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BEAMS: APPLIED COMPUTATION TO SOLVE PROBLEMS OF SOLID MECHANICS

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Abstract. This paper aims to present the program BEAMS, initials in Portuguese for Exercise Bank Applied to Solid Mechanics. It is an open source software-in-progress that intends to available a bank of editable examples about different issues of Solid Mechanics. It has been developed in State University of Minas Gerais, using Scilab and LaTeX. Scilab language has some advantages: It is open software, in continuous improvement and evolution; Obtaining high-resolution graphics is simple; working with vectors and matrices is easy; Good Graphical User Interface with icons, selection buttons, editable input fields, checkboxes and others. Regarding to the LaTeX, it is a high quality typesetting system that produces technical and scientific documentation, allowing to write equations in a mathematical format, by using symbols and specific characters. In this way, the output file obtained via BEAMS displays the results in a pdf format file, with all detailed calculations, adding formatted texts to the explanations, math equations, figures and tables. It is possible because Scilab uses the LaTeX syntax, through of JLaTeXMath library. The implementation developed until here of BEAMS lets the calculation of unsymmetrical bending of the cross-section. Since the whole process is automated, it is enough to modify the input data so the calculations are updated according to each type of cross section. In this way the solution of different examples can be done quickly, allowing teachers and students to conference the calculations and visualizing how to the dimensions and each cross section induce the value of the calculated stresses.

Keywords: Solid Mechanics; Applied Computation; Free Software

1 Introduction

Solid Mechanics is an essential subject in engineering courses, practiced within of the materials science, civil, mechanical and aerospace engineering. It is the branch of physics and mathematics that concerns the behavior of solid matter under external actions.

Sometimes, the calculations involved in the Solid Mechanics are complex and extensive, and they need tables, images and graphics to their understanding. Several researches reported difficulties of students and teachers in the teaching and learning process of this discipline (Silva et al. 2017; Karim, 2011; Kadlowec et al, 2002) [1-3]. Because of this difficulty, frequently, many students wrongly regard Solid Mechanics as a nonessential part of their learning for professional practice. The utilization of computational tools in the teaching of engineering can be smooth the learning of complex issues. In the case of structural engineering, it is common the use of these tools, especially softwares. However, often, this use restricts itself to confer of the results. In this way, it is important the utilization of computational technologies that create an environment more dynamic, with the solution happens step-by-step, in detail, and the student can interact in the process, learning the methodology.

Solid Mechanics has been important in the project of several structures as buildings, ships, automobiles, railways, petroleum refineries, engines, airplanes, nuclear reactors, composite materials, computers, and medical implants. It has specific applications in many other areas, such as understanding the anatomy of living beings, design of dental prostheses and surgical implants (Bower, 2009) [4].

For example, the courses of Engineering offered by State University of Minas Gerais (UEMG) at the João Monlevade unity, i.e. Environmental, Civil, Mining and Metallurgical Engineering present in their program curriculum compulsory the discipline Solid Mechanics.

The aim of this work-in-progress paper is to present the program BEAMS, that has been development at UEMG, João Monlevade unity. The software BEAMS, acronyms for the Exercise Bank Applied to Solid Mechanics, in Portuguese, i. e. Banco de Exercícios Aplicados à Mecânica dos Sólidos is developed using the free software Scilab and LaTeX. The main goal of the BEAMS is to post a large editable exercise bank, with the solution of each exercise, with explanation in Portuguese, directed to the understanding to fundamental concepts, addressing to the student learning, not being a program focused essentially on results.

The scilab language has some advantages for mechanical computation, especially to engineering: i) it is open software, in continuous improvement and evolution. ii) It presents simple language, facilitating to develop programs, particularly for engineers who, in general, do not know advanced computing languages. iii) Obtaining high-resolution graphics is simple; iv) working with vectors and matrices is easy. v) It is possible to build a good Graphical User Interface (GUI) with icons, selection buttons, editable input fields, checkboxes and other visual indicators. These characteristics make scilab an interesting language to develop a program for solving problems, which often involve complex calculations that need to use images, graphs and tables for their understanding, like Solid Mechanic ones (Baudin, 2010; Mathieu & Roux, 2016) [5, 6].

