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NONLINEAR NUMERICAL MODELING OF TRUSSES

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Abstract. Trusses are a type of structural system widely applied in Engineering, performing, in many cases, as an agile and economically advantageous solution. In Structural Engineering, computational modeling consists of a powerful tool for representing the behavior of structural elements and systems, especially if a non-linear approach is utilized, allowing the evaluation of its mechanical performance both in the design phase and throughout its lifetime. Metal alloys, specially the steel, are currently employed on the construction of trusses. This kind of ductile material presents a pronounced non-linear mechanical behavior, more specifically an elastoplastic one. Therefore, it is of importance the consideration of the hardening effect in the structural analysis of structural systems constituted by ductile materials, for allowing a better understanding of the structural behavior of these systems and the study of their resistance in the post-yield regime. This work aims to develop numerical models of trusses, considering different elastoplastic models – e.g., perfectly elastoplastic, isotropic hardening and Ramberg-Osgood – in order to verify its influence on the response of the trusses, regarding serviceability limit state (SLS) failure modes, in terms of allowable displacements and stresses. The models are implemented in Python language and validated according to examples presented in the literature.

Keywords: Truss, Plasticity, Ramberg-Osgood, Isotropic Hardening, Direct Stiffness Method

1 Introduction

Over time, engineers have become increasingly bold in building projects, developing larger and more complex structures, subjecting the structural materials to high stress levels. Thus, the trusses arise as a structural solution to extend the dimensions of structures, while also allowing them to bear elevated loads. Since then, trusses have become a type of structural system widely applied in Engineering, performing, in many cases, as an agile and economically advantageous solution.

Moreover, computational modeling consists of a powerful tool for representing the behavior of structural elements and systems, especially if a non-linear approach is addressed, allowing the evaluation of its mechanical performance both in the design phase and throughout its lifetime. Metal alloys are currently employed on the construction of trusses. This kind of ductile material presents a pronounced elastoplastic behavior. Therefore, the consideration of the hardening effect stands out, for allowing a better understanding of the behavior of these structures, including in the post-yield regime.

In this paper, the direct stiffness method is implemented and combined with different elastoplastic models – perfectly elastoplastic, isotropic hardening and Ramberg-Osgood – in order to analyze plane trusses in terms of force-displacement curves. The results obtained are verified by using analytical solutions and softwares. Finally, a comparative analysis on the different plasticity models is made, based on serviceability limit states.

2 Numerical modeling of plane trusses

The method utilized in this paper for the linear analysis of plane trusses is the Direct Stiffness Method. In this method, according to Papadrakakis and Sapountzakis [1], the implementation in Python 3 language can be made in four stages: firstly, the data of the structure is provided to the program (nodes coordinates, connectivity of members, mechanical properties of the material and boundary/loading conditions). Then, the computation of each element's stiffness matrix is performed, and its stiffness terms are properly placed in the global stiffness matrix. In the third stage, the global matrix representing the equilibrium condition is solved with the appropriate numerical method, providing the nodal displacements. Lastly, the resultant stresses are computed, which subsequently can be used for dimensioning of the structural element.

2.1 Perfectly elastoplastic model

According to Proença [2], the perfectly elastoplastic model can be characterized by the non-existence of the hardening phenomenon. Therefore, once the yield point stress is reached, the material yields indefinitely in a viscoplastic behavior. As such, Eq. (1) and Eq. (2) are valid.

$$\varepsilon = \varepsilon_e + \varepsilon_p \tag{1}$$

$$\sigma = E\varepsilon_e = E(\varepsilon - \varepsilon_p) \tag{2}$$

Where E is Young's modulus, σ is the stress on the material, ε is the strain and ε_e and ε_p are the elastic and plastic components, respectively.

2.2 Isotropic hardening model

In the elastoplastic model with linear isotropic hardening, after the yield stress of the material is reached, the total stress grows with the increase of strain. Such behavior characterizes the positive hardening, as affirmed by Silva and Lima Jr. [3].

The yield criterion for the linear isotropic hardening, whose positive value indicates that the material is in process of hardening, is represented in Eq. (3).

$$f(\sigma, \alpha) = |\sigma| - (\sigma_y + K\alpha) \tag{3}$$

where σ_y is the stress at yield point, K is the isotropic hardening modulus and α is a parameter that measures the accumulated plastic strain, whose variation is defined through Eq. (4). Additionally, the stress-strain relation and the plastic strain evolution law are shown in Eq. (5) and Eq. (6), respectively.

