RELIABILITY-BASED ANALYSIS OF BURST STRENGTH OF CASING TUBES: THE INFLUENCE OF CORRELATION BETWEEN DESIGN VARIABLES

Jonathan H. Cordeiro Nunes
Lucas P. de Gouveia
Eduardo T. de Lima Junior
João Paulo N. de Araújo
Thiago S. Barbosa
William Wagner M. Lira
jonathanhenrique000@gmail.com
lucasgouveia@lccv.ufal.br
limajunior@lccv.ufal.br
joaopna@lccv.ufal.br
thiago.barbosa@lccv.ufal.br
william@lccv.ufal.br
Scientific Computing and Visualization Laboratory, Federal University of Alagoas
Campus A. C. Simões, Tabuleiro dos Martins, 57072-900, Maceió/Alagoas, Brazil
Felipe L. de Oliveira
Jorel Lopes Rodrigues dos Anjos
flimao@petrobras.com.br
jorel.anjos@petrobras.com.br
Petróleo Brasileiro S.A. - Petrobras
Av. Horácio Macedo, 950 – Cidade Universitária da Universidade Federal do Rio de Janeiro, 21941-915, Rio de Janeiro/Rio de Janeiro, Brazil

Abstract. In structural design normative codes, safety factors are used to reduce the value of the resistance and the probability of failure of the structure due to project uncertainties. Regarding oil exploration, there is a concern about the risk of oil well failure, considering the new challenges of this sector, especially in offshore exploration. Even applying safety factors, these uncertainties leads to a non-zero probability that the structural system does not meet the requirements for which it was designed. In order to improve the decision-making process in structural safety analysis, the structural reliability methods arise with the possibility of calculating the probability of failure of the structure, taking into account the inherent uncertainties of the design variables. However, in order to assure the success of the reliability analysis, it is necessary to have the statistical description of the design variables and a mathematical model to determine the failure functions of interest. In the context of oil well design, the API 5C3 [1] recommends the use of data with independent random variables (r.v.) in the reliability analysis of well casing tubes, because this consideration simplifies the problem and is beneficial for safety. However, the present work aims to analyze the influence of the use of correlation between the r.v. in the results obtained through structural reliability methods. The limit state of ductile rupture under internal pressure is defined, in which the tube strength is defined by the Klever-Stewart [2] model, as determined by API 5C3 [1]. The First Order Reliability Method (FORM) and the Monte Carlo simulation are applied for extreme correlation values of a real sample of casing tubes manufactured in industry. Finally, the results obtained are discussed, in order to evaluate the importance of the use of correlation between r.v. on the casing integrity analysis.

Keywords: Structural reliability, Burst, OCTG.

1 Introduction

Accompanied by advances in the field of structural materials and analysis models, it is noted that there has been an increase in structures complexity, where a better assessment of their safety levels is required. In addition, a large number of random variables are linked to the design, such as the dimensional variability of parts, loads and material strength, generating uncertainties regarding the performance of structures and demanding new techniques and methodologies for their analysis.

In the context of oil well design, casing tubes are important components due to their structural function, providing safe passage for running wireline equipment and serving as a high strength flow conduit to surface for both drilling and production fluids. Therefore, they are the focus in present work, where its probability of failure is analyzed.

Furthermore, computational tools appeared in order to realize reliability analysis, where national and international standards are starting to recommend it. The probability of failure can be assessed during structural design by the imposition limit state equations. Thus, Klever-Stewart's Burst Theory characterized burst capacity through the von Mises yield criterion and Tresca yield criterion, where the limit state of ductile rupture under internal pressure is defined and used in this work.

API 5C3 [1] recommends the use of independent random variables in the reliability analysis for casing design in oil wells, because this consideration simplifies the problem and is beneficial for safety. In this work, mathematical models of failure probability estimation that take into account the correlation between the random variables (ρ) are applied in order to verify the influence of this kind of approach.

2 Methodology

According to Beck [3], failure represents an undesirable state of the structure in relation to its established basic requirements, where this failure is characterized by the violation of an ultimate limit state (safety requirements). Thus, the probability of this event occurring is determined with the aid of the structural reliability theory.

According to Gouveia [4], there are three basic requirements for assessing the probability of failure:

a) the design variables and how they behave individually and together;

b) a limit state design;

c) a reliability method.