Related to the language LaTeX, it is useful because is a high quality typesetting system. It includes features designed to produce technical and scientific documentation. It makes possible to write equations in a mathematical format, by using all the symbols and specific characters, producing esthetically beautiful documents of output. In this way, the output file obtained via BEAMS displays the results in a pdf format file, generated from the LaTex, with all detailed calculations, adding formatted texts to the explanations, math equations, figures and tables. It is possible because Scilab allows to usage of Latex syntax, through of JLaTeXMath library that is an implementation of the math mode of LaTeX (Knuth, 1984, Lamport, 1994) [7, 8].

The implementation developed until here of BEAMS allows the calculation of unsymmetrical bending of different cross sections like Sample Problems 4.9 and 4.10 of Beer & Johnston (2011) [9] or the examples 6.19 and 6.20 of Hibbeler (2004) [10] and other cases. It can be observed the high potential of the program BEAMS, since the whole process is automated, it is enough to modify the input data so that the calculations are updated according to each type of cross section. In this way the solution of different examples can be done quickly, allowing teachers and students to conference the calculations as well as the results and visualizing how to the dimensions and each cross section induce the value of the calculated stresses.

2 General Case of Unsymmetrical Bending

Stress is a physical quantity very important in the evaluation of the strength capacity of a structural element. It represents a force distributed on the cross section area of the element. To determinate the stress is necessary to known the properties of geometric figures that compose the cross section of the structural element. The most important geometric properties of an area used in Solid Mechanics are: Geometric Center, Static Moment, Moment of Inertia, Product of Inertia, Elastic Section Modulus and Radius of Gyration.

2.1 Geometric Properties of an area

Geometric Center or Centroid of an area is a geometric concept while the center of mass and the center of gravity are related to the physical properties of the body. For the centroid to be coincident to the center of mass and the center of gravity, the body should has uniform distribution of the matter and it be submitted an uniform gravitational field (Hibbeler, 2009) [11]. The coordinates x and y of the centroid can be obtained by the Eqs. (1) and (2) respectively, where the integrals ∫xdA and ∫ydA are the First moments of area and represent the Static Moment about the axes x and y axes, respectively.

$$\bar{x} = \frac{\int x \, dA}{\int dA} \tag{1}$$

$$\bar{y} = \frac{\int y \, dA}{\int dA} \tag{1}$$

Figure 1 present the infinitesimal area of surface A and the distances x and y used in the calculation of the Geometric Center.

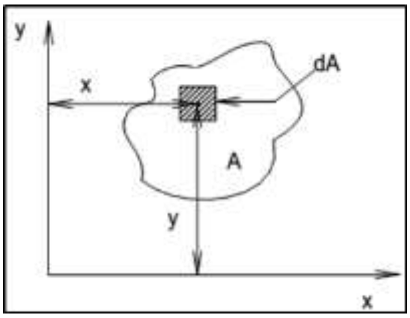


Figure 1 - Infinitesimal area of surface A.

Moment of inertia is a quantity that measures the strength that an area offers when it is submitted to rotatiom about a given axis. Therefore, greater moment of inertia lesser will be the bending stress. The integrals $\int x^2 dA$ and $\int y^2 dA$, as presented in the Eqs. (2) and (3) respectively, are the Second moments of area and represent the Moment of Inertia I_x and Iy about the x and y axes, respectively.

$$I_x = \int_A y^2 dA \tag{2}$$

$$I_y = \int_A x^2 dA \tag{3}$$

For problems involving unsymmetrical cross-sections, an important property is the inertia product I_{xy} presented in the Eq. (4). It can be positive, negative or zero, depending on the location and orientation of the coordinate axes. For example the I_{xy} inertia product for an area will be zero if the x axis or y axis is a symmetry axis.