$$\dot{\alpha} = |\dot{\varepsilon_p}| \tag{4}$$

$$\dot{\sigma} = E(\dot{\varepsilon} - \dot{\varepsilon_p}) \tag{5}$$

$$\dot{\varepsilon_p} = \frac{E\dot{\varepsilon}}{E+K} \tag{6}$$

2.3 Ramberg-Osgood equation

Ramberg and Osgood [4] proposed an equation to approximate the stress-strain curve in terms of three parameters: Young's modulus and two secant yield strengths. The exponential term in the equation is used to describe the behavior of the material regarding its plastic strain. In short, if we consider the strain at yield point to be 0.2%, which is a widely made assumption, the equation can be represented as follows (Eq. (7)).

$$\varepsilon = \frac{\sigma}{E} + 0.002\left(\frac{\sigma}{\sigma_y}\right)^n \tag{7}$$

If we consider E_0 the initial Young's modulus and differentiate Eq. (7) with respect to strain, we can relate the current Young's modulus with the stress, as shown in Eq. (8). This equation can be used in incremental algorithms that provide force-displacement curves of the Ramberg-Osgood method.

$$E = \frac{d\sigma}{d\varepsilon} = \frac{1}{\frac{1}{E_0} + 0.002\frac{n}{\sigma_y}\left(\frac{\sigma}{\sigma_y}\right)^{n-1}} \tag{8}$$

And finally, by knowing the material's yield stress, ultimate stress, its associated strains and the Young's modulus, we can determine the value for the constant n, as shown in Eq. (9).

$$n = \frac{log(\varepsilon_u - \varepsilon_y)}{log(\sigma_u - \sigma_y)} \tag{9}$$

3 Results and discussion

The plane truss analyzed in this paper can be seen in Fig. 1.

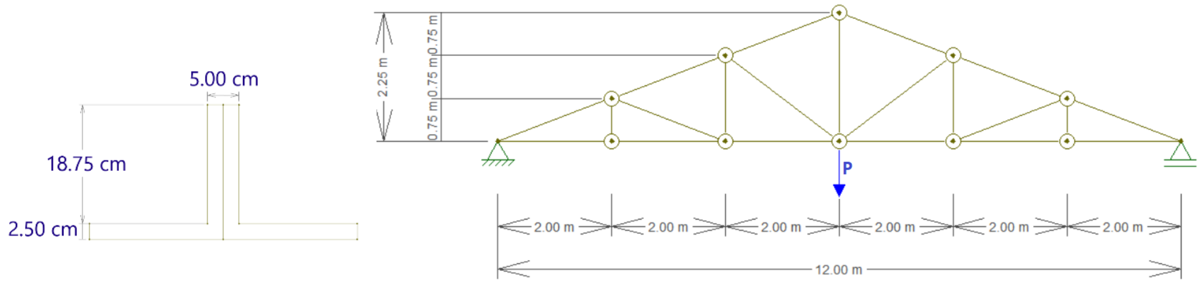

Figure 1. Structural scheme of the analyzed truss with loading applied to the central node

The bars utilized in the analyzed truss have custom-dimension double angle profile and, in the force-displacement analysis, the loading is applied to the central node of the bottom chord, as shown in Fig. (1). Information regarding the properties of the bars and parameters of the applied methods can be seen in Table 1.

Table 1. Terms utilized in the structural analysis

Name	Nomenclature	Value
Isotropic hardening modulus	K	$77.5\ \mathrm{kN/cm^2}$
Young's modulus	E	$20000\ \mathrm{kN/cm^2}$
Cross section area	S	$0.02\ \mathrm{m^2}$
Stress at yield point	σ_y	$25\ \mathrm{kN/cm^2}$
Strain at yield point	ε_y	0.002
Stress at ultimate point	σ_u	$40\ \mathrm{kN/cm^2}$
Strain at ultimate point	ε_u	0.2
Hardening behavior constant	n	9.798159

In order to obtain the stress-strain curves of the material, an incremental algorithm was implemented in Python 3 language, in which small increments of stress ($\Delta\sigma = 0.005\ kN/cm^2$) were applied to the analyzed member and its strain evaluated for each new accumulated stress, according to the criteria of each of the plasticity models addressed in this paper, which resulted in Fig. 2. The results were validated with the analytical equations. It is worth mentioning that, according to Euler's critical load formula, none of the bars buckle before collapse.