In the present work, the correlation between design variables will be considered, in this case, the geometric parameters of the casing tubes. The correlation coefficients between each pair of random variables X_i and X_j are represented by Eq. (1) and arranged in a correlation matrix $\boldsymbol{R_X}$.

$$\rho_{ij} = \frac{Cov[X_i, X_j]}{\sigma_i \sigma_j}, \tag{1}$$

$$\boldsymbol{R_X} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & & \rho_{2n} \\ \vdots & & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix}, \tag{2}$$

where $Cov[X_i, X_j]$ is the covariance between this pair of r.v., σ_i and σ_j are the standard deviation of the variables. Also, n is the number of design variables.

As regards the ultimate limit state design for casing tubes ($G(\boldsymbol{X})$), the normative expression present in ISO TR 10400 [5] for ductile rupture is used, being it the Klever-Stewart [2] formulation. This equation was estimated for burst strength resistant pressure (for internal pressure greater than external pressure), assuming that the tube is under cappped-end conditions, being deduced by a equilibrium tube, with known minimum wall thickness and outer diameter, and considering the imperfections (cracks) generated by the production process. Through ISO TR 10400 [5] it is found that design resistance is **89.38 MPa**.

$$G(\boldsymbol{X}_{KS}) = R_{iKS}(\boldsymbol{X}_{KS}) - 89.38 = mu_{KS}\frac{2k_{dr}f_u(t_{min} - k_a a)}{D - (t_{min} - k_a a)} - 89.38, \quad (3)$$

The variable mu_{KS} is the uncertainty of the Klever and Stewart [2] model, f_u is the ultimate tensile strength of steel, D is the nominal outer diameter, t_{min} is the minimum wall thickness, a is the maximum depth of a crack in the tube wall due to the production process, k_a is an amplification factor of this crack given by 1.0 or 2.0 depending on the material and k_{dr} is a steel deformation correction factor due to hardening, given by Eq. (4):

$$k_{dr} = \left(\frac{1}{2}\right)^{\eta+1} + \left(\frac{1}{\sqrt{3}}\right)^{\eta+1}, \quad (4)$$

where η is a value determined as a function of the steel yield strength, in psi:

$$\eta = 0.1693 - 8.12 * 10^{-7} f_y. \quad (5)$$

The reliability methods used for failure probability analysis were Monte Carlo simulation and the First Order Reliability Method (FORM). In addition, analyzes were performed without the use of correlation between design variables in order to compare the impact on the results.

The Monte Carlo simulation consists of generating N random scenarios with correlated random variables to be evaluated in the $G(\boldsymbol{X})$ limit state design, being characterized the structure failure when $G(\boldsymbol{X}) < 0$. To obtain a correlated sample, it is necessary to generate a vector ($y_i = \{y_1, \ldots, y_n\}$) formed by independent standard normal random numbers, where they are subsequently transformed into space Z by the Eq. (6):

$$Z_i = J_{zy} y_i, \quad (6)$$

where J_{zy} is the Jacobian matrix of transformation of $Y \rightarrow Z$.

Therefore, Eq. (7) is used to determine the accumulated probability vector.

$$u_i = \Phi[Z_i], \quad (7)$$

where Φ is the standard normal cumulative distribution function (CDF).

Finally, the correlated sample is obtained through Eq. (8):

$$x_i = F_X^{-1}(u_i) = F_X^{-1}\left(\Phi\left[J_{zy} y_i\right]\right), \quad (8)$$

where F_X^{-1} is the inverse of CDF.

However, the First Order Reliability Method (FORM) consists in constructing a joint probability distribution function, whose statistical probability distribution of random variable (r.v.) are any and dependent on each other or not, and transforming it into a standard normal distribution with independent r.v..

3 Results

The design variables evaluated in the present work have their statistical data presented in Table 1, where they are obtained from ISO TR 10400 [5]:

Table 1. Statistical and deterministic parameters of design variables

Design Variables	Mean	COV	Distribution	Unit
Ultimate Tensile Strength (f_u)	985.95	0.04320	Gaussian	MPa
Yield Strength (f_y)	925.97	0.00893	Gaussian	MPa
Outer Diameter (D)	0.3766	0.00181	Gaussian	m
Minimum Wall Thickness (t_{min})	0.0194	0.02130	Gaussian	m
Uncertainty of KS (mu_{KS})	1.01	0.03564	Gaussian	-

The correlations between the design variables were determined and the correlation matrix was subsequently defined to analyze the probability of failure. In this case, the scenario with extreme correlation values are analyzed (between all evaluated); this scenario was produced using a real sample of casing tubes manufactured in industry.