$$I_{xy} = \int_A x\, y\, dA \tag{4}$$

The principal moments of inertia are the maximum and minimum values for a section and they occur about the principal axes. Inertia product of area about the principal axes are zero. An axis of symmetry will always be a principal axis. With a knowledge of I_x, I_y, and I_{xy}, for a given section, the principal values of moment of Inertia I_{X1} and I_{Y1} may be determined using either Mohr's circle construction as presented in Eqs (5) and (6):

$$I_{X1} = \frac{I_X+I_Y}{2} + \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 - I_{xy}^2} \tag{5}$$

$$I_{Y1} = \frac{I_X+I_Y}{2} - \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 - I_{xy}^2} \tag{6}$$

The geometric properties presented before for the composite area is equal to the sum of the moments of inertia of the individual property of all its parts. The moment of inertia of a composite area A about a given axis is obtained by adding the moments of inertia of the component areas with respect to the same axis.
The parallel-axis theorem can be used to find the moment of inertia of an area about any axis that is parallel to an axis passing through the Geometric Center and about it the moment of inertia is known. Eqs (7) and (8) present the expressions to calculate the moment of inertia I_x and I_y, respectively of any axis using the parallel-axis theorem, where the distances dx and dy are identified in the Fig. 2.

$$I_x = I_{x_{CG}} + A\, d_y^2 \tag{7}$$

$$I_y = I_{y_{CG}} + A\, d_x^2 \tag{8}$$

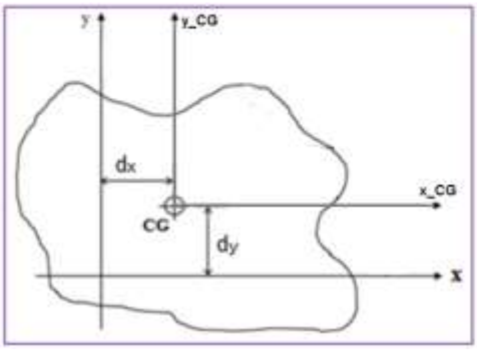


Figure 2. Distances dx and dy of the parallel-axis theorem

In a similar way the product of inertia can also be calculates by parallel-axis theorem as presented in Eq. (9):

$$I_{XY} = I_{xy_{CG}} + A d_x d_y \tag{9}$$

2.2 Unsymmetrical Bending

Different special cases can be determined by the position of the plane of the bending. Pure symmetrical bending occurs if one of the principal axis of the cross-section is on the plane of bending as illustrate in Figure. In this case the neutral axis coincides with the other principal axis and due to the elastic behaviour of the cross-section, the stress is linear on both sides of the neutral axis. The general formula for the stress in pure symmetrical bending is presented in Eq (10):

$$\sigma_z = \frac{M_x}{I_x} y \tag{10}$$

- where M_x is the bending moment about axis x (acting in the plane perpendicular to the axis x)
- I_x is the moment of inertia about axis x;
- y is the orthogonal distance of the given point from axis y.

When the plane of bending is not the principal axes of the cross-section occurs unsymmetrical bending. If the principal planes and the principal moments of inertia are known, unsymmetrical bending can be calculated as a composition of two symmetrical bending, according to Eq. (11) where x_1 and y_1 are the principal axes.

$$\sigma_z = \frac{M_{x1}}{I_{x1}} y_1 - \frac{M_{y1}}{y1} x_1 \tag{11}$$

The Eq (11) can be rewrite in the general form using any centroidal axes, according to Eq. (12). Using these both equations, it would be possible to calculate the bending stress at any point on the cross section regardless of moment orientation or cross-sectional shape.

$$\sigma_z = \frac{\left(M_x I_y + M_y I_{xy}\right) y - \left(M_y I_x + M_x I_{yx}\right) x}{I_x I_y - I_{xy}^2}$$

3. Software BEAMS

BEAMS is a software in-progress focused in solving problems of Solid Mechanics, on what the solution happens step by step, in detail. BEAMS has been developed at State University of Minas Gerais, unity João Monlevade, using free software scilab for processing calculations and image design, and the TeX language through the free system miktex for printing results (Michel et al, 2019).

Scilab is a programming language associated with a powerful collection of numerical algorithms covering many aspects of scientific computing problems. From a software standpoint, Scilab is an interpreted language that enables faster development processes because the user directly accesses a high level language with many features provided by the library. From a license point of view, Scilab is free and open source software provided under the Cecill license (BAUDIN, 2010).

