

## **EVALUATION OF NORMATIVE MODELS INTEGRATED TO NONLINEAR COMPUTATIONAL MODELS FOR SIMULATION OF STEEL-CONCRETE COMPOSITE BEAMS WITH PARTIAL INTERACTION**

**Tawany A. Carvalho**

**Samuel S. Louzada**

**Ígor J. M. Lemes**

*tawanyacarvalho@gmail.com*

*samuel.l.s@estudante.ufla.br*

*igor.lemes@ufla.br*

*Departamento de Engenharia, Universidade Federal de Lavras*

*Campus Universitário, Lavras, 37200-000, Minas Gerais, Brasil*

**Ricardo A. M. Silveira**

*ricardo@em.ufop.br*

*Departamento de Engenharia Civil, Universidade Federal de Ouro Preto*

*Campus Morro do Cruzeiro, Ouro Preto, 35400-000, Minas Gerais, Brasil*

**Abstract.** The present study refers to the composite steel-concrete beams analysis considering the various nonlinear effects inherent to the structural typology. These effects generate complexity to the design requiring computational methodologies for the accurate measurement of structural behavior. With good numerical efficiency, the Refined Plastic Hinge Method also stands out for its simplicity. This methodology will be used considering rotational pseudo-springs at the finite elements ends for the simulation of plasticity. However, this approach was developed for isotropic materials with elastic-perfectly-plastic behavior, implying loss of precision in the analysis of structures containing concrete in their composition. Furthermore, the effects of partial interaction can not be simulated by the inherently rotational behavior of the pseudo-springs. Thus, the introduction of the cracking and partial interaction effects will be approached through effective moment of inertia defined by normative criteria. The validation of the implementations will be done based on the comparison with numerical and experimental data present in the literature through several parameters extracted from the program.

**Keywords:** Cracking, Partial interaction, Steel-concrete composite beams, Concentrated plasticity.

## 1 Introduction

In the design of structural engineering projects, the appropriate choice of materials is fundamental for the elaboration of a good project. Among the most used materials in civil construction, steel and concrete stand out, and their association leads to better physical and mechanical use of materials. However, the steel-concrete composite structural system, despite having several advantages, can be unused because of the complexity of calculations and analysis. Thus, the use of computational methodologies is highlighted [1, 2], because they are directly linked to the design codes simplifications [3], facilitating the checks required for a safe structure project.

In the computational context, several methodologies are used to promote efficient analysis. Among them, the Refined Plastic Hinge Method (RPHM) [4], which deals with plasticity in a concentrated form, is one of the most used to evaluate the inelastic effect.

In the present work, it was chosen to use the platform *Computational System for Advanced Structural Analysis* (CS-ASA). Lemes [2] recently included the numerical changes that made possible the study of several typologies, such as reinforced concrete and steel-concrete composite structures.

The study done in the present paper aims to introduce normative equations in the CS-ASA program, to enable the analysis of steel-concrete composite beams with partial shear connection more simply and quickly, but without losing numerical precision. In this case, the process described in [3] for reducing the moment of inertia due to partial interaction, and the described in [5] for the degradation of this same property when cracking is considered, will be studied here.

## 2 Finite element formulation

In the present work, the displacement-based formulation with concentrated plasticity in the nodal points is applied. In this case, the axial and flexural stiffness degradation occurs exclusively at the FE nodes. Then, the method is presented, introducing the material nonlinearity only. Some considerations and simplifications of this formulation can be seen in [2, 4].

In the structural system modelling, the hybrid beam-column finite element of length  $L$ , delimited by nodal points  $i$  and  $j$  (Figure 1), is used. This element has zero-length pseudo rotational springs at its ends, which are responsible for the plasticity simulation by means of the parameter  $S_p$ , discussed in Section 3. The finite element is referenced to the co-rotational system where the degrees of freedom are the rotations at nodes  $i$  and  $j$ , given by  $\theta_i$  and  $\theta_j$ , and the axial displacement in  $j$ ,  $\delta$ . The terms  $M_i$ ,  $M_j$  and  $P$  represent the bending moments and the axial force in the respective degrees of freedom.

