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USE OF DIGITAL IMAGE CORRELATION AND FINITE ELEMENT METHOD TO SOLVE INVERSE PROBLEM USING DISPLACEMENT FIELDS.

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Abstract. One of the most common problems in solids mechanics is the determination of displacement, strain and/or stress fields of a sample, given the geometry, constitutive parameters and boundary conditions. This problem is known as Direct Problems, being solved numerically by techniques such as the Finite Element Method (FEM). In this work, however, it is explored how to solve the inverse problem. To calculate the constituent parameters of the sample, we use the displacement fields obtained with a Digital Image Correlation (DIC) algorithm. Then, FEM simulations are performed using the same geometry, loading, boundary conditions and an arbitrary set of elastic parameters (Young's Modulus and Poisson's Coefficient). Displacement data measured via DIC serves as a reference in an optimization algorithm that minimizes the difference between the FEM and DIC offset data set using the initial FEM simulation as kickstart and updating the set of constituent material parameters for subsequent iterations. Material parameters are obtained when the optimization is completed, and the two displacement fields are close enough. At the end of the work, numerical examples are presented and compared with those obtained by the Finite Element Method Updating (FEMU), which works with deformations states.

Keywords: Digital Image Correlation, Finite Element Method Updating, Inverse Problem, Constitutive Materials Properties.

1 Introduction

A common problem in solid mechanics is the estimation of the behavior of a specimen under stress given its geometric and constitutive parameters along with its boundary conditions. Behavior consists of displacement, strain and stress data, being of utmost importance to structural analysis. This problem is known as the direct problem and it is numerically solved with computational techniques such as the Finite Elements Method (FEM).

Experimentally, the specimen's reactions can be measured via a variety of methods. Some techniques are intrusive to the specimen, for instance speckle interferometry [1], moiré interferometry [2] and shearography [3]. Others are considered non-intrusive, like speckle [4] and grid method [5]. Between the latter, Digital Image Correlation (DIC) [6, 7, 8] stands out as a simple and inexpensive yet accurate way of measuring full-field displacement data. The method uses pairs of images taken from the specimen's plane surface under controlled lightning and other factors during different stages of loading.

The problem approached in this paper, however, is called the inverse problem or the identification problem [9, 10], as it utilizes a set of data measured by full-field methods like DIC to identify the elastic parameters of the specimen's material, being essentially the opposite of the direct problem. Evaluation of results is advantageous for both materials scientists and constructors who need to verify unknown aspects of a material or to validate a batch of construction materials.

Many solutions for this problem have been developed along the years, like the Virtual Fields Method [11], the Equilibrium Gap Method [12] and the Reciprocity Gap [13]. For this paper, the inverse problem is solved utilizing the Finite Elements Method Updating (FEMU) [14, 15, 16] in a displacement setup, supported by DIC measuring. It consists in an optimization of the difference between two sets of displacement data, the first measured by DIC and the second calculated by FEM using an arbitrary set of material parameters. The parameters are updated via Newton-Raphson model up to an iteration where the sets of displacement are sufficiently close to one another, meaning the adopted elastic parameters represent the real ones.

As a better way to validate the results, pairs of images used as input for DIC are computationally generated using an analytical model where the loading, geometric and elastic parameters are determinated.

2 Methodology

The procedure used in this work uses three stages: image generation, displacement measurement and mechanical parameters identification.

It starts with predetermining the loading and the constitutive parameters of the material. A pair of 1024x1024 images are generated via numerical model simulating a test. The images represent states before and after the loading. Both are created with a random speckled pattern in gray scale by a plugin in the software *Itom* [17].

Then, the images are analyzed with DIC algorithm, which is a full-field displacement nonintrusive measuring technique. Essentially, it can be translated as a minimization problem of a correlation coefficient between intensity functions of the original and deformed image plus an updating displacement field, according to the equation:

$$C_{ZNSSD} = \sum_{\Omega} \left(\frac{I(x,y) - I_m}{\Delta I} - \frac{I'(x+u, y+v) - I'_m}{\Delta I'} \right)^2 \tag{1}$$

also called Zero Normalized Sum of Squared Differences objective function. Intensity functions $I(x,y)$ and $I'(x+u, y+v)$ represent the color intensity in gray scale of a pixel given its position in the image, I_m and I'_m are the average intensity, ΔI and $\Delta I'$ are the standard deviation of the intensities in domain Ω as seen in [18]. Vector (u,v) is the displacement field for the coordinates (x,y), given by the interpolation of the nodal displacements using Q4 elements shape function. Each iteration updates those nodal displacements until the minimization process ends, resulting in the displacement field of the pair of images. The updating is evaluated by Newton-Raphson equation:

$$p = p_o - \frac{\nabla C(p_o)}{\nabla\nabla C(p_o)} \tag{2}$$

where p_o and p are the old and new nodal displacement vectors.

After DIC evaluation of the displacement field, parameters estimation process starts with a normalized sum of squared differences minimization process:

$$C_{NSSD}(p) = \sqrt{\sum_{i=1}^{n}\left(\left(\frac{u_i^{FEM}(p) - u_i^{DIC}}{u_i^{DIC}}\right)^2 + \left(\frac{v_i^{FEM}(p) - v_i^{DIC}}{v_i^{DIC}}\right)^2\right)} \tag{3}$$

between the displacement sets from DIC and FEM simulation with similar boundary conditions and a set of arbitrary constitutive parameters.