Table 2. Extreme correlation coefficients between the pairs of design variables

Design Variables	f_u	f_y	D	t_{min}	mu_{KS}
Ultimate Tensile Strength (f_u)	1	0.8	0	0	0
Yield Strength (f_y)	0.8	1	0	0	0
Outer Diameter (D)	0	0	1	0.6	0
Minimum Wall Thickness (t_{min})	0	0	0.6	1	0
Uncertainty of KS (mu_{KS})	0	0	0	0	1

Applying the structural reliability methods (FORM and Monte Carlo) and with the correlation matrix, the probability of failure with the correlated design variables using Klever-Stewart model was determined as well as considering the independent variables. In order to maintain the coefficient of variation (error) at a maximum of 1%, it is defined a Monte Carlo simulations range between 3560000 to 4200000. The probability of failure is contained in the Table 3:

Table 3. Structural reliability results with correlation x independent design variables

Reliability Method	Design Variables	Probability of failure (P_f)	Variation (%)
FORM	Correlated	$10^{-2.383}$	39.4
	Independent	$10^{-2.165}$	
Monte Carlo	Correlated	$10^{-2.269}$	22.8
	Independent	$10^{-2.156}$	

According to the results obtained, it is observed that the use of the correlation between the design variables reduced the probability of failure, which is detrimental to safety and consistent with API TR 5C3 [1] recommendation. Thus, the use of data with independent random variables (r.v.) in the reliability analysis of well casing tubes contributes to a better casing design, where the overall cost of the project may be reduced.

Regarding the accuracy of reliability methods, Monte Carlo simulation is more accurate with a large number of scenarios (smaller error), but it causes a considerable increase in the computational cost. As such, given the shorter simulation time and good result consistency, FORM is a better choice.

Finally, analyzing the probability of failure variation (Table 03), it is noted that FORM with independent variables was 39.4% higher than considering the correlations, as in the case of Monte Carlo simulation (22.8%). Moreover, it is noted that when comparing the probability of failure between FORM and Monte Carlo with correlated variables, there is a difference of 23.12%, but when the same analysis is performed for independent variables, the difference is 1.95%. However, the use of correlated design variables results in a more real analysis and probability of failure of the problem, reducing the design cost and the robustness of the dimensions and mechanical characteristics of structural elements given the lower probability of failure (at least 22.8% in this case).

4 Conclusions

API TR 5C3 [1] consider conservative to omit the effect of variable correlation and treat the input variables as independent. From the results presented above, it is noted that there is an increase in probability of failure when the design variables were considered independent, by the Monte Carlo simulation as well as the FORM transformation method. In addition, it is noted that when the correlation between design variables increases, the probability of failure for the Klever-Stewart [2] burst resistance model decreases. Therefore, API TR 5C3 [1] shows an appropriate recommendation for structural reliability analyses, because it is in favor of safety. However, the use of correlated design variables results in a more real analysis and probability of failure, reducing the probability of failure in 22.8% (according to results), causing the reduction of project cost due to the need for less sophisticated structural elements. Finally, choosing a solution that is safe and economically viable is at the discretion of the designer, where both independent and correlated design variables can be used in the analysis.

Acknowledgements

To LCCV/UFAL for the technical support provided and to CENPES/PETROBRAS for their technical and financial support.

References

[1] API TR 5C3. Technical Report on Equations and Calculations for Casing, Tubing, and Line Pipe Used as Casing or Tubing; and Performance Properties Tables for Casing and Tubing. 1st. ed. Washington, D.C, 2008.

[2] F. Klever and G. Stewart, 1998. Analytical burst strength prediction of octg with and without defects. In: Proceedings of SPE Applied Technology Workshop on Risk Based Design of Well Casing and Tubing. The Woodlands, Texas: Society of Petroleum Engineers. ISBN 9781555633868.

[3] A. T. Beck. *Curso de confiabilidade estrutural: notas de aula*. Escola de Engenharia de São Carlos – Universidade de São Paulo, 2011.

[4] L. P. Gouveia. *Avaliação da Confiabilidade em Tubos de Revestimento de Poços de Petróleo*. Masters dissertation, Centro de Tecnologia - Universidade Federal de Alagoas, 2014.

[5] ISO TR 10400. Petroleum, petrochemical and natural gas industries - Equations and calculations for the properties of casing, tubing, drill pipe and line pipe used as casing or tubing. 1st. ed. Washington, D.C, 2007.