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AN ALGEBRAIC DYNAMIC MULTILEVEL (ADM) METHOD FOR THE SIMULATION OF TWO-PHASE FLOWS THROUGH HIGHLY HETEROGENEOUS PETROLEUM RESERVOIRS

João Paulo Rodrigues de Andrade
Darlan Karlo Elisiário de Carvalho

jprandrade2@gmail.com
dkarlo101@gmail.com
Federal University of Pernambuco UFPE
Av. Prof. Moraes Rego, 1235 - Cidade Universitária, Recife, 50670-901, Pernambuco, Brasil

Abstract. Nowadays, large reservoir geocellular static models may size up to 10^6 - 10^9 cells. In practice, this might turn impossible the dynamic multiphase flow simulation, even using the most powerful High-Performance Computing techniques. To overcome this problem, industry standards to apply some kind of homogenization (upscaling) technique, in order to reduce the original fine scale problem into a coarser one, containing less degrees of freedom. Despite the gains in CPU performance, these homogenization methods imply in scale information losses. An alternative approach is to use multiscale/multilevel methods, such as the Multiscale Finite Volume Method (MsFVM). In the MsFVM, nested solutions are calculated in coarser scales then projected back to the fine scale, by using restriction and prolongation operators. These operators are constructed using auxiliary primal and dual coarser grids. Recent developments in multiscale techniques led to the Algebraic Dynamic Multilevel (ADM) method. This methodology allows for the use of more than one auxiliary grid levels in a fully automatic adaptive scale approach. In this paper, we use the classical IMPES (Implicit Pressure Explicit Saturation) strategy for the modeling of oil-water flows on highly heterogeneous porous media using the ADM methodology. The ADM solver is used for the implicit solution of the elliptic pressure equation and is constructed using the Two-Point Flux Approximation scheme (TPFA). The explicit solution of the hyperbolic saturation equation is calculated using a First Order Upwind Method (FOUM). As a preprocessing stage, a pressure error estimator is performed to define regions to keep the mesh more refined, such as wells and geological features (channels, barriers, etc.), to define a Static Multilevel Grid (SMG). For the simulation of multiphase flows an algorithm that detects high gradients in the saturation solution (e.g., the saturation front) is used, in order to define the Dynamic Multilevel Grid (DMG), nested to the SMG. In order to evaluate our methodology, we have solved some benchmark problems found in literature. Our results, are very promising, as we are able to obtain solutions almost as accurate as using the fine grid solution, with a significant reduction in computational efforts.

Keywords: Multiscale technique, Reservoir Simulation, Numerical Methods, Heterogeneous porous media, Adaptative Multilevel Method

1 Introduction

Numerical simulation is one of the main tools used in reservoir engineering for the production and management of petroleum engineering projects. These tools enable the modeling of multiphase flow behavior which is of great importance for field development under optimal conditions. According to Zhou [1], due to the large number of information used in dynamic simulations, even with the technological advances, some problems still can not be evaluated using geocellular meshes (even with use of HPC), making essential the use of methods of scale transfer. In general, upscaling techniques are applied in order to reduce the size of the problems and make the simulation feasible on a coarser grid. Although this method reduces the computational costs, there is always the loss of accuracy, due to the homogenization of reservoir properties.

A different approach consists in applying multiscale techniques, in order to mitigate the accuracy loss inherent from upscaling strategies. The MsFVM, which allows a locally conservative fine scale solution to be obtained, is of particular interest for reservoir simulations. With better precision than Upscaling methods and with a lower computational cost, this methodology obtains a coarse solution that is projected directly on the fine scale through defined operators called restriction (responsible for performing the upscaling operations) and prolongation (responsible for performing the downscaling operations).

Different from traditional Multiscale techniques, the Algebraic Dynamic Multilevel method allows to use more than one level of coarsening, by means of nested meshes operations through all mesh levels. The Multilevel Method presented in this work is based on the works of Cusini et al. [2], Santos et al. [3] and Wang et al. [4], which describe an algebraic method to obtain the prolongation operator, where two meshes need to be constructed: one referred to as the primal coarse mesh and the other as the dual coarse mesh.

Due to the easiness of implementation, in this paper we use the classical IMPES methodology for modeling oil-water flows on highly heterogeneous porous media using the ADM methodology. The ADM solver is used for the elliptic pressure equation and is constructed using the Two-Point Flux Approximation scheme (TPFA). The explicit solution of the hyperbolic saturation equation, is calculated using a First Order Upwind Method (FOUM).