The software currently allows the calculation of unsymmetrical bending for 15 different cross.The data input of the software is made by box selections and box with editable input fields. Images are used in these boxes data input to facilitate the understanding of the data that to be entered. Figure 3 present the window of BEAMS where the user selected the cross section to be calculated

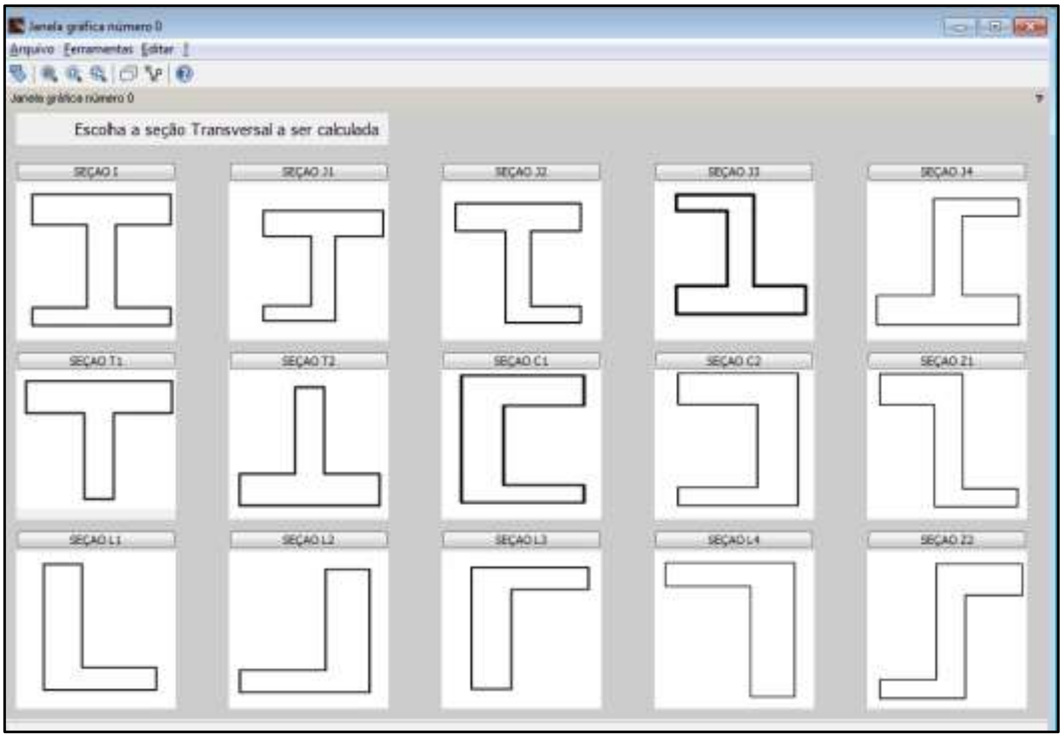


Figure 3. Window of program BEAMS to select the cross-section to be calculated.

The screen to enter the dimensions of selected the cross-section is presented in Fig. 4. It is possible to selected the unity of dimensions (mm, cm or dm) in a previous window. As it is presented in figure, in this case, it has been selected the unity cm and cross section in shape of T, where is possible to adopt unsymmetric table and different widths for table and web.

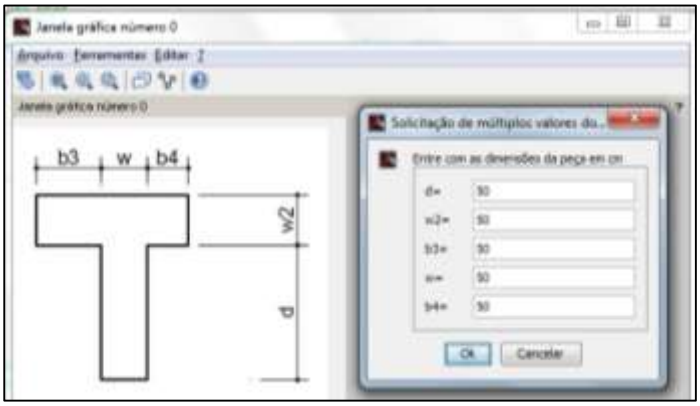


Figure 4. Window of program BEAMS to enter the dimensions of selected the cross-section.

