to observe the phenomenon of "veering" and coalescence, where the system becomes unstable, thus showing the importance of the study of wind towers in the coupled system. Furthermore, it has been shown that when a harmonic wind force is applied, the transverse displacements of the tower increase and decrease depending on the wind frequency ratio as a function of the natural frequency of the system.

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