2 Mathematical Model

We assume the immiscible and isothermal two-phase fluid flow of oil and water. We also neglect the effects of the capillary pressure and gravity. The equations describing the movement of fluids in porous media are obtained from the Mass Conservation Law (Eq. (1)) and the Darcy's Law

$$\frac{\partial \phi \rho_\alpha S_\alpha}{\partial t} = -\nabla \cdot (\rho_\alpha \vec{v}_\alpha) + Q_\alpha \tag{1}$$

$$\vec{v}_\alpha = -\lambda_\alpha \underset{\sim}{K} \nabla p_\alpha, \tag{2}$$

where S_α, ρ_α, $\vec{v}_\alpha$, Q_α, $\lambda_\alpha = {kr_\alpha}/{\mu_\alpha}$, p_α, kr_α and μ_α are, respectively: the saturation, density, velocity, source or sink terms (injection or production wells), mobility, relative permeability, and viscosity of each phase α. $\underset{\sim}{K}$ is the absolute permeability tensor and ϕ is the porosity, defined as the ratio of void volume to total rock volume. After the manipulation of the equations above, the following system of equations is obtained:

$$\begin{cases} \nabla \cdot \vec{v}_T = q_T \\ \phi \dfrac{\partial S_w}{\partial t} = -\nabla \cdot f_w \vec{v}_T + q_w, \end{cases} \tag{3}$$

where $q_\alpha = {Q_\alpha}/{\rho_\alpha}$, with $\alpha =$ oil (o) or water (w), $q_T = q_w + q_o$, $f_w = {\lambda_w}/{\lambda_T}$, $\lambda_T = \lambda_w + \lambda_o$, $p = p_w = p_o$. $\vec{v}_T = \vec{v}_w + \vec{v}_o = \lambda_T \underset{\sim}{K} \nabla p$, λ_T and q_T are, respectively: the total velocity, the total mobility and the total specific flow rate. From this a linear system is obtained for the first equation of

Eq. (3), as:

$$T^f p^f = q^f \tag{4}$$

where T^f is the transmissibility matrix, p^f is the pressure vector and q^f is the source term of the fine mesh. The system of equations, Eq. (3), is solved following the IMPES strategy which first solves the elliptic pressure equation assuming an initial saturation distribution, and after calculating velocities, the hyperbolic saturation equation is solved for the next time step, in what follows we will concentrate on presenting the pressure solver using the ADM Method.

3 Multiscale Method

In order to handle highly heterogeneous media, we use an operator based algebraic variant of the Multiscale Finite Volume Method (MsFVM). In the OBM (Operator Based Multiscale)-MsFVM according to Zhou [1], it is necessary to construct two operators: the prolongation (P_p) and the restriction (R_p) operators, in order to obtain the coarse scale equations from the fine scale equations by simple algebraic operations. Then, the coarse scale variables are calculated and it is then possible to reconstruct the fine scale solution.

The prolongation operator is obtained from the base functions ϕ_i^I (ϕ_i^I is the influence of the coarse volume I on the fine volume i), calculated locally in each volume of the dual coarse mesh. The base functions are given according to Barbosa et al. [5]:

$$-\nabla \cdot (\lambda \underset{\sim}{K} \cdot \nabla \phi_i^I) = 0 \text{ in } \Omega_d^c, \tag{5}$$

$$\frac{\partial}{\partial \tau}\left(\lambda \underset{\sim}{K} \frac{\partial \phi_i^I}{\partial \tau}\right) = 0 \text{ in } \partial\Omega_d^c, \text{ with } \phi_i^I(x^p) = \delta_{Ip} \tag{6}$$

where Ω_d^c are the volumes of the dual coarse mesh, $\partial\Omega_d^c$ is the contour of volume Ω_d^c, x^p are the vertices of dual coarse mesh, δ_{Ip} is the Kroenecker delta applied in x^p and τ is the tangential direction at $\partial\Omega_d^c$.

In this work, the AMS method was used to obtain P_p where the volumes are classified into: internal, face, edge and vertex, into the dual coarse mesh. Once the transmissibility matrix (T) is defined, the volumes are reordered following the wirebasket pattern, according to Wang et al. [4], where is explained in details how to get the AMS prolongation operator. In the MsFVM, the restriction operator (R_p) only maps which fine scale volumes are contained in the coarse volumes.