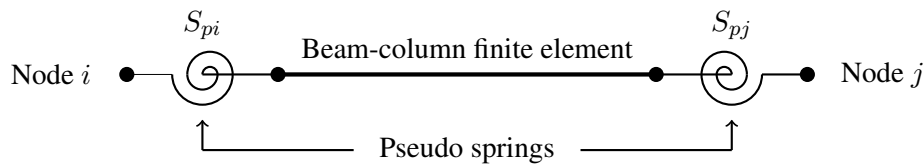


Figure 1. Finite element with pseudo-springs

$$\begin{Bmatrix} \Delta N \\ \Delta M_{pi} \\ \Delta M_{pj} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & S_{pi} - \frac{S_{pi}^2 (S_{pj} + k_{33})}{\beta} & \frac{S_{pi} k_{23} S_{pj}}{\beta} \\ 0 & \frac{S_{pj} k_{32} S_{pi}}{\beta} & S_{pj} - \frac{S_{pj}^2 (S_{pi} + k_{22})}{\beta} \end{bmatrix} \begin{Bmatrix} \Delta \delta \\ \Delta \theta_{pi} \\ \Delta \theta_{pj} \end{Bmatrix} \quad (1)$$

in which  $\beta = (S_{pi} + k_{22})(S_{pj} + k_{33}) - k_{32}k_{23}$ .

The terms  $k_{11}$ ,  $k_{22}$ ,  $k_{23}$ ,  $k_{32}$ , and  $k_{33}$  are components of the beam-column stiffness matrix element, without the pseudo-springs, described as [2]:

$$\begin{aligned} k_{11} &= \frac{E_s A}{L} & k_{22} &= \frac{E_s (3I_{eff,i} + I_{eff,j})}{L} \\ k_{23} = k_{32} &= \frac{E_s (I_{eff,i} + I_{eff,j})}{L} & k_{33} &= \frac{E_s (I_{eff,i} + 3I_{eff,j})}{L} \end{aligned} \quad (2)$$

where  $E_s$  is the steel modulus of elasticity,  $A$  is the homogenized area of the section,  $I_{eff}$  is the modulus of inertia as discussed on Section 4, measured in nodes  $i$  and  $j$ , and  $L$  is the finite element length.

### 3 Pseudo springs flexural stiffness

The limits of uncracked, elastic or plastic states are defined by the moment-curvature relationship [2]. In this nonlinear procedure, the initial cracking moment  $M_{cr}$ , the initial yield moment  $M_{er}$  and the full yield moment  $M_{pr}$  can be easily obtained.

According to the classical RPHM, three equations define the pseudo-spring stiffness for the previously mentioned bending moments. For a given axial force-bending moment combination,  $S_p$  is defined as follow:

$$\begin{aligned} \text{if } M \leq M_{er} : & \quad S_p = 1 \times 10^{10} \quad (\text{Elastic Range}) \\ \text{if } M_{er} \leq M \leq M_{pr} : & \quad S_p = \frac{E_s I_{eff}}{L} \left( \frac{M_{pr} - M}{M - M_{er}} \right) \quad (\text{Flexural Stiffness Degradation}) \\ \text{if } M_{pr} \leq M : & \quad S_p = 1 \times 10^{-10} \quad (\text{Fully Plastified Section}) \end{aligned} \quad (3)$$

in which  $L$  is the finite element length and  $E_s I_{eff}$  is the section's flexural stiffness, considering the cracking, as discussed below.

Note that, by the value described in Eq. 3, there is no possibility of simulating cracking and partial shear connection in the elastic regime. This adjustment is made in the following section.

### 4 Moment of inertia

Branson and Metz [6] proposed a simple equation for the effective moment of inertia evaluation of RC sections in a cracking state. This equation is used by some design codes, such as NBR 6118 [5]. The effective moment of inertia,  $I_{eff,c}$ , is given by:

$$\begin{aligned} \text{if } M \leq M_{cr} : & \quad I_{eff,c} = I_c \\ \text{if } M > M_{cr} : & \quad I_{eff,c} = \left( \frac{M_{cr}}{M} \right)^3 I_c + \left[ 1 - \left( \frac{M_{cr}}{M} \right)^3 \right] I_{cr} \quad I_{eff} \leq I_c \end{aligned} \quad (4)$$

where  $M_{cr}$  and  $M$  are, respectively, the initial cracking bending moment and the bending moment acting on the section,  $I_c$  is the intact section moment of inertia, and  $I_{cr}$  is the cracked moment of inertia of the section evaluated in the critical point of moment-curvature relationship [2].