The point where it will be calculated the stress is defined by the coordinates x and y with the reference to the axes passing by the origin O, as illustrated in Fig. 5.

Figure 5. Window of program BEAMS to enter the coordinates of the point where will be calculated the stress.

The direction of applied moment is defined by the angle with the vector of moment according to the illustration presented in Fig. 6. This angle is measured with respect to the x-axis in the counter-clockwise.

Figure 6. Window of program BEAMS to enter the value and the direction of the concentrate moment acting in the cross section.

The program BEAMS has been developed in independent modulus, with the subroutine *Calcula_Propriedades_Geometricas.sce* that calculates the area, the coordinates of centroid, moments of inertia Ix and Iy, and the product of inertia I_{xy}. The subroutine *Calcula_Tensao.sce* executes the calculations relating to the bending stress. The figures used in the explanation of the execise solved by BEAMS are plotted in the subroutine *Desenha_Imagens.sce*. Finally the subroutine *Impressao_Resultados.sce* generates the file Resultado.tex in language TeX and the command of scilab dos("pdflatex " + "Resultado.tex") runs the tex file and generates the pdf file with the results. Figure 7 presents part of the code in TeX language that generates the printing results.

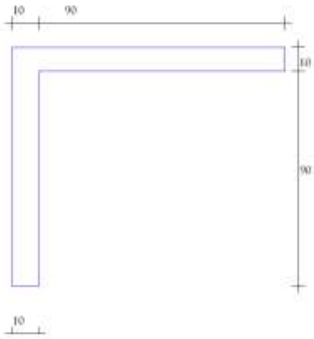
Figure 7. Part of the subroutine *Impressao_Resultados.sce* with the source in language TeX..

4. RESULTS

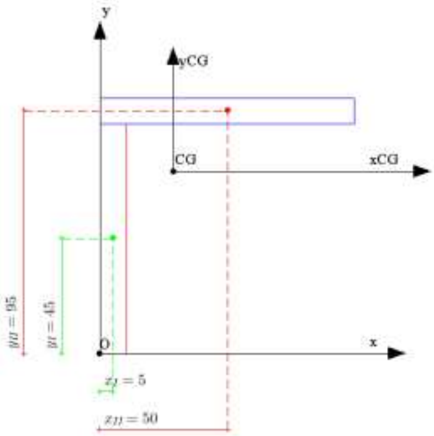
The program BEAMS is directed to the teaching-learning of Solid Mechanics. Although it has not yet been tested in classrooms, it is already possible at the current stage of the program to identify points that will facilitate student learning, such as the ease of viewing the variation dimensions of the cross section influencing the geometric properties and consequently the value of the bending stress.

BEAMS is still a program-in-progress, in the process of testing and improvement. However, some potential results already achieved during the project should be highlighted: process automation; updating calculations and texts according to each type of cross-section; possibility to include figures and tables in the file results; resizing the image to fit the dimensions that the user entered. Figures 8-11 present the pages of results like is printed in the pdf file with the results:

PROPRIEDADES GEOMÉTRICAS CALCULADAS
ÁREA, CENTRO DE GRAVIDADE E MOMENTO DE INÉRCIA. Dimensões em mm



ÁREA) A área é obtida considerando a área dos dois retangulos que formam a figura:
A =90×10+100×10
A =1900mm^2



CENTRO DE GRAVIDADE)As distâncias xCG e yCG do Centro de Gravidade (CG) são obtidas tendo como referência o ponto O. Como a figura não possui simetria em nenhum dos eixos, as distâncias horizontal (xCG) e vertical (yCG) são obtidas por:

$$x_{CG} = \frac{\sum_{i=1}^{n} x_i \times A_i}{\sum_{i=1}^{n} A_i} = \left(\frac{x_I \times A_I + x_{II} \times A_{II}}{A_I + A_{II}}\right)$$

$$x_{CG} = \left[\frac{(5 \times 900)_I + (50 \times 1000)_{II}}{900 + 1000}\right]$$