Considering an isotropic material, the set of elastic parameters p, comprised by the Young modulus (E) and the Poisson's ratio (ν), is updated each iteration by the following Newton-Raphson expression:

$$\Delta p = (S^t S)^{-1} S^t (u^{DIC} - u^{FEM}(p^k)) \tag{4}$$

where p^k is the former set of parameters and S is the sensitivity matrix, given by:

$$S = \begin{bmatrix} \frac{\partial u}{\partial E} & \frac{\partial u}{\partial \nu} \\ \frac{\partial v}{\partial E} & \frac{\partial v}{\partial \nu} \end{bmatrix} \tag{5}$$

Overall, the FEMU procedure implemented for this paper works according to the flow chart:

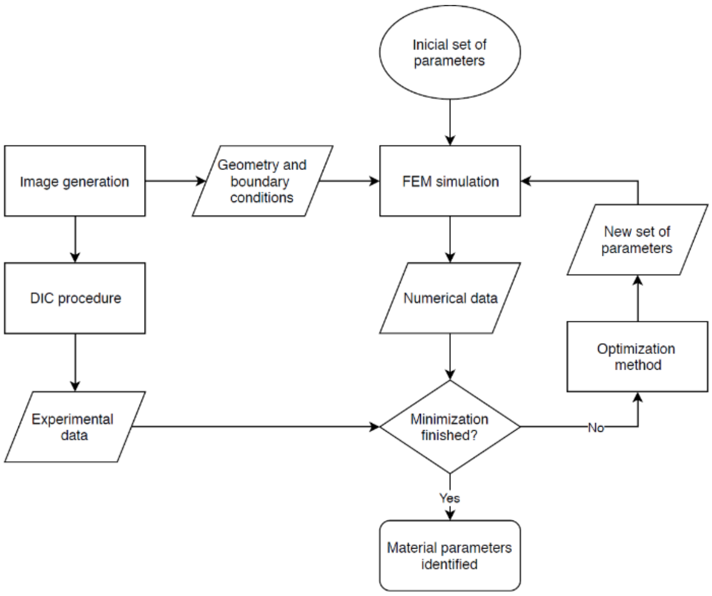

Figure 1: Flow chart describing the functioning of the FEMU algorithm. Source: Adapted from Martins et al. (2018) [9].

3 Results and Validation

By the end of the FEMU process, material's constitutive parameters are identified. They are compared to the values used for the image generation for method validation. The data acquired using the formulation proposed in this paper is also compared to the set of data coming from a formulation based on [19], where the minimization is done using deformation data. The test was executed multiple times with a different element discretization for each case.

The software simulates an uniaxial tension test, where the specimen is under a 0,01 KN/mm traction load, having predeterminated Young modulus E (kN/cm²) and Poisson's ratio υ. Using these values as model for validation, the following table compares them with both FEMU acquired set of elastic parameters, using displacement and deformation:

Table 1. Comparison between constitutive parameters for the uniaxial tension test.

Nº of elements	Predeterminated		Present work		Error (%)		Def. formulation		Error (%)	
	E	υ	E	υ	E	υ	E	υ	E	υ
100	15	0,3	14,647	0,321	2,4	-6,9	13,998	0,273	6,7	8,9
400			14,333	0,299	4,5	0,3	13,997	0,272	6,7	9,4
900			14,952	0,292	0,3	2,8	14,002	0,270	6,7	9,9
1600			14,186	0,286	5,4	4,8	14,004	0,269	6,6	10,3
2500			14,158	0,286	5,6	4,7	13,996	0,272	6,7	9,5
100	28	0,2	28,367	0,239	-1,3	-19,5	26,988	0,181	3,6	9,7
400			27,696	0,216	1,1	-8,1	27,022	0,186	3,5	7,0
900			28,839	0,209	-3,0	-4,3	26,987	0,186	3,6	6,8
1600			27,378	0,204	2,2	-1,9	26,996	0,186	3,6	7,0
2500			27,287	0,202	2,5	-1,2	26,957	0,186	3,7	6,9
100	31	0,15	31,643	0,181	-2,1	-20,8	29,982	0,131	3,3	12,8
400			30,838	0,164	0,5	-9,3	30,002	0,140	3,2	6,9
900			32,120	0,158	-3,6	-5,3	30,008	0,140	3,2	6,4
1600			30,474	0,154	1,7	-2,7	30,018	0,143	3,2	4,7
2500			30,421	0,152	1,9	-1,0	30,035	0,145	3,1	3,5

As seen in Table 1, the data acquired by the FEMU algorithm developed for this paper had quite satisfactory results for the uniaxial tension test, if the image was divided in at least 900 elements. After this threshold the results doesn't seem to get significantly more accurate with more elements. Also, the algorithm doesn't seem to perform differently for different values of predeterminated parameters.

The average error relative to the original parameters for the displacement FEMU algorithm running with at least 900 elements was 2,9% for the Young modulus and 3,2% for the Poisson's ratio. The displacement formulation presented in this paper seems to perform slightly better than its deformation counterpart in every situation considered. Thus, the utility for this method applied to this test for constitutive parameter identification is noticeable.

4 Conclusion

The identification problem is one of the most common in modern structural engineering. Among many solutions to it, Finite Elements Method Updating (FEMU) shines as a simple yet accurate form of acquiring material parameters through experiments.

This paper presented a procedure of using FEMU together with Digital Image Correlation (DIC) for an easy, inexpensive and accessible technique for constitutive parameter identification, and utilizes simulations made by *Itom* with a set of parameters for better validation.

Applying this method for uniaxial tension test resulted in an accurate identification of the parameters, if at least 900 elements were used. The output of the displacement FEMU algorithm had an average error of 2,9% for the Young modulus and 3,2% for the Poisson's ratio relative to the original set of parameters, being more accurate than its deformation counterpart.

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