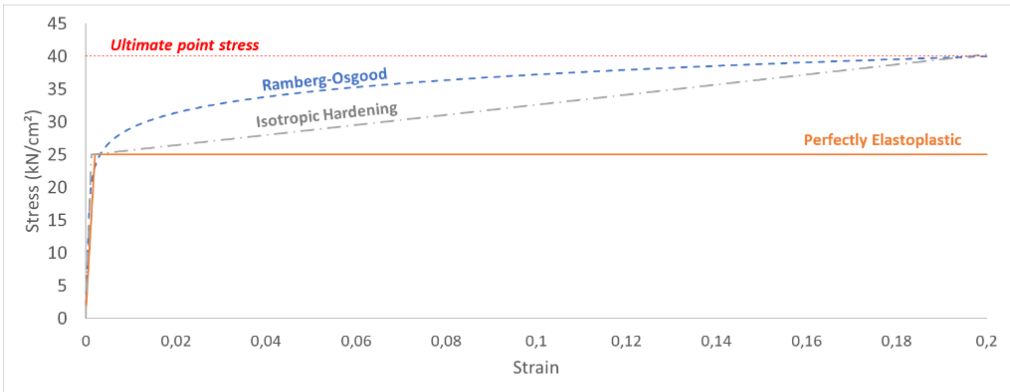

Figure 2. Stress-strain curve of the adopted material according to different plasticity chord

The Direct Stiffness Method algorithm directly provides the force-displacement relation for the load applied to the node. In the perfectly elastoplastic method, successively larger values of loading are applied, and its respective displacements evaluated. In the Ramberg-Osgood method (and, similarly, in the isotropic hardening method), an incremental approach is added, in which steps of loading are applied to the structure in its initial configuration and the respective displacements evaluated, and, after each step, its Young modulus updated to correspond to the new accumulated loading. The force-displacement curves obtained from the adopted methods can be seen in Fig. (3).

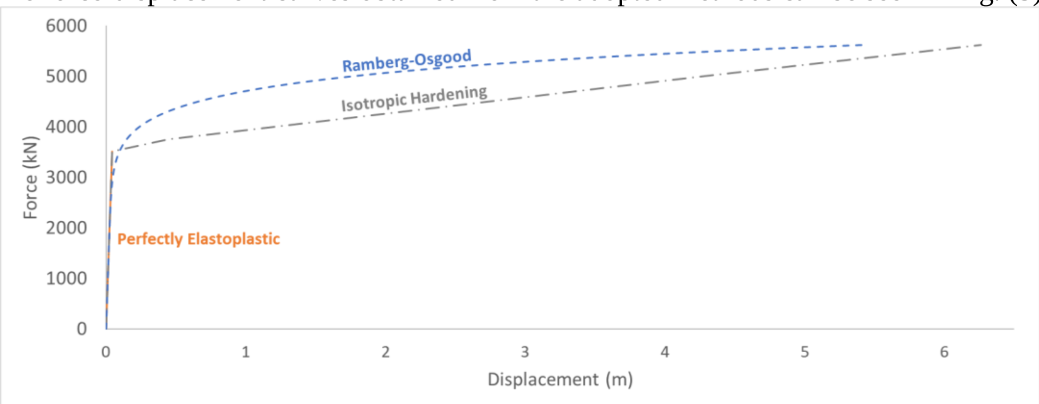

Figure 3. Force-displacement curve for loading applied to the central node of the bottom chord

The structure collapses in the perfectly elastoplastic model by yielding of the top chord, whereas, in both Ramberg-Osgood and isotropic hardening methods, the structure collapses by tensile rupture of the top chord. The corresponding force-displacement values for each method are shown in Table 2.

Table 2. Maximum force and displacement values for different plasticity models

Method	Maximum Force	Maximum Displacement
Perfectly elastoplastic	$3511.23\ kN$	$4.35\ cm$
Isotropic hardening	$5617.98\ kN$	$625.67\ cm$
Ramberg-Osgood	$5617.98\ kN$	$541.00\ cm$

The inferior maximum displacement of the Ramberg-Osgood method, in relation to the isotropic hardening model, can be explained by the smoother process of hardening, which can be seen in Fig. 3, in contrast to the abrupt hardening of the isotropic hardening model.

According to the [5], the serviceability limit state for maximum allowable displacement for roof beams is given by the Eq. (10).

$$\delta = \frac{Length\ of\ beam}{250} \tag{10}$$

In the present case, the allowable displacement is $4.80\ cm$, which is barely in the plastic section of the material's stress-strain curve.

4 Conclusions

From the force-displacement analysis of the studied case, it is evident that the serviceability limit state proposed by the NBR8800 stands out as relatively conservative if compared to the maximum displacement observed in the ultimate limit state, as it sticks to a behavior region where the material's response can be easily predicted with relatively high conviction.

If we analyze the force-displacement graphs past the serviceability limit state, we can confirm how the non-linearity of constitutive properties of the material can greatly influence the response of the structural system.

The authors suggest for future works, as a continuity of this study, the consideration of geometric non-linearity in the structural analysis, as well as adaptation of the algorithm to process spatial trusses.

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