. x_{CG} =28.684211mm

$$y_{CG} = \frac{\sum_{i=1}^{n} y_i \times A_i}{\sum_{i=1}^{n} A_i} = \left(\frac{y_I \times A_I + y_{II} \times A_{II}}{A_I + A_{II}}\right)$$

$$y_{CG} = \left[\frac{(45 \times 900)_I + (95 \times 1000)_{II}}{900 + 1000}\right]$$

. y_{CG} =71.315789mm

1

Figure 8 - Page 1 of results obtained via BEAMS

Table 1: Resumo cálculo xCG

Elem	x mm	A mm^2	xA mm^3
I	5	900	4500
II	50	1000	50000
$\sum$	–	1900	54500
x_{CG} =	28.684211	mm	

Table 2: Resumo cálculo yCG

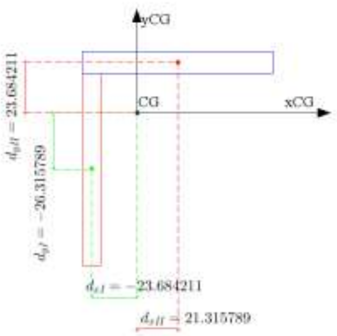
Elem	y mm	A mm^2	yA mm^3
I	45	900	40500
II	95	1000	95000
$\sum$	–	1900	135500
y_{CG} =	71.315789	mm	

MOMENTO DE INÉRCIA E PRODUTO DE INÉRCIA

O momento de inércia e o produto de inércia são obtidos usando o Teorema dos Eixos Paralelos. As distâncias do centro de gravidade de cada elemento ao centro de gravidade da figura estão calculadas a seguir

$$d_{xI} = x_I - x_{CG} = 5 - 28.684211 => d_{xI} = -23.684211mm$$
$$d_{xII} = x_{II} - x_{CG} = 50 - 28.684211 => d_{xII} = 21.315789mm$$

$$d_{yI} = y_I - y_{CG} = 45 - 71.315789 => d_{yI} = -26.315789mm$$
$$d_{yII} = y_{II} - y_{CG} = 95 - 71.315789 => d_{yII} = 23.684211mm$$



Momento de inécia em relação ao eixo centroidal xCG

$$I_x = I_{xI} + I_{xII} + I_{xIII}$$
$$I_x = \left(\frac{b_I \times h_I^3}{12} + d_{yI}^2 \times A_I\right) + \left(\frac{b_{II} \times h_{II}^3}{12} + d_{yII}^2 \times A_{II}\right)$$
$$I_x = \left[\frac{10 \times 0^3}{12} + -26.315789^2 \times 900\right]\left[\frac{10 \times 90^3}{12} + 21.315789^2 \times 1000\right]$$
$$I_x = 1800043.5mm^4$$

Momento de inécia em relação ao eixo centroidal yCG

$$I_y = I_{yI} + I_{yII}$$
$$I_y = \left(\frac{b_I^3 \times h_I}{12} + d_{xI}^2 \times A_I\right) + \left(\frac{b_{II}^3 \times h_{II}}{12} + d_{xII}^2 \times A_{II}\right)$$
$$I_y = \left[\frac{10^3 \times 0}{12} + -23.684211^2 \times 900\right]\left[\frac{10^3 \times 90}{12} + 21.315789^2 \times 1000\right]$$
$$I_y = 1800043.5mm^4$$

2

Figure 9 - Page 2 of results obtained via BEAMS

Produto de Inércia

$I_{xy} = 0$

$I_{XY} = I_{xy} + Adxdy$

$I_{XY} = I_{XYI} + I_{XYII}$

$I_{XY} = I_{xy} + (A_I \times d_{xI} \times d_{yI} + A_{II} \times d_{xII} \times d_{yII})$

$I_{XY} = 0 + (900 \times -23.684211 \times -26.315789 + 1000 \times 21.315789 \times 23.684211)$

$I_{XY} = 1065789.5mm^4$

Table 3: Resumo do Produto de Inércia

Elem	dx mm	dy mm	A mm^2	Ixy mm^4
I	-23.684211	-26.315789	900	0
II	21.315789	23.684211	1000	560941.83
$\sum$	–	–	1900	1065789.5