Thus the pressure of the fine mesh p^f is approximated by the multiscale pressure p^{ms} where $p^{ms} = P_p p^c$, being p^c the pressure of coarse scale. Exchanging p^{ms} for p^f and applying the restricion operator on both sides of Eq. (4), we get the system of equations: $T^c p^c = q^c$, where $T^c = R_p T^f P_p$ and $q^c = R_p q^f$ are the transmissibility and source term of coarse scale. According to Barbosa et al. [5], in order to recover a conservative flux on the fine scale we solve the following "Neumann" problem:

$$\begin{cases} (\underset{\sim}{K}\lambda\nabla p'') \cdot \vec{n} = (\underset{\sim}{K}\lambda\nabla p^{ms}) \text{ in } \partial\Omega_i^c \\ \nabla \cdot (\underset{\sim}{K}\lambda\nabla p'') = q \text{ in } \Omega_i^c \end{cases} \tag{7}$$

where $\partial\Omega_i^c$ is the boundary of Ω_i^c, p'' is the locally computed pressure and $\vec{n}$ is the normal vector that points outside the boundary of Ω_i. For further details see Barbosa et al. [5]

4 Multilevel Method

The Multilevel Method allows to obtain the pressure approximation from several levels of coarsening, leaving in the scale of better resolution the regions that present some important physics, as in regions near the wells that present high pressure gradients. The level 0 is the fine mesh.

Consider the sets: Π^l: set of all volumes of the level l; Ω^l: set of volumes of the ADM mesh of the level l; Γ^l: $\Omega^l \bigcap \Pi^l$; $\hat{\Pi}^l$: set of the volumes wich are vertex of the dual mesh of the level l. The ADM

operators are constructed as follows:

$$(\hat{P}_p)^l_{l-1}(i,j) = \begin{cases} (P_p)^l_{l-1}(i,j) \text{ if } i \in \Gamma^{l-1} \text{ and } j \in \hat{\Pi}^l \\ \delta_{ij} \text{ otherwise} \end{cases} \tag{8}$$

$$(\hat{R}_p)^{l-1}_l(i,j) = \begin{cases} (R_p)^{l-1}_l(i,j) \text{ if } i \in \Gamma^l \text{ and } j \in \Gamma^{l-1} \\ \delta_{ij} \text{ otherwise} \end{cases} \tag{9}$$

where $\hat{P}_p$ and $\hat{R}_p$ are the ADM operators, according to Cusini et al. [2]. To approximate the ADM solution p^{ADM}, the operators are applied successively in order of the levels as follows in:

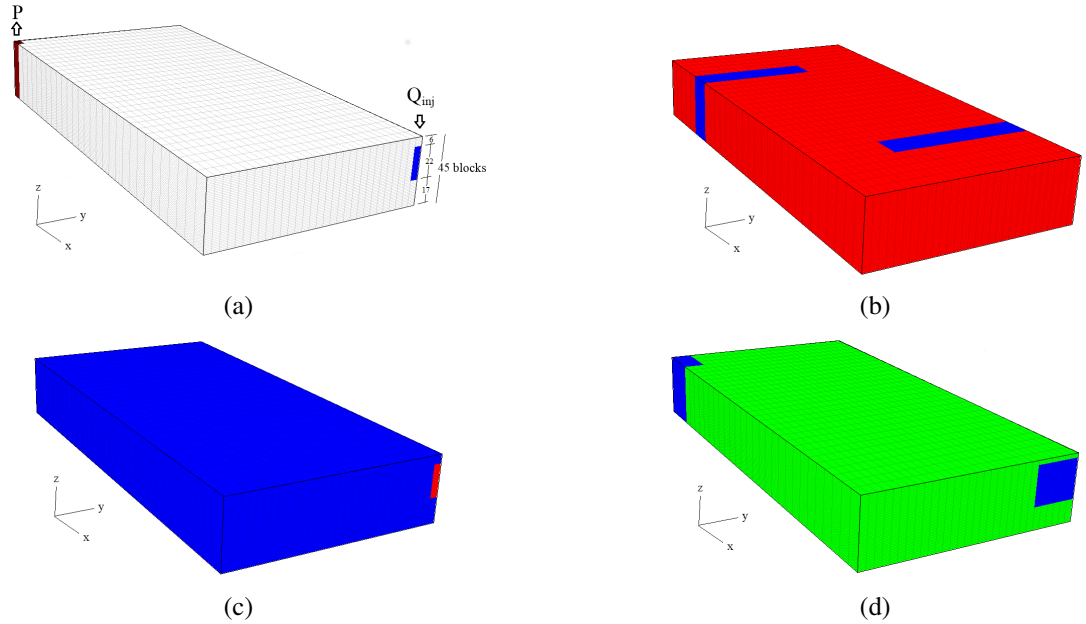
$$(\hat{R}_p)^{l-1}_l \dots (\hat{R}_p)^f_1 T^f (\hat{P}_p)^1_f \dots (\hat{P}_p)^l_{l-1} p^{ADM} = (\hat{R}_p)^{l-1}_l \dots (\hat{R}_p)^f_1 q^f \tag{10}$$