Considering the partially conjunct action with concrete slab and steel section, the effective moment of inertia,  $I_{eff}$ , can be determined as a directly function of degree of interaction,  $\eta_i$ . Thus [3]:

$$I_{eff} = I_{steel} + \sqrt{\eta_i} (I_{tr} - I_{steel}) \quad (5)$$

in which  $I_{steel}$  and  $I_{tr}$  are moment of inertia of steel and homogenized cross sections, respectively. The homogenized moment of inertia is calculated by the direct relation of  $I_{eff,c}$  and  $I_{steel}$ .

## 5 Numerical application

In this section the numerical procedure described in this paper will be tested. Chapman and Balakrishnan [7] tested simply supported composite beams with partial interaction. In this analysis, the  $E1$  beam [7], illustrated in Fig. 2, is simulated using the proposed formulation. In this same figure, loads, geometry, FE meshes and the cross-section are showed. The partial shear connection is made by equally spaced 50 rows with a couple of stud-bolt connectors per row. The material data of this beam are presented in Tab. 1.

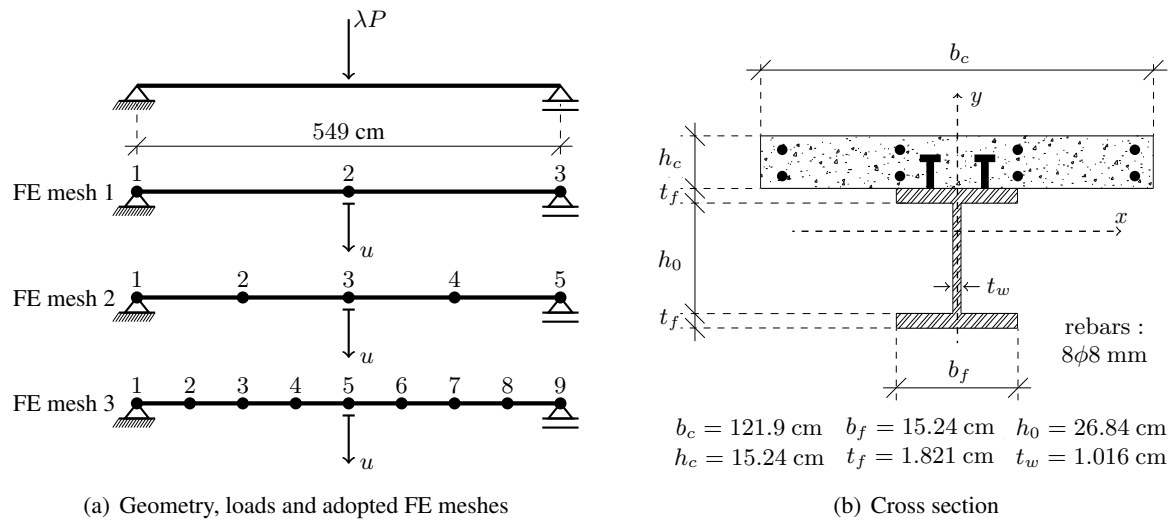


Figure 2. Simply supported beam with partial shear connection

Table 1. Material data of simply supported composite beam with partial interaction (in kN, cm)

Concrete		Connectors	Steel	Rebars
$f_c$	$\varepsilon_{ci}$	$\varepsilon_{cu}$	$\eta_i$	$f_{yr}$
3.268	-0.0022	-0.00395	0.929	32
			$E_s$	$E_{sr}$
			20200	20500

In Figure 3 the equilibrium paths for finite element meshes 1,2 and 3 are plotted and compared with the experimental results [7]. As can be seen in this figure, this formulation present a low mesh sensibility, being in all cases a good precision in initial stiffness and final bearing capacity. In this same figure it can be observed that after the beginning of the stiffness degradation the most refined meshes present a more rigid behavior.