Descobrindo o eixo principal

$$tan2\theta = \frac{2I_{xy}}{I_X - I_Y} = \frac{2 \times 0}{1800043.5 - 1800043.5}$$

$\theta = 45^o$

Momento de inécia em relação ao eixo principal X1

$$I_{X1} = \frac{I_X + I_Y}{2} + \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + I_{XY}^2}$$

$$I_{X1} = \frac{1800043.5 + 1800043.5}{2} + \sqrt{\left(\frac{1800043.5 - 1800043.5}{2}\right)^2 + 1065789.5^2}$$

$I_{X1} = 2865833.0mm^4$

Momento de inécia em relação ao eixo principal Y1

$$I_{Y1} = \frac{I_X + I_Y}{2} - \sqrt{\left(\frac{I_X - I_Y}{2}\right)^2 + I_{XY}^2}$$

$$I_{Y1} = \frac{1800043.5 + 1800043.5}{2} - \sqrt{\left(\frac{1800043.5 - 1800043.5}{2}\right)^2 + 1065789.5^2}$$

$I_{Y1} = 734254.03mm^4$

Coordenadas de um ponto Com referência ao Eixo Principal

$X1 = (Xcos\theta) + (Ysen\theta) = (100 \times cos(45)) + (28.684211 \times sen(45)) = 70.710678mm$

$Y1 = (Ycos\theta) - (Xsen\theta) = (28.684211 \times cos(45)) - (100 \times sen(45)) = -30.145079mm$

Table 4: Coordenadas de P com referência ao eixo principal

Ponto	X	Y	X1	Y1
P	100	28.684211	70.710678	-30.145079

1)Tensão de flexão considerando o eixo principal X1 e Y1
Momentos atuantes nos eixos principais X1 e Y1

$M_{X1} = M_s cos(360 - \alpha - \theta) = 3 \times cos(295) = 1.2678548kN.m$

$M_{Y1} = M_s sen(360 - \alpha - \theta) = 3 \times sen(295) = 2.7189234kN.m$

$$\sigma = \frac{M_{X1}}{I_{X1}} Y1 + \frac{M_{Y1}}{I_{Y1}} X1$$

$$\sigma = \frac{1267.8548}{2865833.0} \times -30.145079 + \frac{2718.9234}{734254.03} \times 70.710678$$

$\sigma = 248.50349MPa$

3

Figure 10 - Page 3 of results obtained via BEAMS

2)Tensão de flexão considerando os eixos centroidas x e y
Momentos atuantes nos eixos centroidas x e y

$$M_x = M_s cos\alpha = 3 \times cos20 = 2.8190779 kN.m$$
$$M_y = M_s sen\alpha = 3 \times sen20 = 1.0260604 kN.m$$

$$\sigma = \frac{(M_x I_y + M_y I_{yx})\, y - (M_y I_x + M_x I_{yx})\, x}{I_x I_y - I_{yx}^2}$$

$$\sigma = \frac{(2819.0779 \times 1800043.5 + 1026.0604 \times 1065789.5)\, 28.684211 + (1026.0604 \times 1800043.5 + 2819.0779 \times 1065789.5)\, 100}{1800043.5 \times 1800043.5 - 1065789.5^2}$$

$$\sigma = 248.50349 MPa$$

Figure 11 - Page 4 of results obtained via BEAMS

4. CONCLUSIONS

The implementation developed until here of BEAMS lets the calculation of unsymmetrical bending of the cross-section. Since the whole process is automated, it is enough to modify the input data so the calculations are updated according to each type of cross section. In this way the solution of different examples can be done quickly, allowing teachers and students to conference the calculations and visualizing how to the dimensions and each cross section induce the value of the calculated stresses.
BEAMS already presents some requirements for your use in the classrom as a pleasing look and easy data input through the use of boxes selection, check boxes, box with editable input fields and push buttons. Thus, the application in the classroom is an important step for the further development of the program and confirmation of its potentiality.
The program is being implemented in independent modules so that it can be developed simultaneously by other teachers, as well as other educational institutions, allowing a potential for growth in research area for the institution.

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