The Static Multilevel Grid (SMG) is obtained in the pre-processing stage and it is defined as the mesh that allows the minimum amount of volumes in the fine level for a given tolerance. These regions, according to Santos et al. [3] are identified by a pressure error estimator that takes into account an approximation of the fine scale pressure solution obtained from Eq. (4) towards an iterative method, using as initial approximation the fine scale ADM pressure $(\hat{P}_p)^1_f \dots (\hat{P}_p)^l_{l-1} p^{ADM}$.

The saturation front region must remain on the fine scale, due to abrupt saturation variations. The method used to identify these regions is to check a maximum saturation variation ($\Delta S = S_{max} - S_{min}$), as in Cusini et al. [2], in each coarse volume of level 1 (Π^1). If ΔS is greater than a previously defined value, the volume is held at level 0 and his neighbors at level 1.

5 Results

The reservoir consists of 30x30x45 volumes in cartesian directions of the fine mesh, each volume with 6.08x3.04x0.61 m. The value of 0.1 was used as the refinement limit due to the saturation field (ΔS) and also for the pressure error estimator in pre-processing stage. For the residual oil and water saturations we used $S_{or} = S_{wr} = 0.2$, and as mobility ratio $\mu_o/\mu_w = 5$. The densities used for water and oil were $\rho_w = 1{,}000\ Kg/m^3$ and $\rho_o = 900\ Kg/m^3$ and null flux at the external boundaries.



(a) (b)

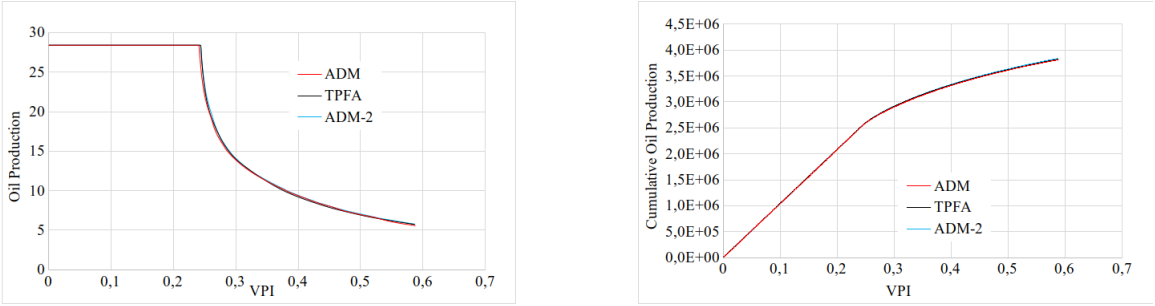
(c) (d)

Figure 1. a) Wells Configuration. Blue (injector): $Q_{inj} = 28.4m^3/d$. Red (productor): $P = 27.6MPa$. b) Permeability field. Blue: 0.001 md. Red: 1 md. c) Initial Saturation. Red: $S_w = 1$. Blue: $S_w = 0.2$. d) Static mesh. Blue: level 0, Green: level 1.

Two barriers are inserted inside the reservoir positioned in regions near the wells (Fig. 1b), with

configuration shown in Fig. 1a. Initial saturation values are 1 in the injector well and 0.2 in the rest of the reservoir (Fig. 1c). The permeability is 1 md in the reservoir and 0.001 md whithin the barriers. The mesh adaptation algorithm identified that the biggest errors were in the level 2 volumes, thus, only 2 levels remained, as shown in Fig. 1d (blue region is the fine scale mesh (level 0) and green at level 1).

In Fig. 2 we present the results obtained from the simulation, in terms of oil production and cumulative oil production, where TPFA (black) is obtained using the fine scale solution, ADM (red) is the ADM method and ADM-2 (blue) is the ADM method without updating the prolongation operators.



(a) (b)

Figure 2. a) Oil Production (m^3/d). b) Cumulative Oil Production (m^3).

6 Conclusions

In this simulation the ADM method was able to approximate the fine scale solution, even without updating the operators, which further improves the computational performance.

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