## 6 Conclusions

This paper presents a concentrated plasticity-based formulation using the finite element method for material nonlinear analysis of steel-concrete composite beams with partial interaction. The classical Refined Plastic Hinge Method was applied considering the explicit modification of the effective moment of inertia. For this, the cracking effect of the slab was introduced by the Branson and Metz [6] propose. Associate to this, the moment of inertia was reduced by the degree of interaction of concrete slab and steel section.

The simply supported beam simulated in this paper presented consistent initial stiffness and final bearing capacity with the experimental data. It is important to highlighted that low refinement meshes were sufficient for a satisfactory global response in the tested example.

Thus the proposal of union of the classical RPHM with the effective moment of inertia equation, considering cracking and partial interaction, provided satisfactory results in the context of material

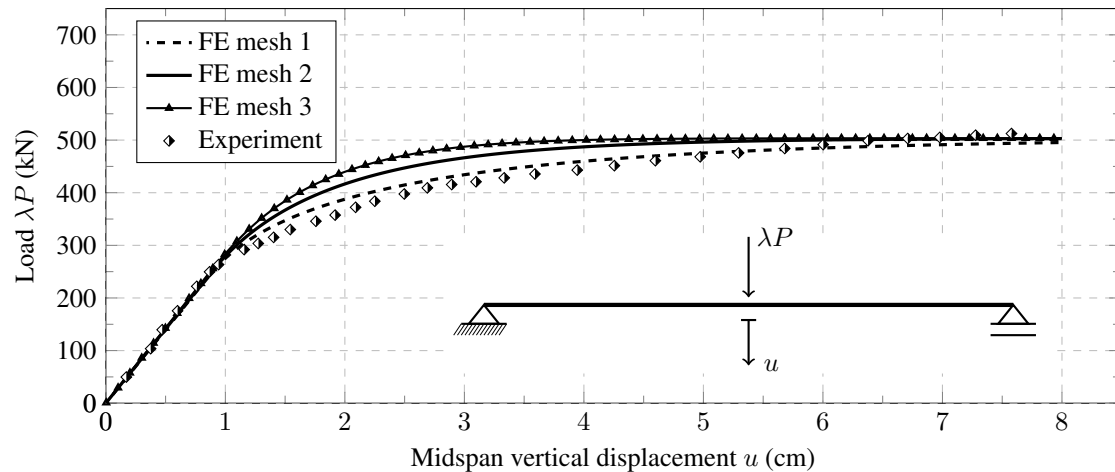


Figure 3. Equilibrium path of simply supported composite beam

nonlinear analysis of steel-concrete composite beams. But this results can be improve by the study of pseudo-springs stiffness degradation. This study will be in a next opportunity.

### Acknowledgements

The authors would like to thank CAPES and CNPq (Federal Research Agencies), Fapemig (Minas Gerais State Research Agency), UFLA and UFOP for their support during the development of this work.

### References

- [1] Liew, J., Chen, H., & Shanmugam, N., 2001. Inelastic analysis of steel frames with composite beams. *Journal of Structural Engineering*, vol. 127, n. 2, pp. 194–202.
- [2] Lemes, 2018. *Advanced numerical study of steel, concrete and steel-concrete composite structures*. PhD thesis, Federal University of Ouro Preto, Ouro Preto, Brazil.
- [3] NBR 8800, 2008. Projeto de estruturas de aço e de estruturas mistas de aço e concreto de edifícios. *Associação Brasileira de Normas Técnicas*.
- [4] Lemes, Í. J. M., Barros, R. C., Silveira, R. A. M., Silva, A. R. D., & Rocha, P. A. S., 2018. Numerical analysis of rc plane structures: a concentrated nonlinear effect approach. *Latin American Journal of Solids and Structures*, vol. 15, n. 2.
- [5] NBR 6118, 2014. Projeto de estruturas de concreto - procedimento. *Associação Brasileira de Normas Técnicas*.
- [6] Branson, D. & Metz, G., 1963. Instantaneous and time-dependent deflections of simple and continuous reinforced concrete beams. Technical report, Auburn: Dept. of Civil Engineering Auburn Research Foundation, Auburn University, Auburn: Dept. of Civil Engineering and Auburn Research Foundation, Auburn University.
- [7] Chapman, J. C. & Balakrishnan, S., 1964. Experiments on composite beams. *Structural engineers*, vol. 42, pp. 369–383.